The University of the State of New York

314TH High School Examination

INTERMEDIATE ALGEBRA

Wednesday, January 23, 1952—9.15 a.m. to 12.15 p.m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II, III and IV (a) name of school where you have studied, (b) number of weeks and recitations a week in intermediate algebra.

The minimum time requirement is four or five recitations a week for half a school year after the completion of elementary algebra.

Part II

Answer three questions from part II.

26. The area of a rectangle is 6 square inches. The length in inches is represented by \( x \) and the width in inches by \( x - 3 \).
   \( a \) Write an equation in \( x \) that can be used to find the length of the rectangle. \([2]\)
   \( b \) Find to the nearest tenth of an inch the length and the width of the rectangle. \([8]\)

27. Solve the following system of equations and check the answers: \([8, 2]\)
   \[
   \begin{align*}
   3x - y &= 1 \\
   4x^2 - xy &= 2
   \end{align*}
   \]

28. \( a \) Draw the graph of the equation \( y = x^2 + 3x - 2 \) from \( x = -4 \) to \( x = +1 \) inclusive. \([6]\)
   \( b \) From the graph made in answer to \( a \), estimate to the nearest tenth
   (1) the roots of the equation \( x^2 + 3x - 2 = 0 \) \([2]\)
   (2) the coordinates of the minimum point \([2]\)

29. Using logarithms, find to the nearest tenth the value of \( \frac{154 \cdot \sqrt{8.888}}{\cos 35^\circ} \) \([10]\)

*30. Solve the following system of equations: \([10]\)
   \[
   \begin{align*}
   x - 2y + 20z &= 1 \\
   3x + y - 4z &= 2 \\
   2x + y - 8z &= 3
   \end{align*}
   \]

*31. \( a \) If \( 5^{x-1} = 25 \), find \( x \). \([3]\)
   \( b \) If \( 5^x = 70 \), find \( x \) to the nearest tenth. \([7]\)

* This question is based upon one of the optional topics in the syllabus.
Part III

Answer one question from part III.

32 Write the equations that would be used in solving the following problems. In each case state what the letter or letters represent. [Solution of the equations is not required.]

a The sum of the digits of a two-digit number is 11. The number obtained by interchanging the digits exceeds twice the original number by 34. Find the original number. [5]

b How many ounces of a 90 per cent solution of disinfectant should be added to 15 ounces of a 10 per cent solution to make the result a 30 per cent solution? [5]

33 A man bought a certain number of shares of stock for $384. Had he bought when a share cost $8 less, he could have purchased four more shares for the same amount of money. How many shares did he buy? [6, 4]

Part IV

Answer one question from part IV.

34 If the blank in each of the following statements is filled by one of the words always, sometimes or never, the resulting statement will be true. Write the numbers (1) through (5) on your answer paper and opposite each write the word that correctly completes the corresponding statement. [Consider only the cases where a, b and c are real numbers other than zero.]

(1) If the graphs of the equations \(y = ax\) and \(y = bx + b\) are drawn on the same set of axes, the graphs will ... intersect. [2]

(2) The graph of the equation \(x^2 + y^2 = a^2\) ... passes through the point \((b, c)\) if \(b^2 + c^2 = a^2\). [2]

(3) The graph of the equation \(ax^2 - by^2 = c\) is ... a hyperbola if \(a, b\) and \(c\) have the same sign. [2]

(4) If the graphs of the equations \(xy = a\) and \(y = bx + c\) are drawn on the same set of axes, they will ... intersect. [2]

(5) The straight line whose equation is \(x = y\) is ... an axis of symmetry of the graph whose equation is \(ax^2 + by^2 = c\). [2]

35 A patrol plane flies at \(r\) miles an hour in still air. On a certain day the plane flew a distance of \(d\) miles from its base against a headwind of \(w\) miles an hour. It returned immediately over the same route, the round trip taking \(t\) hours. The rate and the direction of the wind remained the same during the entire trip.

a Express in terms of \(r, d\) and \(w\) the time required for the flight (1) away from the base, (2) back to the base. [2, 2]

b Write an equation that can be used to solve for \(d\) in terms of \(r, t\) and \(w\). [2]

c Solve for \(d\) the equation written in answer to b. [4]
Fill in the following lines:

Name of pupil........................................Name of school........................................

Part I
Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed.

1 Find the factors of \(12x^2 - 5x - 3\).

2 Express \(\frac{5}{3 - \sqrt{2}}\) as an equivalent fraction with a rational denominator.

3 Solve for \(x\) the equation \(2 + 3\sqrt{x} = 8\).

4 Write the expression \(3\sqrt{-4} - 2i\) as a single term.

5 Solve for \(l\) the equation \(S = \frac{rl - a}{r - 1}\).

6 Write an equation of the straight line whose slope is 5 and which passes through the point \((0, 1)\).

7 Write an equation expressing the relationship between \(x\) and \(y\) shown by the following table:

<table>
<thead>
<tr>
<th>(x)</th>
<th>2</th>
<th>3</th>
<th>5</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>(y)</td>
<td>-1</td>
<td>1</td>
<td>5</td>
<td>11</td>
</tr>
</tbody>
</table>

8 If \(p\) varies inversely as \(v\) and \(p = 2\) when \(v = 20\), find \(p\) when \(v = 8\).

9 Simplify the complex fraction \(\frac{\frac{1}{a}}{\frac{1}{a}}\).

10 Find the value of \(2b^2 + b - 1\) when \(b = 16\).

11 Find \(\log 3.142\)

12 Find the number whose logarithm is \(8.8680 - 10\).

13 The first term of an arithmetic progression is \(\frac{3}{2}\) and the sixth term is 14. Find the common difference.

[3]

[over]
14 Find two numbers that when inserted between 2 and 250 form with those numbers a geometric progression of four terms.

15 Find the sum of the infinite geometric progression whose first term is 3 and whose common ratio is \( \frac{3}{4} \).

16 If the second term in the expansion of \((a + 2b)^n\) is 14a^3b, find \(n\).

17 At a certain time of the day, the school flagpole, 30 feet high, casts a shadow 25 feet long. Find to the nearest degree the angle of elevation of the sun at that time.

18 Find the sum of the roots of the equation \(x^2 - 4x + 7 = 0\).

19 Write the equation of the axis of symmetry of the graph of \(y = x^2 - 4x - 12\).

20 If \(y = \frac{1}{x}\) and \(x\) is positive, does \(y\) increase or does it decrease as \(x\) increases? [Answer increase or decrease.]

Directions (questions 21-25) — Indicate the correct completion for each of the following by writing on the line at the right the letter \(a\), \(b\) or \(c\).

21 If the roots of a quadratic equation are real, rational and unequal, the discriminant of the equation may be \( (a) 0 \quad (b) -4 \quad (c) 1 \).

22 The expression \(r^n = s^2\) is always equal to \( (a) \left(\frac{r}{s}\right)^n \quad (b) \left(\frac{r}{s}\right)^0 \quad (c) \frac{r}{s} \).

23 One man can paint a house in \(r\) days and another man can paint it in \(s\) days. If together they can do the work in \(d\) days, the expression that gives the relationship between \(r\), \(s\) and \(d\) is \( (a) \frac{r + s}{2} = d \quad (b) \frac{1}{r} + \frac{1}{s} = d \quad (c) \frac{1}{r} + \frac{1}{s} = \frac{1}{d} \).

24 The expression \(\log r + \log r^2\) is equal to \( (a) \log (r + r^2) \quad (b) 3\log r \quad (c) r^3 \).

25 The graph of the equation \(x^2 + 4y^2 = 36\) is \( (a) \) a parabola \( (b) \) an ellipse \( (c) \) a circle.