The University of the State of New York
299th High School Examination

INTERMEDIATE ALGEBRA

Wednesday, January 29, 1947 — 9.15 a. m. to 12.15 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II, III and IV (a) name of school where you have studied, (b) number of weeks and recitations a week in intermediate algebra.

The minimum time requirement is four or five recitations a week for half a school year after the completion of elementary algebra.

Part II

Answer three questions from part II.

26 Find, correct to the nearest tenth, the roots of the equation \(2x^2 - 5x - 1 = 0\)\[10\]

27 Solve the following system of equations, group your answers and check one set: \[7, 2, 1\]
\[
x^2 + y^2 = 2x = 3
\]
\[3x + y = 1\]

28 Using logarithms, find, correct to the nearest thousandth, the value of \(\sqrt{\frac{7.53}{0.666 \times 97}}\)\[10\]

29 a Draw the graph of \(y = x^2 - 2x - 6\) from \(x = -2\) to \(x = 4\) inclusive. \[5\]
   b Write the equation of the axis of symmetry. \[2\]
   c Write the coordinates of the minimum point. \[1\]
   d From the graph estimate, correct to the nearest tenth, the roots of the equation \(x^2 - 2x - 6 = 0\) \[2\]

*30 Find the roots of the equation \(2x^3 - x^2 - 27x + 36 = 0\) \[10\]

* This question is based on one of the optional topics in the syllabus.

Part III

Answer one question from part III.

31 Write the equations that may be used in solving each of the following problems. In each case, state what the letter or letters represent. [Solution of the equations is not required.]
   a A man rows 18 miles downstream in 2 hours. He finds it takes him 6 hours to row back. What is his rate of rowing in still water and what is the rate of the stream? \[5\]
   b The length of a rectangle is 6 inches more than the width. If the length is diminished by 2 inches, the area is diminished by 8 square inches. Find the dimensions of the rectangle. \[5\]
32 A man buys a certain number of railway shares for $912. Had he bought when each share cost $19 less, he could have purchased four more shares for the same amount of money. How many shares did he buy? [6, 4]

Part IV

Answer one question from part IV.

33 Each of the following statements may be correctly completed by one or more of the given choices. Write on your answer paper the numbers (1) to (4) and after each indicate the correct answer or answers to the corresponding question by writing one or more of the letters, a, b, c, d, e. [10]

[In each of the four parts of this question, one credit will be allowed for each correct choice made and one credit will be deducted for each incorrect choice. The minimum credit on each part will be 0.]

(1) If the graphs of the equations \(x^2 + y^2 = 5\) and \(xy = 2\) are drawn on the same set of axes, (a) they intersect in four points (b) the point whose coordinates are \((2, 1)\) is on both graphs (c) both graphs intersect the y axis (d) one of the graphs is a straight line

(2) The graph of the equation \(3x - 2y = 10\) (a) is parallel to the graph of \(6x + 4y = 20\) (b) has a positive slope (c) has a negative y intercept (d) forms with the x axis an acute angle which is greater than 45° (e) has -5 for its x intercept

(3) In the equation \(x^2 + px + q = 0\), (a) if \(p^2 - 4q = 0\), the roots are equal (b) if \(p = 0\), the roots are numerically equal but opposite in sign (c) if \(q = 0\), there is only one root (d) if \(p = 0\) and \(q = 0\), the equation has no roots (e) the sum of the roots is \(-p\) and their product is \(q\)

(4) If \(\log a = x\) and \(\log b = y\), (a) \(\log \sqrt[n]{ab} = \frac{x + y}{3}\) (b) \(\log (a + b) = x + y\) (c) \(\log \frac{a}{b^x} = x - 2y\) (d) \(\frac{\log a}{\log b} = x - y\)

34 Three numbers are in arithmetic progression, the common difference being 2. If the first number is diminished by 3, the second is diminished by 2 and the third is doubled, the resulting numbers are in geometric progression. Find the original numbers. [10]
Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

1. Factor \( x^2 - 25 \)

2. Solve the formula \( S = \frac{P + Q}{L} \) for \( L \).

3. Find the negative root of the equation \( 2x^2 + 5x - 12 = 0 \)

4. Write a linear equation expressing the relationship between \( x \) and \( y \) shown in the following table:

<table>
<thead>
<tr>
<th>( x )</th>
<th>0</th>
<th>2</th>
<th>4</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>3</td>
<td>7</td>
<td>11</td>
<td>15</td>
</tr>
</tbody>
</table>

5. Solve the equation \( \sqrt{x^2 - 4} = 4 - x \)

6. If \( x \) varies directly as \( y \) and if \( x = \frac{3}{4} \) when \( y = \frac{1}{3} \), find \( x \) when \( y = \frac{2}{5} \).

7. If the second and fourth terms of an arithmetic progression are 16 and 4 respectively, find the third term of the progression.

8. Find the seventh term of the series 4, 12, 36, ... .

9. Find the sum of the infinite geometric progression 3, 1, \( \frac{1}{3} \), ...

10. Write the first three terms in the expansion of \( (x - y)^6 \)

11. In the equation \( y = \frac{1}{x} \), does \( y \) increase or does it decrease when \( x \) increases from 0?

12. Find the value of \( 8^\frac{1}{3} + 2^{-1} + 3^0 \).

13. Express \( \frac{5}{3 + \sqrt{3}} \) as an equivalent fraction having a rational denominator.

14. From \( \sqrt{-16} \) subtract \( 2\sqrt{-1} \) and express the result in terms of \( i \).

15. Find the sum of \( \frac{3}{a-3} \) and \( \frac{5}{3-a} \).

16. Two trains start from the same place at the same time and travel in opposite directions. If one train travels \( a \) miles an hour and the other travels \( b \) miles an hour, how far apart are they at the end of \( c \) hours?

17. Find the logarithm of 182.6

18. If \( \log x = 1.5240 \), find \( x \) correct to the nearest hundredth.

19. From a point at the foot of a hill, the angle of elevation of the top is 13°. The distance from the foot of the hill to the top is 200 feet. Find the height of the hill.

20. Write the equation of the straight line that passes through the point \( (4, 5) \) and has a slope of \( \frac{1}{2} \).
21. If $4^x \times 8 = 2^n$, then $n$ is equal to (a) 7 (b) 12 (c) 64 (d) 128

22. The sum of the roots of the equation $2x^2 + 5x - 3 = 0$ is (a) 5 (b) $\frac{5}{2}$ (c) $-\frac{5}{2}$ (d) $\frac{3}{2}$

23. The fraction $\frac{4x + 6}{4x}$ is equal to (a) 6 (b) 7 (c) $\frac{4x + 3}{2x}$ (d) $\frac{2x + 3}{2x}$

24. The graph of the equation $x^2 + 4y^2 = 16$ is (a) an ellipse (b) a parabola (c) a hyperbola (d) a circle

25. If the discriminant of a quadratic equation is 17, the roots of the equation are (a) rational and equal (b) rational and unequal (c) irrational and unequal (d) imaginary