

INTERMEDIATE ALGEBRA

Monday, June 15, 1925 — 9.15 a. m. to 12.15 p. m., only

Write at top of first page of answer paper (a) name of school where you have studied, (b) number of weeks and recitations a week in (1) elementary algebra, (2) intermediate algebra.

The minimum time requirement is five recitations a week for half a school year, or the equivalent, after the completion of elementary algebra.

Answer eight questions. Full credit will not be granted unless all operations (except mental ones) necessary to find results are given; simply indicating the operations is not sufficient. Each answer should be reduced to its simplest form.

In the examination in intermediate algebra the use of the slide rule will be allowed for checking, provided all computations with tables are shown on the answer paper.

- 1 a Factor each of the following:

$$\begin{aligned} 3a^2 - 75a^6 \\ h^3 + h + 2 \\ r^4 + 125r \end{aligned} \quad [9]$$

- b The dimensions of a rectangle are $x + 8$ and $x - 7$. The area of a second rectangle exceeds the area of the first by 54. What are the dimensions of the second rectangle in terms of x ? $[3\frac{1}{2}]$

- 2 The numerator and the denominator of the fraction

$$\frac{2a - a^{-1} - 14a^{-2}}{a^{\frac{1}{2}} + 2a - 2a^{\frac{1}{2}} - 4}$$

have the common factor $a - 2$; reduce the fraction to lowest terms. $[12\frac{1}{2}]$

- 3 Simplify and express with rational denominator:

$$\frac{\sqrt{a-2x} + \frac{x}{\sqrt{a-2x}}}{a-x} \quad [12\frac{1}{2}]$$

- 4 a Find the value of each of the following:

$$\begin{aligned} (1) 9^{\frac{1}{2}}; (3) 7^{\frac{1}{2}} \times 7^{\frac{1}{2}}; (5) x^3 - 6x \text{ when } x = 3 - \sqrt{5}; \\ (2) 81^{-\frac{1}{2}}; (4) (a+b)^0 \end{aligned} \quad [1, 1, 1, 1, 4]$$

- b Rationalize the denominator of $\frac{\sqrt{7}-2}{\sqrt{7}+2}$ $[4\frac{1}{2}]$

- 5 Solve for
- x
- and
- y
- and check:

$$\begin{aligned} \frac{2}{x} - \sqrt{y} &= -\frac{5}{3} \\ \frac{3}{x} + 2\sqrt{y} &= 4\frac{1}{2} \end{aligned} \quad [10\frac{1}{2}, 2]$$

- 6 a Solve for
- x
- and
- y
- and group your answers:

$$\begin{aligned} x^2 + y^2 &= 25 \\ x + y^2 &= 5 \end{aligned} \quad [8\frac{1}{2}, 2]$$

- b On the graph of the equation $2x + 3y = 15$, indicate the point whose coordinates are equal. $[2]$

- 7 If a certain number is increased by $\frac{1}{2}$, the square root of this sum exceeds the square root of the original number by $\frac{1}{2}$; find the number. $[6, 6\frac{1}{2}]$

- 8 Two passengers together have 300 pounds of baggage and are charged 10 cents and 40 cents respectively for the excess above the weight allowed free. If the baggage had belonged to one of them, he would have been charged \$1. How much baggage is one passenger allowed without charge? $[8, 4\frac{1}{2}]$

- 9 A and B travel around a circular track in the same direction in 10 minutes and 6 minutes respectively. If they start together after how many minutes will B overtake A ? $[8, 4\frac{1}{2}]$

- 10 a The amount A of P dollars in n years when interest is compounded annually at the rate r is given by the formula $A = P(1+r)^n$. Find by using logarithms the interest earned by \$457 invested at 6% for 10 years if the interest is compounded annually. $[7]$

- b The area of an equilateral triangle is found from the formula $A = \frac{s^2\sqrt{3}}{4}$ where A = area and s = one side. Find by using logarithms the area of an equilateral triangle whose side is 0.523 inches. [Do not leave answer in radical form.] $[5\frac{1}{2}]$

- 11 a By using a formula find the value of the sum of the following geometric progression:

$$1 + x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7 + x^8 + x^9$$

when $x = \frac{1}{2}$ $[8\frac{1}{2}]$

- b Derive the formula used in answer to a. $[4]$

- 12 a Graph the equation

$$y = x^2 - 4x - 1 \text{ from } x = -2 \text{ to } x = 6 \quad [7\frac{1}{2}]$$

- b From the graph read the roots of the following equations: $x^2 - 4x - 1 = 0$; $x^2 - 4x - 1 = 4$ $[2, 1]$

- c Write an equation whose left member is $x^2 - 4x - 1$ and whose right member is such that the roots can not be found from the graph no matter how far this graph is extended. What is the nature of the roots of this equation? $[2]$