The University of the State of New York

294TH HIGH SCHOOL EXAMINATION

INTERMEDIATE ALGEBRA

Wednesday, June 20, 1945 — 9.15 a. m. to 12.15 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II, III and IV (a) name of school where you have studied, (b) number of weeks and recitations a week in intermediate algebra.

The minimum time requirement is five recitations a week for half a school year after the completion of elementary algebra.

Part II

Answer three questions from part II.

26a On the same set of axes, draw the graphs of \( x^2 + y^2 = 25 \) and \( y = x^2 - 1 \) [2, 6]

b From the graphs made in answer to a, determine to the nearest tenth the real values of \( x \) and \( y \) that satisfy both equations. [2]

27 Find, correct to the nearest tenth, the roots of the equation \( 3z^2 - z - 1 = 0 \) [10]

28 Solve the following system of equations: [10]

\[
\begin{align*}
2x^2 - y^2 + 8 &= 0 \\
3x - y - 2 &= 0
\end{align*}
\]

29 The horsepower of a ship is computed from the formula \( H = \frac{V^3 \sqrt{D^2}}{K} \). Using logarithms, find \( H \) when \( V = 32, D = 45,000 \) and \( K = 297 \) [10]

*30 Solve and check the following system of equations: [8, 2]

\[
\begin{align*}
x + y + z &= 5 \\
2x + y &= 6 \\
z - 3y &= 9
\end{align*}
\]

*This question is based on one of the optional topics in the syllabus.
Part III

Answer one question from part III.

31 Write the equations that would be used in solving the following problems. In each case state what the unknown letter or letters represent. [Solution of the equations is not required.]

   a The difference between two numbers is 24. If the larger number is divided by the smaller, the quotient is 4 and the remainder is 3. Find the numbers. [5]

   b A ship sails due east from a certain point at a speed of 16 mph. Two hours later a ship leaves the same point and sails due north at a speed of 20 mph. In how many hours after the first ship starts, will the two ships be 100 miles apart? [5]

32 A man has $5000 invested in a mortgage that pays 5% annually. He buys Series G War Bonds paying 2 1/2%, and now his total investment pays him 3% annually. How much has he invested in Series G War Bonds? [10]

Part IV

Answer one question from part IV.

33 For each of the following statements, indicate whether you have been given (1) too little information, (2) just enough information, or (3) more information than is necessary, to justify the conclusion.

   a If a circle whose center is at the origin passes through the point whose coordinates are (3, 4), then the equation of the circle can be determined. [2]

   b If the first term and the number of terms of an arithmetic progression are known, then the sum of the terms of the progression can be determined. [2]

   c If the slope and the y intercept of a line are known, then the angle which the line makes with the x axis can be determined. [2]

   d If the discriminant of a quadratic equation is positive, then the roots are real, unequal and rational. [2]

   e If the mantissa of the logarithm of a number is known, then the number can be determined. [2]

34 Write the letters a to e inclusive; then indicate the correct answer to each part below by writing opposite the corresponding letter the number (1), (2) or (3).

   a The decimal 0.00027 when written as 2.7 multiplied by a power of 10 is  
       (1) $2.7 \times 10^{-4}$  
       (2) $2.7 \times 10^4$  
       (3) $2.7 \times 10^{-4}$  [2]

   b If $3^x = x$, then $3^{x+1}$ equals  
       (1) $x + 3$  
       (2) $3x$  
       (3) $x + 1$  [2]

   c If $\log \frac{x}{2} = y$, then  
       (1) $x = 2y$  
       (2) $\log x = \log y + \log 2$  
       (3) $\log x = y + \log 2$  [2]

   d If $\sqrt[x]{y} = 16$, then one value of $\sqrt[x]{y}$ is  
       (1) 4  
       (2) 256  
       (3) 32  [2]

   e The maximum number of points in which the graph of $x^2 + y^2 = r^2$ can intersect the graph of $xy = k$ is  
       (1) 4  
       (2) 2  
       (3) 3  [2]
Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

1. Find the logarithm of 104.6
2. If \( \log x = 1.7755 \), find \( x \) correct to the nearest hundredth.
3. Express \( \log \frac{a}{b} \) in terms of \( \log a \) and \( \log b \).
4. Factor \( x^2 - x - 12 \)
5. Form the equation whose roots are 3 and \(-5\).
6. Find the positive root of the equation \( x^2 - 5x - 36 = 0 \)
7. Solve the following system of equations:
   \[
   \begin{align*}
   3x - y &= 22 \\
   x - y &= 10
   \end{align*}
   \]
8. Express \( 3\sqrt{-36} \) in terms of the imaginary unit \( i \).
9. Given \( \sqrt{x - a} = b \); express \( x \) in terms of \( a \) and \( b \).
10. Express \( \frac{3}{\sqrt{5} - \sqrt{3}} \) as an equivalent fraction with a rational denominator.
11. Find the value of \( x \) that satisfies the equation \( \sqrt[3]{x} = -3 \)
12. The second and fourth terms of an arithmetic progression are 12 and 4. Find the third term.
13. Find the 17th term of the progression 3, 7, 11, \ldots
14. Write the \( n \)th term of the geometric progression, the first two terms of which are 1 and 5.
15. Find the value of \( 2 \times 8^3 \)
16. Write the linear equation expressing the relation between \( x \) and \( y \) shown in the table:
   \[
   \begin{array}{c|cccc}
   x & -1 & 0 & 3 & 6 \\
   y & -1 & 1 & 7 & 13
   \end{array}
   \]
17. What is the name of the curve which is the graph of \( 4x^2 - 9y^2 = 36 \)?
18. Does the point \((2, -3)\) lie on the graph of \( y^2 = 3 + xy \)? [Answer yes or no.]
19. Write the first two terms of the expansion \((x - 5)^8\)
20. A plane takes off from a field and rises at an angle of 12° with the horizontal. Find, correct to the nearest foot, the height of the plane after it has traveled a horizontal distance of 2000 ft.
21. If \( x \) varies directly as \( y \) and if \( x = 4 \) when \( y = 7 \), find the value of \( x \) when \( y = 17_\frac{1}{2} \)
Directions (questions 22–25) — Indicate the correct answer to each question by writing on the line at the right the letter a, b or c.

22 The value of $2x^0$ is (a) 0, (b) 1, (c) 2

23 If one quantity varies directly as another, (a) their product is constant, (b) their sum is constant, (c) their ratio is constant

24 Which of the following has one real value? (a) $\sqrt[2]{-4}$, (b) $\sqrt[3]{-8}$, (c) $\sqrt[4]{-16}$

25 The sum of the roots of the equation $2x^2 - 7x - 8 = 0$ is (a) $-7$, (b) $\frac{-7}{2}$, (c) $\frac{3}{2}$