The University of the State of New York
326th High School Examination
INTERMEDIATE ALGEBRA
Wednesday, January 25, 1956 — 9.15 a.m. to 12.15 p.m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one-half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II and III (a) name of school where you have studied, (b) number of weeks and recitations a week in intermediate algebra.

The minimum time requirement is four or five recitations a week for half a school year after the completion of elementary algebra.

Part II

Answer three questions from this part. Show all work.

26 Find the nearest tenth the roots of the equation \(2x^2 + 5x - 1 = 0 \) \([10]\)

27 Solve the following system of equations and check both sets of roots: \([8, 2]\)
\[
\begin{align*}
x^2 - xy - x &= 8 \\
y + x &= -1
\end{align*}
\]

28 a Draw the graph of the equation \(y = x^2 + 2x - 4\) from \(x = -4\) to \(x = 2\) inclusive. \([6]\)

b On the same set of axes used in part a, draw the graph of the equation \(y = x - 2\). \([2]\)

c From the graphs made in answer to parts a and b, find the common solutions of the two equations. \([2]\)

29 Using logarithms, find to the nearest hundredth the value of \(\frac{18.7 \times \sqrt[3]{0.0595}}{(0.861)^2}\) \([10]\)

* The following questions, *30 and *31, are based upon optional topics in the syllabus, and one or more of them may be substituted for any one question in either part II or part III. Therefore one, but not both, of these questions may be included in the total of 5 required questions from parts II and III.

*30 Using the formula \(A = (1 + r)^n\), find \(n\) to the nearest integer when \(r = .04\) and \(A = 3\). \([10]\)

*31 a (1) Find the remainder when \(x^4 + 2x^3 - 5x^2 - 4x + 6\) is divided by \((x - 1)\). \([3]\)

(2) Find the remainder when the polynomial given in part (1) is divided by \((x + 3)\). \([3]\)

b (1) Find the rational roots of the equation \(x^4 + 2x^3 - 5x^2 - 4x + 6 = 0\). \([2]\)

(2) How many irrational roots does this equation have? \([2]\)
32 A dairy company buys raw milk that contains $4\frac{1}{2}$% butterfat. This milk is to be standardized so that it will contain only 4% butterfat by adding skim milk (0% butterfat). How many pounds of skim milk must be added to each 100 pounds of the raw milk? \[7, 3]\n
33 Write the equations that would be used to solve the following problems. In each case state what the letter or letters represent. \[Do not solve the equations.\]

a $A$ and $B$ together received a total of $\$246$ for working 8 days and 10 days respectively. If $A$ had worked for 10 days and $B$ for 8, $A$'s total wages would have equalled $B$'s. Find the daily wages of each. \[5\]

b A train leaves a certain point and travels at a uniform rate of 55 miles per hour. Two hours later another train leaves the same point and travels in the same direction at a uniform rate of 70 miles per hour. How long after the faster train leaves will it be 65 miles behind the slower train? \[5\]

34 A rectangular piece of cardboard is twice as long as it is wide. From each of its four corners a square piece 2 inches on a side is cut out. The flaps are then turned up to form an uncovered box.

a If the length of the shorter side of the original piece of cardboard is represented by $x$, express the volume $V$ of the box in terms of $x$. \[6\]

b If $V = 320$ cubic inches, find $x$. \[4\]

35 Each equation in column $A$ has one of the geometric figures in column $B$ as its graph. List the numbers 1–5 on your answer paper and after each number write the letter that indicates the corresponding graph. \[10\]

<table>
<thead>
<tr>
<th>Column A</th>
<th>Column B</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) $x^2 + y^2 = 4 = 0$</td>
<td>a The point $(0, 0)$</td>
</tr>
<tr>
<td>(2) $4x^2 + y^2 = 1 = 0$</td>
<td>b Two straight lines parallel to the $y$-axis</td>
</tr>
<tr>
<td>(3) $x^2 - y = 4 = 0$</td>
<td>c Two straight lines intersecting at the origin</td>
</tr>
<tr>
<td>(4) $x^2 + 4y^2 = 0$</td>
<td>d A parabola which crosses the $y$-axis at $(0, -4)$</td>
</tr>
<tr>
<td>(5) $x^2 - 4y^2 = 0$</td>
<td>e A circle whose center is the origin and whose radius is 2</td>
</tr>
<tr>
<td></td>
<td>f An ellipse which crosses the $y$-axis at $(0, 1)$ and $(0, -1)$</td>
</tr>
<tr>
<td></td>
<td>g A hyperbola which crosses the $y$-axis at $(0, 2)$ and $(0, -2)$</td>
</tr>
</tbody>
</table>
Fill in the following lines:

Name of pupil..........................................................Name of school..........................................................

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed.

1. Factor completely: \( x^3 + 7x^2 + 12x \)

2. Simplify the complex fraction:

\[
\frac{1}{x} + \frac{1}{y}
\]

3. Perform the indicated operation and express the result in simplest form:

\[
\frac{1}{a + b} - \frac{1}{a - b}
\]

4. Express as a single term in simplest form:

\[\sqrt{27a^3} + a\sqrt{3}\]

5. Express \( \frac{5}{\sqrt{5} - 1} \) as an equivalent fraction with a rational denominator.

6. Find the sum of the roots of the equation \( 2x^2 - 4x + 5 = 0 \).

7. Solve the formula \( S = \frac{a}{1 - r} \) for \( r \).

8. Find the arithmetic mean between 5 and 8.

9. Solve the equation \( \sqrt{x^2 + 27} = 2x \) for the positive value of \( x \).

10. Solve the following pair of equations for \( x \):

\[
\begin{align*}
3x - 2y &= 10 \\
x - y &= 4
\end{align*}
\]

11. Write an equation of the straight line which passes through the point \((0, 5)\), and whose slope is 2.

12. If \( y \) varies directly as \( x \) and if \( y = 8 \) when \( x = 4 \), find the value of \( y \) when \( x = -5 \).

13. Find the sum of the first ten terms of the progression \(-8, -6, -4, \ldots\).

14. Find the logarithm of 0.6383

15. Find the number whose logarithm is 2.4752

16. A diagonal of a rectangle is 8.0 inches. If the diagonal makes an angle of 59° with the shorter side, find to the nearest tenth of an inch the length of the shorter side.

[3] [over]
Directions (17-25): Indicate the correct completion for each of the following by writing on the line at the right the letter a, b or c.

17. The equation of a line tangent to the graph of \( x^2 + y^2 = 81 \) is  
(a) \( x + y = 9 \)  
(b) \( x = 9y \)  
(c) \( x = 9 \)  
17……

18. If \( (x - a) (x - b) = m \), then \( (x - a) = m \) and \( (x - b) = m \) when \( m \) is equal to  
(a) 1  
(b) 0  
(c) -1  
18……

19. When \( x = 4 \), the value of \( x^4 + 2x^3 \) is  
(a) 8  
(b) 9  
(c) 10  
19……

20. The expression \( x^3 + y^3 \) is equal to  
(a) \( (x + y)^3 \)  
(b) \( \frac{1}{x} + \frac{1}{y} \)  
(c) \( \frac{1}{x + y} \)  
20……

21. The roots of the equation \( 6x^2 - 13x + 6 = 0 \) are  
(a) real and rational  
(b) real and irrational  
(c) imaginary  
21……

22. Of the following, the number which is not imaginary is  
(a) \( \sqrt{-1} \)  
(b) \( \sqrt{-1} \)  
(c) \( \sqrt{-1} \)  
22……

23. The graph of the equation \( y = \frac{6}{x} \) is a  
(a) straight line  
(b) parabola  
(c) hyperbola  
23……

24. The graphs of the equations \( 2x + 4y = 7 \) and \( x + 2y = 7 \), when drawn on the same set of axes,  
(a) intersect  
(b) coincide  
(c) are parallel  
24……

25. The sum of the first ten terms of the progression 2, 6, 18, \ldots is  
(a) \( 3^{10} \)  
(b) \( 3^{10} - 1 \)  
(c) \( 3^{10} + 1 \)  
25……

[4]
FOR TEACHERS ONLY

INSTRUCTIONS FOR RATING
INTERMEDIATE ALGEBRA

Wednesday, January 25, 1956 — 9.15 a.m. to 12.15 p.m., only

Use only red ink or pencil in rating Regents papers. Do not attempt to correct the pupil's work by making insertions or changes of any kind. Use check marks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. In problems involving logarithms, answers should be left correct to four significant digits unless directions say otherwise. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow 2 credits for each correct answer; allow no partial credit. Do not allow credit if the answer to question 15 is not expressed to four significant digits. For questions 17–25, allow credit if the pupil has written the correct answer instead of the letter a, b or c.

(1) \( x(x + 3)(x + 4) \)

(2) \( \frac{y - x}{y + x} \)

(3) \( \frac{-2b}{a^2 - b^2} \) or \( \frac{-2b}{(a + b)(a - b)} \)

(4) \( 4a\sqrt{3} \)

(5) \( \frac{5(\sqrt{5} + 1)}{4} \)

(6) \( 2 \)

(7) \( r = \frac{S - a}{S} \) or \( r = 1 - \frac{a}{S} \)

(8) \( 6\frac{1}{2} \)

(9) \( 3 \)

(10) \( 2 \)

(11) \( y = 2x + 5 \)

(12) \(-10\)

(13) \(10\)

(14) \( 9.8050 - 10 \) or \( 1.8050 \)

(15) \( 298.7 \)

(16) \( 4.1 \)

(17) \( c \)

(18) \( b \)

(19) \( c \)

(20) \( b \)

(21) \( a \)

(22) \( b \)

(23) \( c \)

(24) \( c \)

(25) \( b \)