The University of the State of New York
293d High School Examination

INTERMEDIATE ALGEBRA

Wednesday, January 24, 1945 — 9.15 a. m. to 12.15 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II, III and IV (a) name of school where you have studied, (b) number of weeks and recitations a week in intermediate algebra.

The minimum time requirement is five recitations a week for half a school year after the completion of elementary algebra.

Part II

Answer three questions from part II.

26 a Draw the graph of \( y = x^2 - 4x - 5 \) from \( x = -2 \) to \( x = 6 \) \[5\]
   b Using the same set of axes as in a, draw the graph of \( y = 2 \) \[1\]
   c On the graphs made in answer to a and b, indicate by the letters \( P \) and \( Q \) the points whose abscissas are the roots of the equation \( x^2 - 4x - 5 = 2 \) \[2\]
   d From the graph made in answer to a, determine the least value of \( y \) for which the roots of \( x^2 - 4x - 5 = y \) are real. \[2\]

27 Find, correct to the nearest tenth, the roots of the equation \( 2x^2 + 5x - 8 = 0 \) \[10\]

28 Solve the following system of equations, group your answers and check one set:
   \[
   \begin{align*}
   3x^2 - 2xy + 4y^2 &= 148 \\
   x &= -3y
   \end{align*}
   \]
   \[7, 2, 1\]

29 The diameter \( d \) in inches of a wrought-iron shaft required to transmit \( h \) horsepower at a speed of \( n \) revolutions per minute is given by the formula \( d = \sqrt[3]{\frac{65h}{n}} \). Using logarithms, find, correct to the nearest tenth of an inch, the diameter required when 135 horsepower is to be transmitted at a speed of 148 revolutions per minute. \[10\]

*30 Find, correct to the nearest year, the time in which $3500 will amount to $5100 at 3% interest compounded annually. \[Use the formula \( A = P \ (1 + r)^n \) \[10\]

* This question is based on one of the optional topics in the syllabus.
31 Write the equations that would be used in solving the following problems. In each case state what the unknown letter or letters represent. [Solution of the equations is not required.]

a Of 24 pounds of salt water, 8% is salt. Of another mixture, 4% is salt. How many pounds of the second mixture should be added to the first mixture in order to get a mixture that is 5% salt? [5]

b A man worked a certain number of days to earn $240. If he had received $2 less per day, he would have had to work 10 days longer to earn the same amount. How many days did he work? [5]

32 A company contracted to make a certain number of planes. Its two factories together can make the planes in 12 days. Working alone, one factory requires 7 days longer than the other. Find the time in which each factory alone can fulfill the contract. [6, 4]

Part IV

Answer one question from part IV.

33 a $S$ is the sum of a geometric progression of $n$ terms whose first term is $a$ and whose common ratio is $r$.

Derive the formula $S = \frac{a - ar^n}{1 - r}$ [6]

b To what does the formula in a reduce when $r$ is numerically less than 1 and $n$ increases without limit? [2]

c Find the sum of the infinite geometric series 3, $\frac{3}{4}$, $\frac{1}{4}$, ... [2]

34 a Write .00016 as the product of 1.6 times a power of 10. [2]

b Given $\log \frac{P}{6} = 2.3000$; find the value of $P$. [2]

c Given the equation $x + y - 7 + 4k = 0$; find the value of $k$ for which the graph of this equation will pass through the origin. [2]

d It took $m$ army engineers $h$ hours to build a pontoon bridge. If $r$ more engineers had been assigned to the work and they had worked at the same rate, how many hours would it have taken? [2]

e A man travels $m$ miles per hour for $t$ hours and then changes his rate to $s$ miles per hour for $h$ hours. Find his average rate. [2]
Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

1 Express \( \frac{5}{\sqrt{13}} \) as an equivalent fraction with rational denominator.

2 Find the three factors of \( r^3 - 9r \).

3 Solve the following set of equations:
   \[
   \begin{align*}
   3x + 2y &= 13 \\
   x - y &= 1
   \end{align*}
   \]

4 Express \( \sqrt{-49} \) in terms of \( i \).

5 What is the value of \( 3a^0 \)?

6 Find the value of \( 64^{\frac{1}{3}} \).

7 Multiply \( a^4 \) by \( a^{-1} \).

8 What value of \( x \) satisfies the equation \( \sqrt{x^2 - 4} = x - 2 \)?

9 Find the sum of the roots of the equation \( 5x^2 - 4x + c = 0 \).

10 Find the roots of the equation \( 3x^2 - 2x - 8 = 0 \).

11 Using the formula \( T = 2\pi r(r + h) \), find \( h \) when \( T = 572 \) and \( r = 7 \). [Use \( \pi = \frac{22}{7} \).]

12 The shadow of a radio tower is 40 feet long. If the angle of elevation of the top of the tower at one end of the shadow is 70°, find, correct to the nearest foot, the height of the tower.

13 Find the logarithm of 63.48

14 Find the number whose logarithm is 3.3648

15 Given \( A = \frac{r^2}{s} \); express \( \log A \) in terms of \( \log r \) and \( \log s \).

16 Which term of the progression 3, 6, 12, ... is 96?

17 Insert three arithmetic means between 8 and 44.

18 Find the sum of the infinite geometric progression 2, 1, \( \frac{1}{2} \), ...

19 A straight line has a slope of 2 and a \( y \) intercept of 3. Write the equation of the line.

20 Write the first two terms of the expansion of \( (a - 2b)^6 \)

21 If the price of a bus ticket varies directly as the mileage involved and a ticket to travel 135 miles costs $2.97, how much will a ticket for 30 miles cost?

22 The results (in round numbers) of a series of experiments are shown in the table below. Write a linear equation expressing the relation between \( r \) and \( s \).

<table>
<thead>
<tr>
<th>( r )</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>9</th>
<th>...</th>
</tr>
</thead>
<tbody>
<tr>
<td>( s )</td>
<td>5</td>
<td>11</td>
<td>17</td>
<td>26</td>
<td>...</td>
</tr>
</tbody>
</table>
Directions (questions 22–25) — Indicate the correct answer to each question by writing on the line at the right the letter a, b or c.

23 The graph of \(2x^2 + 2y^2 = 49\) is \((a)\) a circle \((b)\) a parabola \((c)\) an ellipse

24 If the discriminant of a quadratic equation is 1, the roots of the equation are \((a)\) rational and unequal \((b)\) rational and equal \((c)\) irrational and unequal

25 Of the following, the value that most nearly satisfies the equation \(x^2 + 3x = 5\) is \((a)\) 0.8 \((b)\) 1 \((c)\) 1.2