The University of the State of New York
305TH HIGH SCHOOL EXAMINATION

INTERMEDIATE ALGEBRA

Wednesday, January 26, 1949 — 9:15 a.m. to 12:15 p.m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II, III and IV (a) name of school where you have studied, (b) number of weeks and recitations a week in intermediate algebra.

The minimum time requirement is four or five recitations a week for half a school year after the completion of elementary algebra.

Part II

Answer three questions from part II.

26 Find to the nearest tenth the roots of the equation
   \[3x^2 - 5x - 5 = 0\]  \[\text{[10]}\]

27 Solve the following set of equations, group your answers and check:
   \[x^2 - 2xy + 8 = 0\]
   \[y = x + 1\]  \[\text{[7, 1, 2]}\]

28 The formula for the radius of a sphere, in terms of the volume \(V\), is \(r = \sqrt[3]{\frac{3V}{4\pi}}\). If \(V = 487\) and \(\pi = 3.14\), find \(r\) to the nearest tenth.  \[\text{[10]}\]

29 a On the same set of axes, draw the graphs of the following equations:
   \[x^2 + y^2 = 25\] and \(xy = 12\)  \[\text{[3, 5]}\]
   b From the graphs made in answer to a, find the common solutions of the two equations.  \[\text{[2]}\]

*30 Find the three roots of the equation \(2x^3 + x^2 - 13x + 6 = 0\) \[\text{[10]}\]

*31 Solve the following set of equations:
   \[2x + 2y - z = 10\]
   \[x + 3y + 3z = 4\]
   \[5x + z = 3\]  \[\text{[10]}\]

* This problem is based on one of the optional topics in the syllabus.

[1]
Part III

Answer one question from part III.

32 Write the equations that would be used in solving the following problems. In each case state what the letter or letters represent. [Solution of the equations is not required.]
   a A 20-quart solution of alcohol and water is 10% alcohol. How much pure alcohol must be added to make it a 40% solution? [5]
   b Two boys working together can shovel snow from a driveway in 4 hours. If the larger boy works twice as fast as the smaller boy, how long would it take the larger boy to do it alone? [5]

33 A number of boys agreed to contribute equally toward purchasing a basketball for $14. Later, four more boys joined the group, causing the individual contribution to be 40 cents less. How many boys were originally in the group? [6, 4]

Part IV

Answer one question from part IV.

34 a (1) Given the equation \(2x^2 + bx + 8 = 0\). Assign a value to \(b\) that will make the roots of the equation
   (a) Real and unequal [2]
   (b) Imaginary [2]

   (2) Assign a value to \(b\) that will make the graph of the equation \(y = 2x^2 + bx + 8\) tangent to the \(x\)-axis. [2]

b Given \(y = \frac{1}{x^2} - x\). Indicate whether each of the statements below is true or false:
   (1) If \(x\) is a positive number greater than 1, then \(y\) is negative. [1]
   (2) If \(x\) is less than 0, \(y\) is positive. [1]
   (3) If \(x\) is greater than 0 but less than 1, \(y\) is positive. [1]
   (4) As \(x\) increases, \(y\) decreases. [1]

35 A boy is going to make an inclosed rectangular yard for his rabbits. He wants the yard to be subdivided into three sections by two fences parallel to the shorter side. He has 30 yards of fencing to use for inclosing and dividing the yard.
   a Representing the shorter side of the yard by \(x\) and the longer side by \(y\), express the total length of fencing in terms of \(x\) and \(y\). [2]
   b Write the equation for the area \(A\) of the yard in terms of \(x\). [3]
   c Draw the graph of the equation found in answer to b. [4]
   d From the graph made in answer to c, estimate the value of \(x\) which would make the area greatest. [1]
Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

1. Find the factors of $2x^2 - 8$

2. Express $\frac{4}{x + 2} - \frac{3}{x - 1}$ as a single fraction.

3. Find the number whose logarithm is 3.6467

4. Find the logarithm of 0.8462

5. Find the slope of the straight line whose equation is $y = 2x - 1$

6. Find the ordinate of the point at which the graph of the equation $y = x^2 - 3$ intersects the y-axis.

7. Write a linear equation expressing the relationship between $x$ and $y$ shown in the following table:

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>-2</td>
<td>1</td>
<td>4</td>
<td>7</td>
<td>10</td>
</tr>
</tbody>
</table>

8. Find the sum of the first 10 terms of the series $-8, -2, 4, 10, \ldots$

9. The first term of a geometric series is 2 and the fourth term is $-16$. Find the common ratio.

10. Write in simplest form the first three terms in the expansion of $(x + 1)^7$

11. Find the value of $(8)^{\frac{1}{3}} + 2^0$

12. Find the value of $x$ which satisfies the equation $\sqrt{x^2 + 7} = x + 1$

13. Express $\frac{1}{3 + \sqrt{3}}$ with a rational denominator.

14. The area of a triangle is equal to $\frac{1}{2}bh$. If $h = \frac{b\sqrt{3}}{2}$, express the area of the triangle in terms of $b$.

15. The rental charge for an automobile is $5 for the first hour and $3 for each additional hour. Write an expression for the cost of using the car $h$ hours, $h$ being greater than 1.

16. Solve the following set of equations:

$x + 2y = 8$

$x - y = 2$

17. Simplify the complex fraction $\frac{1 - \frac{4}{x}}{1 - \frac{2}{x^2}}$

18. If $y$ varies directly as $x$ and if $y = 4$ when $x = 10$, find the value of $y$ when $x = 15$
19 Express $9.8 \times 10^{-4}$ as a decimal.

20 Find to the nearest tenth the altitude of an isosceles triangle whose base is 6 and whose base angle is $30^\circ$.

Directions (questions 21–25) — Indicate the correct answer to each question by writing on the line at the right the letter a, b or c.

21 Log $A$ — log $B$ is equal to

(a) $\log (A - B)$  (b) $\log \frac{A}{B}$

(c) $\frac{\log A}{\log B}$

22 $(x^2)^3$ is equal to

(a) $x^3$  (b) $x^6$  (c) $x^5$

(b) 2  (c) $-2$

23 The sum of the roots of the quadratic equation $2x^2 - 4x - 5 = 0$ is

(a) $-4$  23.....

(b) $\frac{5}{2}$  (c) $\frac{3}{2}$

24 The graph of the equation $2x^2 - 3y^2 = 5$ is

(a) a circle  (b) an ellipse  24.....

(c) a hyperbola

25 The roots of the equation $x^2 + 16 = 0$ are

(a) real and equal  (b) real  25.....

and unequal  (c) imaginary