The University of the State of New York
278th High School Examination
INTERMEDIATE ALGEBRA
Wednesday, June 19, 1940 — 9.15 a. m. to 12.15 p. m., only

Instructions

Do not open this sheet until the signal is given.

Group I

This group is to be done first and the maximum time allowed for it is one and one half hours. Merely write the answer to each question in the space at the right; no work need be shown.

If you finish group I before the signal to stop is given you may begin group II. However, it is advisable to look your work over carefully before proceeding, since no credit will be given any answer in group I which is not correct and in its simplest form.

When the signal to stop is given at the close of the one and one half hour period, work on group I must cease and this sheet of the question paper must be detached. The sheets will then be collected and you should continue with the remainder of the examination.

Groups II, III and IV

Write at top of first page of answer paper to groups II, III and IV (a) name of school where you have studied, (b) number of weeks and recitations a week in intermediate algebra.

The minimum time requirement is five recitations a week for half a school year after the completion of elementary algebra.

The use of the slide rule will be allowed for checking but all computations with tables must be shown on the answer paper.
Fill in the following lines:

Name of school..........................Name of pupil..........................

Detach this sheet and hand it in at the close of the one and one half hour period.

Group I

Answer all questions in this group. Each correct answer will receive 2 credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

1. Express the fraction \( \frac{2}{\sqrt{3}} \) with a rational denominator.
2. Find the product of \((\sqrt{5} - 2)\) and \((\sqrt{5} + 2)\).
3. Write the slope of the line whose equation is \( y = -2x + 3 \).
4. Express \( \sqrt{-16} \) in terms of \( i \).
5. The formula for the sum \( S \) of an infinite geometric series, whose first term is \( a \) and whose common ratio \( r \) is less than 1, is \( S = \ldots \).
6. Factor \( 3x^2 - 10x + 3 \).
7. Solve for \( \epsilon \) the formula \( P = \frac{c - d}{a} \).
8. Write the product of the roots of the equation \( 2x^2 + px + 3 = 0 \).
9. Find the logarithm of .06386.
10. Find, correct to the nearest tenth, the number whose logarithm is 2.9358.
11. In the triangle \( ABC \), angle \( C = 90^\circ \), angle \( A = 37^\circ \) and \( AC = 10 \). Find the length of \( BC \), correct to the nearest tenth.
12. Insert two geometric means between 5 and 135.
13. If \( x = \frac{4 - y}{y} \), does \( x \) increase or decrease as \( y \) increases from +1 to +4?
14. Find the 15th term of the series 2, 5, 8, \ldots
15. Solve for \( x \): \( \sqrt{2x - 3} = 4 \).
16. Simplify: \( -\frac{1}{2a} - \frac{a}{2} \div (\frac{1}{a} + 1) \).
17. Write the product of \( a^{2n} \) and \( a \).
18. Write the first three terms in the expansion of \((x - 2)^6 \).
19. Find the value of \( 4 \times (\frac{1}{2})^0 + 2^{-1} \).
20. Write the equation expressing the relation between \( x \) and \( y \) as shown in the table at the right.

<table>
<thead>
<tr>
<th>( x )</th>
<th>-1</th>
<th>1</th>
<th>3</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>

21. Write in the form \( x^2 + px = 0 \) the equation whose roots are -3 and 0. 

[3]
Directions (questions 22–25) — Indicate the correct answer to each question by writing on the dotted line at the right the letter a, b or c.

22 If the discriminant of a quadratic equation is 17, the roots are (a) real, rational and unequal, (b) real, rational and equal or (c) real, irrational and unequal.

23 The graph of the equation $xy = 12$ is (a) a circle, (b) an ellipse or (c) a hyperbola.

24 The fraction $\frac{4 + \sqrt{8}}{2}$ is equal to (a) $2 + \sqrt{2}$, (b) $2 + \sqrt{8}$ or (c) $3 \sqrt{2}$.

25 Log $a^3$ is equal to (a) log $3a$, (b) $\frac{1}{3}$ log $a$ or (c) 3 log $a$. 
See instructions for groups II, III and IV on page 1.

Group II

Answer three questions from this group.

26 Find, correct to the nearest tenth, the roots of the equation $3x^2 - 4x - 5 = 0$ [10]

27 Solve the following set of equations, group your answers and check one set:
   \[5x^2 - 3xy = 14\]
   \[y = 4x - 7\] [7, 1, 2]

28 Using logarithms, find, correct to the nearest thousandth, the value of
   \[\sqrt[3]{3.14 \times \sin 41^\circ \div 79.3}\] [10]

29 a Using the same set of axes, draw the graphs of the equations
   \[x^2 + y^2 = 16\] and \[y = x^2\] [3, 5]

b From the graphs made in answer to a, estimate, correct to the nearest tenth, two solutions common to the equations. [2]

30 Given the equation $ax^2 + bx + c = 0$; derive the formula for the roots of this equation in terms of $a$, $b$ and $c$. [10]

*31 Solve the following set of equations:
   \[2x - 5y + 6z = 11\]
   \[3x - 2y + 3z = 9\]
   \[2x + 4y - 9z = -3\] [10]

Group III

Answer one question from this group.

32 When a certain number consisting of two digits is multiplied by the sum of its digits, the product is 63. If the tens digit is twice the units digit, what is the number? [7, 3]

33 Write the equations that would be used in solving the following problems. In each case state what the unknown letter or letters represent. [Solution of the equations is not required.]
   a Find the dimensions of a rectangle if its perimeter is 56 inches and its diagonal is 20 inches. [5]

b Three numbers are in the ratio 2 : 5 : 7. If 7 is subtracted from the second number, the resulting numbers will be in arithmetic progression. Find the three numbers. [5]

Group IV

Answer one question from this group.

34 Each of the following statements is sometimes true and sometimes false. In each case give one illustration in which it is true and one illustration in which it is false.
   a The positive square root of a number is less than the number. [2]

b The graphs of two equations of the first degree intersect in one point. [2]

c If \(a, b\) and \(c\) are each greater than 1, the graph of the equation \(ax^2 + by^2 = c\) is a circle. [2]

d A root of a negative number is an imaginary number. [2]

e If \(y\) is a function of \(x\), \(y\) increases as \(x\) increases from 0. [2]

35 Two points move at different but constant rates along a circle whose circumference is 150 feet. Starting at the same time and from the same point, when they move in opposite directions they coincide every 5 seconds; when they move in the same direction they coincide every 25 seconds. Find their rates in feet per second. [10]

* This question is based on one of the optional topics in the syllabus. [2]