Intermediate Algebra

Monday, June 15, 1908—9:15 a.m. to 12:15 p.m., only

Answer eight questions, selecting at least two from each group.

Group I

1. Simplify the following:
   \[
   \sqrt{-64} + \sqrt{-25} + \sqrt{-121} - \sqrt{225}.
   \]
   \[
   \sqrt{-\frac{1}{9}} \times \sqrt{-\frac{9}{4}} + \sqrt{-\frac{9}{4}}.
   \]

2. Solve the equation \(ax^2 + bx + c = 0\); show the condition under which the roots are (a) real and unequal, (b) real and equal, (c) conjugate imaginaries, (d) rational, (e) surds.

3. Solve as a quadratic \(3x^2 - 7 + 3\sqrt{3x^2 - 16x + 21} = 16x\)

4. Solve \(x^3 + y^3 = 243,\)
   \(xy(x + y) = 162\)

Group II

5. Find the cube root of
   \(a^3 - 2a^6 + \frac{22a^4}{3} - \frac{224a^3}{27} + \frac{44a^2}{3} - 8a + 8\)
   State how the trial divisor is formed and give reasons.

6. If \(a:b = c:d\), prove (a) \(ma:nb = mc:nd\), (b) \(ma + nb:ma - nb = mc + nd:mc - nd\)

7. Three numbers are in arithmetic progression; the first divided by the second equals \(\frac{1}{4}\) and the sum of the second and third equals 7. Find the numbers.

8. Derive the formula for the sum of a geometric progression. Adapt this formula to the infinite decreasing series.

Group III

9. Plot the graph of (a) \(\begin{cases} 4x + 5y = 1 \\ 5x - 4y = 9 \end{cases}\)
   \(b) y = x^2 - 3x + 12\)

From the graphs determine the approximate values of the roots of (a).

10. Explain the meaning of (a) negative integral exponents, (b) fractional exponents, (c) zero exponents.

11. Form the equation whose roots are \(a\) and \(b\) and show the relation between these roots and the coefficients of the equation formed.

12. The sum of the terms of a fraction is 5, and the product of this fraction by another fraction whose numerator and denominator exceed the numerator and denominator of the first fraction by 4 and 5 respectively is \(\frac{1}{4}\). Find the fraction.