The University of the State of New York
275th High School Examination
INTERMEDIATE ALGEBRA
Wednesday, June 21, 1939 — 9.15 a. m. to 12.15 p. m., only

Instructions

Do not open this sheet until the signal is given.

Group I

This group is to be done first and the maximum time allowed for it is one and one half hours. Merely write the answer to each question in the space at the right; no work need be shown.

If you finish group I before the signal to stop is given you may begin group II. However, it is advisable to look your work over carefully before proceeding, since no credit will be given any answer in group I which is not correct and in its simplest form.

When the signal to stop is given at the close of the one and one half hour period, work on group I must cease and this sheet of the question paper must be detached. The sheets will then be collected and you should continue with the remainder of the examination.

Groups II, III and IV

Write at top of first page of answer paper to groups II, III and IV (a) name of school where you have studied, (b) number of weeks and recitations a week in intermediate algebra.

The minimum time requirement is five recitations a week for half a school year after the completion of elementary algebra.

The use of the slide rule will be allowed for checking but all computations with tables must be shown on the answer paper.
Intermediate Algebra

Fill in the following lines:

Name of school........................................Name of pupil........................................

Group I

Detach this sheet and hand it in at the close of the one and one half hour period.

Answer all questions in this group. Each correct answer will receive 2 credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

Directions (questions 1–17) — Write on the dotted line at the right of each question the expression which when inserted in the corresponding blank will make the statement true.

1. The slope of the line whose equation is \( y = 2x - 3 \) is ....
2. The graph of \( y = 2x - 4 \) crosses the \( x \) axis at the point whose abscissa is ....
3. The sum of the roots of the equation \( x^2 - 2x + q = 0 \) is ....
4. \( a^m \div a^n = \ldots \)
5. The value of \( x \) which satisfies the equation \( \sqrt{x + 5} = 3 \) is ....
6. The factors of \( 3x^2 + x - 2 \) are ....
7. The fraction \( \frac{1}{\sqrt{5} - 1} \) expressed with a rational denominator is ....
8. Expressed in terms of \( i \), \( \sqrt{-2} \) is ....
9. The sum of the fractions \( \frac{a}{b} \) and \( \frac{b}{a} \) is ....
10. The first two terms of the expansion \( (1 + a)^6 \) are ....
11. The positive geometric mean between 4 and 9 is ....
12. The formula for \( l \), the last term of a geometric series, in terms of the first term \( a \), the common ratio \( r \), and the number of terms \( n \), is \( l = \ldots \)
13. The formula \( R = \sqrt{\frac{S}{4\pi}} \) when solved for \( S \) is \( S = \ldots \)
14. The value of \( (8)^6 + (8)^{-\frac{1}{2}} \) is ....
15. The logarithm of 55.25 is ....
16. If the logarithm of a number is 1.9038, the number expressed to the nearest hundredth is ....
17. In triangle \( ABC \), angle \( C = 90^\circ \), angle \( A = 40^\circ \), \( AC = 20 \); the length of \( BC \), correct to the nearest tenth, is ....
18. Solve the following set of equations for \( x \):
   \[ x + 2y = 7 \]
   \[ 2x + y = 5 \]
19. Write in the form \( x^2 + px + q = 0 \), the equation whose roots are 2 and \(-5\).
20. If \( x \) is positive and \( y = 1 - \frac{1}{x} \), does \( y \) increase or decrease as \( x \) increases?

21. Write the equation which expresses the relation between \( x \) and \( y \) shown in the table at the right.

\[
\begin{array}{|c|c|c|c|c|}
\hline
x & 0 & 1 & 2 & 3 \\
\hline
y & 3 & 5 & 7 & 9 \\
\hline
\end{array}
\]

22. The graph of the equation \( 2x^2 + 2y^2 = 5 \) is (a) a circle, (b) an ellipse or (c) a hyperbola.

23. \( 10 \log a \) is equal to (a) \( \log 10a \), (b) \( \log (10 + a) \) or (c) \( \log a^{10} \).

24. If the discriminant of a quadratic equation is zero, the roots of the equation are (a) real and equal, (b) real, unequal and rational or (c) real, unequal and irrational.

25. Which of the following statements is an identity?

(a) \( a^2 - b^2 = 4 \)

(b) \( a^2 - b^2 = c^2 \)

(c) \( a^2 - b^2 = (a + b)(a - b) \)
26 Find, correct to the nearest tenth, the roots of the equation $x^2 - 4x - 3 = 0$.

27 Solve the following pair of equations and check one set of answers:
\[ x^2 + y^2 + 2x = 33 \]
\[ y = x - 1 \]
\[ [8, 2] \]

28 Using logarithms, find, correct to the nearest hundredth, the value of
\[ \sqrt[3]{\frac{324 \times 0.372}{21.5}} \]
\[ [10] \]

29 a Draw the graph of the equation $y = -x^2 + x + 6$ from $x = -3$ to $x = +4$ inclusive. [6]

b On the same set of axes used in answer to a, draw the graph of the equation
\[ y = 2x + 4 \] [2]

c From the graphs drawn in answer to a and b, determine the values of $x$ and $y$ common to the two equations. [2]

30 Derive the formula for $S$, the sum of an arithmetic series, in terms of the first term $a$, the number of terms $n$, and the last term $l$. [10]

*31 Solve for $x$ correct to the nearest tenth the equation $6^x = 72$ [10]

Group III

Answer one question from this group.

32 The hypotenuse of a right triangle is one inch longer than one leg and eight inches longer than the other. Find the three sides of the triangle and the angle which the hypotenuse makes with the longer leg. [10]

33 Write the equations that would be used in solving the following problems. In each case state what the unknown letter or letters represent. [Solution of the equations is not required.]

a A motorist traveled a distance of 265 miles. He drove at 40 miles per hour during the first part of his journey and at 35 miles per hour during the remaining part. He reached his destination after traveling 7 hours. For how many hours did he travel 40 miles per hour? [5]

b A painter can paint the side of a barn in 8 hours. His helper can do the same work in 12 hours. The painter begins the job and after one hour is joined by his helper. How many hours will it take them to finish the job? [5]

Group IV

Answer one question from this group.

34 a The base of a triangle is $b$ and its altitude is $h$. If the base of the triangle is increased by an amount $c$, find, in terms of $b$, $h$ and $c$, the amount $x$ by which the altitude must be decreased so that the area of the triangle shall remain the same. [7]

b From the relationship obtained in answer to a, show that if the base of a triangle is doubled, then the altitude must be halved in order that the area shall remain the same. [3]

35 A river runs along one side of a field. A plot is to be fenced off in the form of a rectangle, the river serving as one of its sides. The total amount of fencing to be used is 20 rods.

a Express the area $A$ of the plot in terms of its length $l$. [4]

b Make a graph of the equation obtained in answer to a and from this graph determine the dimensions of the plot so that its area may be a maximum. [6]

* This question is based on one of the optional topics in the syllabus. [2]