The University of the State of New York

296th HIGH SCHOOL EXAMINATION

INTERMEDIATE ALGEBRA

Wednesday, January 30, 1946 — 9.15 a. m. to 12.15 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II, III and IV (a) name of school where you have studied, (b) number of weeks and recitations a week in intermediate algebra.

The minimum time requirement is five recitations a week for half a school year after the completion of elementary algebra.

Part II

Answer three questions from part II.

26. Solve the equation $2x^2 - 3x - 1 = 0$ for values of $x$ correct to the nearest tenth. [10]

27. Solve the following system of equations, group your answers and check one set: [7, 2, 1]
   
   $4x^2 + y^2 = 40$
   
   $2x + y = 8$

28. Using logarithms, find, correct to the nearest hundredth, the value of
   
   $\frac{(2.53)^2 \times \sqrt{15.2}}{0.851}$ [10]

29. $a$ On the same set of axes, draw the graphs of $y = x^2 - 5x$ and $y = -2x + 3$ [6, 2]
   
   $b$ From the graphs made in answer to $a$, find, correct to the nearest tenth, the values of $x$ and $y$ common to the two equations. [2]

*30. Solve the following set of equations for $x$, $y$ and $z$: [10]
   
   $2x + z = 4a$
   
   $x + y = -a$
   
   $y + z = -3a$

* This question is based on one of the optional topics in the syllabus.

[1]
31 Write the equations that would be used in solving each of the following problems. In each case state what the letter or letters represent. [Solution of the equations is not required.]

a How many quarts of water must be added to 12 quarts of a mixture that is 15 per cent alcohol to make a mixture that is 10 per cent alcohol? [4]

b If 2 is added to both the numerator and the denominator of a certain fraction, the value of the fraction becomes 9. If 1 is subtracted from the numerator of the original fraction, the value of the fraction becomes 4. Find the fraction. [3]

c A boy walked out into the country at the rate of 4 miles an hour and returned over the same road at the rate of 3 miles an hour. If the entire trip took 3 1/2 hours, how far into the country did he walk? [3]

32 A man bought some cows for $750. If they had cost $25 apiece less, he could have bought one more for the same amount. How many cows did he buy? [6, 4]

Part IV

Answer one question from part IV.

33 Each of the following statements is sometimes true and sometimes false, depending on the values of \( a, b \) and \( c \). In each case assign to \( a, b \) and \( c \) numerical values that will make the statement true and also numerical values that will make the statement false.

(1) The graph of the parabola \( y = ax^2 + bx + c \) passes through the origin. [2]

(2) The graph of the equation \( ax + by = c \) is parallel to the \( x \) axis. [2]

(3) The formula \( S = \frac{a}{1-r} \) is used in finding the sum of the terms of the geometric progression \( a, b, c, \ldots \) [2]

(4) The roots of the equation \( ax^2 + bx + c = 0 \) are equal. [2]

(5) \( \log a = \log b + \log c \) [2]

34 If the roots of the quadratic equation \( ax^2 + bx + c = 0 \) are represented by \( r_1 \) and \( r_2 \)

a Express \( r_1 \) and \( r_2 \) in terms of \( a, b \) and \( c \). [2]

b Prove that \( r_1 + r_2 = -\frac{b}{a} \) [3]

c Prove that \( r_1 \times r_2 = \frac{c}{a} \) [5]
Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

1 Factor \( 2x^4 + 4x^3 - 48x^2 \)

2 Write a linear equation expressing the relation between \( x \) and \( y \) shown in the following table:

<table>
<thead>
<tr>
<th>( x )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>1</td>
<td>4</td>
<td>7</td>
<td>13</td>
</tr>
</tbody>
</table>

3 The formula for the volume of a circular cone is \( V = \frac{1}{3} \pi r^2 h \). Find \( r \) if \( V = 132 \), \( \pi = \frac{22}{7} \), \( h = 14 \)

4 What is the name of the graph of the equation \( ax^2 + by^2 = c \) in which \( a \), \( b \) and \( c \) are positive and \( a \) is not equal to \( b \)?

5 A can do a piece of work in 8 days and B can do the same piece of work in 12 days. If \( x \) represents the number of days it would take A and B working together to do the work, write the equation that should be used to find \( x \).

6 Find the value of \( x \) that satisfies the equation \( \frac{x + 6}{\sqrt{x}} = 4\sqrt{x} \)

7 Write the discriminant of the equation \( 2x^2 - 5x - 2 = 0 \)

8 Express \( \frac{5}{3 - \sqrt{2}} \) as a fraction with a rational denominator.

9 Find the value of the expression \( 2^a \times 2^{-1} + 2^0 \)

10 Express the sum of \( 3 \sqrt{-4} \) and \( \sqrt{-9} \) in terms of \( i \).

11 What is the greatest positive integral value of \( a \) for which \( \sqrt{a - 4} \) is imaginary?

12 If \( \log x = 8.2363 - 10 \), find \( \log \sqrt[3]{x} \)

13 Find the logarithm of 18.24

14 The sides of a rectangle are 4 and 5. Find, correct to the nearest degree, the angle which a diagonal makes with the longer side.

15 The first term of an arithmetic progression is \( -4 \) and the twelfth term is 10. Find the sum of these 12 terms.

16 The first term of a geometric progression is 4 and the second term is \( -8 \). Find the sixth term.

17 Find the third term in the expansion of \( (x + y)^6 \)

18 If \( y \) varies directly as \( x \) and if \( y = 5 \) when \( x = 7 \), what is the value of \( x \) when \( y = 3 \)?

19 Write the equation of the straight line which passes through the origin and whose slope is 3.
Directions (questions 20–25) — Indicate the correct answer to each question by writing on
the line at the right the letter a, b, c or d.

20 The value of the fraction \( \frac{3x + 12}{3x} \), when reduced to lowest terms, is

(a) 13 \hspace{1cm} (b) \( \frac{4}{x} \) \hspace{1cm} (c) \( \frac{x + 4}{x} \) \hspace{1cm} (d) none of these answers

21 The product of the roots of the equation \( 2x^2 + 7x + 8 = 0 \) is

(a) 8 \hspace{1cm} (b) \( -8 \) \hspace{1cm} (c) 7 \hspace{1cm} (d) none of these answers

22 The roots of the equation \( 3x^2 - 5x - 2 = 0 \) are

(a) real, rational and equal \hspace{1cm} (b) real, rational and unequal \hspace{1cm} (c) real and irrational \hspace{1cm} (d) imaginary

23 The product of \( 3^x \) and \( 3^z \) is

(a) equal to \( 3^{z+x} \) \hspace{1cm} (b) equal to \( 3^{x+z} \) \hspace{1cm} (c) equal to \( 9^{z+x} \) \hspace{1cm} (d) none of these answers

24 \( \log ab^3 \) is

(a) \( 3(\log a + \log b) \) \hspace{1cm} (b) \( \frac{1}{3}(\log a + \log b) \) \hspace{1cm} (c) \( \log a + 3 \log b \) \hspace{1cm} (d) none of these answers

25 The value of \( 64^{-\frac{2}{3}} \) is

(a) \( -512 \) \hspace{1cm} (b) \( \frac{1}{4} \) \hspace{1cm} (c) \( \frac{1}{2} \) \hspace{1cm} (d) none of these answers