The University of the State of New York
215TH HIGH SCHOOL EXAMINATION
INTERMEDIATE ALGEBRA

Monday, June 19, 1916—9.15 a. m. to 12.15 p. m., only

Write at top of first page of answer paper (a) name of school where you have studied, (b) number of weeks and recitations a week in (1) elementary algebra, (2) intermediate algebra.
The minimum time requirement is four recitations a week for half a school year, after the completion of elementary algebra.

Answer eight questions, including the eleventh. Each answer should be reduced to its simplest form.

1 a Simplify \( \left( a - \frac{2b^2 - a^2 + ab}{2b} \right) \times \left( \frac{6a^2 + 5ab - b^2}{6a^2 + 11ab - 2b^2} \right) \div \left( \frac{a^3}{b} - ab \right) \)

b Factor \( 16a^4 + 36a^2b^2 + 81b^4 \)
\[ 64 - 64a^3 - a^6 + a^9 \]
\[ 64a^6 - b^6 \]

2 Solve \( \frac{5}{x} + \frac{1}{\sqrt{x}} - \frac{9}{3} = 0 \)

Compute the value of the roots to two decimal places.

3 Find the diameter of a circle circumscribed about a triangle whose sides are 5, 7 and 8, using the formula
\[ d = \frac{abc}{2\sqrt{s(s-a)(s-b)(s-c)}} \]
where \( d \) is the diameter, \( a, b \) and \( c \) represent the sides and \( s \) represents \( \frac{a+b+c}{2} \). Find the result correct to two decimal places.

4 Solve for all values of \( x \) and \( y \):
\[ x^2 + xy = 12 \]
\[ xy - y^2 = 2 \]

5 Solve for values of \( x \) and check the results:
\[ 2x^2 - 6x + 3 = 0 \] [Solution 4, checking 8]

6 By plotting the graphs of
\[ 3x - 2y = 6 \]
\[ x^2 - 2y = 10 \]
determine the values of \( x \) and \( y \).
7 \(a\) Solve for \(x\) and \(y\):
\[
\begin{align*}
x^{\frac{1}{2}} + 3y^{\frac{1}{2}} &= \frac{4}{5} \\
2x^{\frac{1}{2}} - y^{\frac{1}{2}} &= \frac{1}{5}
\end{align*}
\] [6]

\(b\) Simplify \((a^2 \sqrt{-3} + a^{-2} \sqrt{-2})^2\) [6]

8 A man made two investments totaling $710. On the first he received 5\(\frac{1}{2}\)% interest and on the second 4%. At the close of a year he received 10 cents more interest on the former investment than he did on the latter. What were the sums invested? [12]

9 Each member of a Christmas club deposits in a bank 2 cents the first week and increases the deposit each succeeding week 2 cents; what is the 50th deposit of each member and what is the total number of dollars deposited by \(n\) members after each has made the 50th deposit? [12]

10 \(a\) Derive the formula for the sum of a finite geometric series in terms of \(a, r\) and \(n\). [8]

\(b\) Find the number which 1.212121 \ldots approaches when the number of decimal places is increased without limit. [4]

11 \(a\) Without solving, determine the nature of the roots of the following equations:
\[
\begin{align*}
7x^2 + 15x + 2 &= 0 \\
9x - 7 - 3x^2 &= 0
\end{align*}
\] [4]

\(b\) Write the quadratic equation whose roots are
\[
\frac{1 + \sqrt{-3}}{-2} \quad \text{and} \quad \frac{1 - \sqrt{-3}}{-2}
\] [6]

\(c\) Find \(c\), if the difference between the roots of the following equation is 3: \(x^2 + 7x + c = 0\) [6]
The direction, "Less than 60% of the credit should be granted when an error in computation occurs," should be followed in rating all incorrect answers to questions which fall under the topics mentioned in "Suggestions on the Rating of Regents Examination Papers in Mathematics" under "General 3."

In all problems solved with two unknowns, no credit should be given for one equation correctly formed if the other is not given or is inaccurate.

Teachers are urged to mark papers cumulatively, that is, to add the credits earned by each answer to the total credits earned by preceding answers so that the mark given to the last answer is the per cent to which the paper is entitled, e.g. consecutive answers earning 5, 7, 4 etc. respectively should be marked 5, 12, 16 etc. respectively.

1 12 credits
   a 6 credits
      Allow 1 credit for correctly reducing each mixed number to fractional form.
      Allow 3 credits for correct factoring (1 for each fraction).
      Allow 1 credit for correct result.
   b 6 credits
      Allow 1 credit for correct factors of first.
      Allow 3 credits for correct factors of second.
      Allow 2 credits for correct factors of third.

2 12 credits
   Allow 3 credits for clearing equation of fractions.
   Allow 5 credits for solution.
   Allow 4 credits for values correct to 2 decimal places.

3 12 credits
   Allow 6 credits for correct substitution.
   Allow 2 credits for correct reduction to form \( \frac{14}{\sqrt{3}} \)
   Allow 4 credits for correct result to 2 decimal places.

4 12 credits
   Allow 6 credits for one pair of correct values.
   Allow 2 credits for each of the other three pairs.
   Or
   If solved by "\( \nu x = y \)" method, allow 4 credits for correct values of \( \nu \).

5 12 credits
   Allow 4 credits for correct solution.
Allow 4 credits for each correct check.
Allow no credit unless checks are made in original equation.

6 12 credits
Allow 3 credits for straight line.
Allow 5 credits for parabola.
Allow 1 credit each for correct values of $x$ and $y$.

7 12 credits
a 6 credits
Allow 4 credits for one correct value of unknown.
Allow 2 credits for the other correct value of unknown.

b 6 credits
Allow 2 credits for each correct term of answer.

8 12 credits
Allow 8 credits for correct equation.
Allow 4 credits for correct solution.
Allow 2 credits for each correct value.
Allow no credit for solution by arithmetic.

9 12 credits
Allow 1 credit for correct formula for $I$.
Allow 5 credits for correct substitution and reduction.
Allow 1 credit for correct formula for $S$.
Allow 4 credits for finding amount deposited by each member.
Allow 1 credit for amount deposited by $n$ members.

10 12 credits
a Allow 8 credits.
b Allow 4 credits.
Allow 1 credit for correct formula.
Allow 3 credits for correct substitution and reduction.

11 16 credits
a Allow 4 credits (2 each).
Allow no credit on either if incorrect.
b Allow 6 credits.
c Allow 6 credits.
Allow 4 credits for correct equation.
Allow 2 credits for correct solution. Assume that $c$ can be found mentally and allow full credit for correct answer if no work is given, but allow no credit in such case if the answer is incorrect.