INTERMEDIATE ALGEBRA

Wednesday, January 28, 1948 — 9.15 a. m. to 12.15 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II, III and IV (a) name of school where you have studied, (b) number of weeks and recitations a week in intermediate algebra.

The minimum time requirement is four or five recitations a week for half a school year after the completion of elementary algebra.

Part II

Answer three questions from part II.

26 Find to the nearest tenth the roots of the equation $2x^2 + 9x + 3 = 0$ \[10\]

27 Solve the following set of equations, group your answers and check one set: \[7, 2, 1\]

\[
\begin{align*}
4x^2 + y^2 &= 12 \\
2x^2 - 3y^2 &= -8
\end{align*}
\]

28 By logarithms, find to the nearest thousandth the value of \[10\]

\[
\frac{0.0012 \times \tan 53\degree \sqrt[3]{4.35}}{3}
\]

29 a Draw the graph of $y = x^2 - 6$ from $x = -4$ to $x = +4$ inclusive. \[5\]

b On the set of axes used in a, draw a circle whose center is the origin and which passes through the turning point of $y = x^2 - 6$ \[2\]

c Write the equation of the circle drawn in answer to b. \[2\]

d From the graphs, find the set of positive values of $x$ and $y$ that satisfy both equations. \[1\]

[Estimate any approximate value to the nearest tenth.]

*30 Find the number of years \(n\) it will take $550$ to amount to $670$ at $2\%$ interest compounded annually. \[Use the formula $A = P(1 + r)^n$ and express your answer to the nearest year.\] \[10\]

*31 Find the three roots of $2x^3 - 3x^2 - 11x + 6 = 0$ \[10\]

* This question is based on one of the optional topics in the syllabus.
32 Write the equations that would be used in solving the following problems. In each case, state what the letter or letters represent. [Solution of the equations is not required.]

a A man is planning to drive 168 miles at his usual rate. If he were to increase his rate by 6 miles per hour, he would make the trip in one-half hour less time. Find his usual rate. [5]

b The sum of the digits of a two-digit number is 9. If the digits are reversed, the resulting number exceeds the original number by 27. Find the original number. [5]

33 A solution of salt in water is 12% salt. Another solution is 4% salt. How many pounds of the second solution must be added to 24 pounds of the first solution to make a 10% solution? [7, 3]

Part IV

Answer one question from part IV.

34 For each of the following statements indicate whether you are given too little information, just enough information or more information than is needed to justify the conclusion. [Consider only the cases in which a, b and c represent integers.]

1. The graphs of \( y = ax \) and \( y = bx + c \) will intersect if \( a \) and \( b \) are unequal. [2]

2. The graph of \( ax + by = c \) passes through the origin if \( c = 0 \). [2]

3. The graph of \( ax^2 + by^2 = c \) is an ellipse if \( a \) and \( b \) are unequal. [2]

4. The graph of \( ax^2 + by^2 = c \) is a hyperbola if \( a \) and \( b \) have unlike signs and \( c \) is positive. [2]

5. The axis of symmetry of the graph \( y = ax^2 + bx + c \) is the \( y \) axis if \( b = 0 \). [2]

35 a If \( n \) articles can be bought for \( c \) cents, express in terms of \( n, c \) and \( d \) the number of articles at the same price that can be bought for \( d \) dollars. [2]

b The length of a certain rectangle is \( l \) and the width is \( w \). If the length is increased by an amount \( a \), by how much must the width be decreased so that the area will remain the same? [4]

c (1) Express in terms of \( n \) the sum \( S \) of \( n \) consecutive positive even integers beginning with 2. [2]

(2) Express in terms of \( n \) the sum \( S' \) of \( n \) consecutive positive odd integers beginning with 1. [1]

(3) Show that the difference between \( S \) and \( S' \) is \( n \). [1]
Fill in the following lines:

Name of pupil........................................Name of school........................................

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

1 Reduce to lowest terms: \( \frac{x - 4}{x^2 - x - 12} \)

2 Combine and express the result in terms of \( i \):

\( \sqrt{-36} - \sqrt{-16} \)

3 Factor \( x^2 - 4 \)

4 Solve the equation \( \sqrt{x - 1} = 2 \)

5 Find the logarithm of 1.516

6 Find to the nearest tenth the number whose logarithm is 2.3824

7 Write in simplest form the first three terms of the expansion of \( (a - b)^2 \)

8 Find the positive geometric mean between 2 and 32.

9 The first term of an arithmetic progression is 3 and the sixth term is 23. Find the common difference.

10 Find the sum of the infinite series 6, 3, 1\( \frac{1}{2} \), . . .

11 Write the fraction \( \frac{2}{\sqrt{6} - 1} \) with a rational denominator.

12 Write a linear equation expressing the relationship between \( x \) and \( y \) shown in the following table:

<table>
<thead>
<tr>
<th>( x )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>...</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>1</td>
<td>4</td>
<td>7</td>
<td>...</td>
</tr>
</tbody>
</table>

13 Find the slope of the line whose equation is \( 2y - x = 4 \)

14 If a point whose abscissa is 2 is on the line whose equation is \( y = 3x + 2 \), find the ordinate of the point.

15 Find the value of \( 9^{\frac{1}{2}} + 3^0 \)

16 If \( y = x - \frac{1}{x} \) and \( x \) is positive, does \( y \) increase or does it decrease as \( x \) increases?

17 If \( x \) varies directly as \( y^2 \) and \( x = 8 \) when \( y = 2 \), what is the value of \( x \) when \( y = 3 \)?

18 Solve the formula \( S = \frac{1}{2} gt^2 \) for the positive value of \( t \).

19 A leg of an isosceles triangle is 16 and a base angle is 64°. Find to the nearest integer the altitude on the base.

20 Simplify: \( \left( \frac{1}{y} - \frac{1}{x} \right) \div \left( 1 - \frac{y}{x} \right) \)
21 The cost of sending a telegram is 35 cents for the first 10 words and 3 cents for each additional word. Express in cents the cost of sending a telegram of \( n \) words, where \( n \) is greater than 10.

Directions (questions 22–25) — Indicate whether each of the following statements is true or false by writing the word true or false on the line at the right.

22 A quadratic equation in one unknown has two roots.
23 If the roots of a quadratic equation are equal, the discriminant is zero.
24 The sum of the roots of \( 2x^2 + 4x - 1 = 0 \) is \(-2\).
25 The graph of \( y = x^2 - 8x + 16 \) has no point in common with the \( x \) axis.