The University of the State of New York

236TH HIGH SCHOOL EXAMINATION

INTERMEDIATE ALGEBRA

Thursday, June 17, 1926 — 9.15 a. m. to 12.15 p. m., only

Write at top of first page of answer paper (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in (1) elementary algebra, (2) intermediate algebra.

The minimum time requirement is five recitations a week for half a school year, or the equivalent, after the completion of elementary algebra.

Answer eight questions. Full credit will not be granted unless all operations (except mental ones) necessary to find results are given; simply indicating the operations is not sufficient. Each answer should be reduced to its simplest form.

In the examination in intermediate algebra the use of the slide rule will be allowed for checking, provided all computations with tables are shown on the answer paper.

1 *a* Factor *each* of the following:

$5(a-b)^2 - 3(b-a)$; $1 - 36(x-y)^2$ [6]

b Without actual division show that $x+3$ is a factor of $x^3 - 7x + 6$. [3]

c Write as a fraction and reduce to lowest terms: The difference between the cubes of x and y divided by the cube of their difference. [1½, 2]

2 *a* The perimeter of an isosceles triangle is $\frac{2a+3}{a^2-4}$; one of the equal sides is $\frac{1}{8a+16}$. What is the length of the third side? [8½]

b Write *two* fractions whose quotient is $\frac{5a-5c}{2a-2y}$, the denominator of one of the fractions being 2. [4]

3 *a* Find the value of $x^{\frac{1}{3}} + 3x^{-\frac{1}{3}} - 2$ when $x = \frac{1}{8}$. [4½]

b Find the *third* term of the geometric progression $\sqrt[3]{x}, \sqrt{x}, \ldots$ [4]

c In the equation $5^{x-2} = 1$, what is the value of x? [4]

4 Using the formula $S = \frac{a}{1-r}$, when $a = 1 + \sqrt{5}$ and $r = 3 - \sqrt{5}$, find the value of S expressed (*a*) as a fraction with rational denominator, (*b*) as a decimal correct to the *nearest tenth.* [7½, 5]

5 *a* In a right triangle having one arm equal to a and the hypotenuse equal to c, the area K is given by the formula $K = \frac{1}{2}a\sqrt{(c-a)(c+a)}$; find K when $c = 9.73$ and $a = 5.47$. [6½]

b In the formula $A = P(1+r)^n$, find P when $A = 1000$, $r = 0.06$, $n = 10$ [6]

[Use logarithms in the solution of both *a* and *b*.]

6 *a* For what values of m will the roots of the equation $x^2 - (m-3)x + 2m - 9 = 0$ be equal? [6½]

b Determine the nature of the roots of $25x^2 + 75x - k = 0$ when k is a positive number. [6]

7 The arms of a right triangle are 5 feet and x feet.

a Express the hypotenuse, the perimeter and the area in terms of x. [3]

b If the number of linear feet in the perimeter equals the number of square feet in the area, find the numerical value of x. [3½, 6]

8 A man bought a lot for $280; he then sold it for $60 an acre, thereby gaining as much as 3½ acres of the lot had cost him. How many acres were there in the lot? [7½, 5]

9 A man saves $*a* the first year and increases by $*d* each year the amount saved the preceding year.

a How much will he save the *n*th year? [2½]

b Using the proper formula, find how many years it will take him to save $4500 if $*a* = $100 and $*d* = $50. [10]

10 *a* By substitution in the proper formula find the expression for the sum of the first 10 terms of the progression $1, 1.05, (1.05)^2, \ldots$ [6½]

b Using logarithms, find the value of the expression $\frac{(1.04)^{11}}{.04}$ [6]

11 *a* If x is a positive number, determine without solving whether *each* of the following statements is true or false: [Copy each statement and after it write the word true or false, giving a reason in each case.]

$\frac{50}{x+7} = -4$; $\sqrt[3]{2x+1} = -1$; $\frac{200}{x+2} - 5 = \frac{200}{x}$ [6]

b Given $y = mx + c$; if $y = -1$ when $x = 1$, and $y = 5$ when $x = 4$, find the values of m and c. [6½]

12 The dimensions of a rectangle are 5 and 3. The longer dimension is decreased by x and the shorter dimension is increased by x; the area of the new figure is represented by y.

a Express y in terms of x. [3]

b Graph the relation between y and x for integral values of x from -3 to 5 inclusive. [6½]

c From the graph determine x and y when the area is the largest possible. [3]