The University of the State of New York
277th High School Examination
INTERMEDIATE ALGEBRA
Wednesday, January 24, 1940 — 9.15 a. m. to 12.15 p. m., only

Instructions

Do not open this sheet until the signal is given.

Group I

This group is to be done first and the maximum time allowed for it is one and one half hours. Merely write the answer to each question in the space at the right; no work need be shown.

If you finish group I before the signal to stop is given you may begin group II. However, it is advisable to look your work over carefully before proceeding, since no credit will be given any answer in group I which is not correct and in its simplest form.

When the signal to stop is given at the close of the one and one half hour period, work on group I must cease and this sheet of the question paper must be detached. The sheets will then be collected and you should continue with the remainder of the examination.

Groups II, III and IV

Write at top of first page of answer paper to groups II, III and IV (a) name of school where you have studied, (b) number of weeks and recitations a week in intermediate algebra.

The minimum time requirement is five recitations a week for half a school year after the completion of elementary algebra.

The use of the slide rule will be allowed for checking but all computations with tables must be shown on the answer paper.
Fill in the following lines:

Name of school........................................................................ Name of pupil........................................................................

Detach this sheet and hand it in at the close of the one and one half hour period.

Group I

Answer all questions in this group. Each correct answer will receive 2 credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

Directions (questions 1–17) — Write on the dotted line at the right of each question the expression which when inserted in the corresponding blank will make the statement true.

1. The three factors of \( x^2 - 9x \) are ....

2. Expressed in terms of \( i, \sqrt{-9} \) is ....

3. The slope of the line whose equation is \( y = 2x + 3 \) is ....

4. The formula for \( S \), the sum of an arithmetic series, in terms of the first term \( a \), the last term \( l \) and the number of terms \( n \), is \( S = \ldots \)

5. The sum of \( \frac{a}{b} \) and 1, expressed as a single fraction, is ....

6. The value of \( x \) which satisfies the equation \( \sqrt{x - 2} = 5 \) is ....

7. The fraction \( \frac{1}{\sqrt{3} + 1} \) expressed with a rational denominator is ....

8. The product of the roots of the equation \( x^2 + px - 3 = 0 \) is ....

9. The arithmetic mean between 4 and 7 is ....

10. The positive root of the equation \( x^2 - 2x - 3 = 0 \) is ....

11. The logarithm of 234.3 is ....

12. The number whose logarithm is 1.6518, expressed to the nearest hundredth, is ....

13. In triangle \( ABC \), angle \( C = 90^\circ \), angle \( A = 35^\circ \), \( AB = 100 \); the length of \( BC \) correct to the nearest integer is ....

14. \( a^{2x} + a = \ldots \)

15. The first three terms of the expansion \( (a + b)^5 \) are ....

16. The value of \( (16)^{-\frac{1}{4}} \times 4(3)^0 \) is ....

17. The formula \( S = \frac{an}{1 - r} \) when solved for \( r \) is \( r = \ldots \)

18. Write the equation of the circle whose center is at the origin and whose radius is 7.

19. Write the equation which expresses the relation between \( x \) and \( y \) shown in the table at the right.

<table>
<thead>
<tr>
<th>( x )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>...</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>1</td>
<td>4</td>
<td>7</td>
<td>10</td>
<td>...</td>
</tr>
</tbody>
</table>

20. Write in the form \( x^2 + px + q = 0 \), the equation whose roots are 2 and \(-1\).
22 If \( y \) varies directly as \( x \) and if \( x = 2 \) when \( y = 8 \), find the value of \( y \) when \( x = 7 \).

Directions (questions 23–25) — Indicate the correct answer to each question by writing on the dotted line at the right the letter \( a \), \( b \) or \( c \).

23 The graphs of the equations \( 2x + 3y = 12 \) and \( 2x + 3y = 6 \) are straight lines which \( (a) \) coincide, \( (b) \) intersect or \( (c) \) are parallel.

24 If the discriminant of a quadratic equation is \(-9\), the roots of the equation are \( (a) \) real and equal, \( (b) \) real and unequal or \( (c) \) imaginary.

25 The expression \( \log \sqrt{a} \) is equal to \( (a) \) \( 2 \log a \), \( (b) \) \( \frac{1}{2} \log a \) or \( (c) \) \( \log \frac{1}{2}a \).
26 Find, correct to the nearest tenth, the roots of the equation \(2x^2 - 4x - 1 = 0\) \([10]\)

27 Solve the following pair of equations, group the answers, and check one set of answers:
\[
x^2 + y^2 = 13 \\
3x^2 + 2y^2 = 30
\] \([7, 2, 1]\)

28 Using logarithms, find, correct to the nearest hundredth, the value of
\[
a \sqrt{1632} \quad [6] \\
b \tan 42^\circ \quad [4] \\
c \frac{\sqrt{261}}{12} \quad [4]
\]

29 Derive the formula for \(S\), the sum of a geometric series, in terms of the first term \(a\), the common ratio \(r\) and the number of terms \(n\). \([10]\)

30 a Draw the graph of the equation \(y = x^2 - 4x\) from \(x = -1\) to \(x = 5\) inclusive. \([6]\)

b Write the equation of the axis of symmetry. \([1]\)

c Write the coordinates of the minimum point. \([1]\)

d Using the graph made in answer to \(a\), estimate, correct to the nearest tenth, the roots of the equation \(x^2 - 4x = 4\) \([2]\)

*31 Find the three roots of the equation \(x^3 - 2x^2 - x - 6 = 0\) \([10]\)

**Group III**

Answer one question from this group.

32 A piece of wire 40 inches long is bent into the form of a right triangle whose hypotenuse is 17 inches. Find the other two sides of the triangle. \([6, 4]\)

33 Write the equations that would be used in solving the following problems. In each case state what the unknown letter or letters represent. \(\text{[Solution of the equations is not required.]}\)

\(a\) The sum of the numerator and the denominator of a certain fraction is 14. If the numerator is increased by 3, the resulting fraction exceeds the original fraction by \(\frac{3}{8}\). Find the fraction. \([5]\)

\(b\) An airplane flew a distance of 480 miles in 2 hours when traveling with the wind. Returning against the wind, it was able to travel the same distance in 3 hours. Find the velocity of the wind. \([5]\)

**Group IV**

Answer one question from this group.

34 Explain why each of the five following statements is in general false:

\(a\) \(\sqrt{a^2 + b^2} = a + b\) \([2]\)

\(b\) \(\frac{a}{c} = \frac{c}{b}\) \([2]\)

\(c\) \(\log a = \log a - \log b\) \([2]\)

\(d\) \(\sqrt{-a} = ia\) \([2]\)

\(e\) \(3^a \times 3^b = 9^{a+b}\) \([2]\)

35 The perimeter of a parallelogram is 70 feet.

\(a\) If one side of the parallelogram is represented by \(y\), represent the adjacent side in terms of \(y\). \([2]\)

\(b\) If the altitudes on two adjacent sides of the parallelogram as bases are in the ratio 3:4, represent these two altitudes in terms of the letter \(x\). \([1]\)

\(c\) In terms of \(x\) and \(y\) write two expressions for the area \(K\) of the parallelogram. \([2]\)

\(d\) If \(K = 240\) square feet, find the sides and the altitudes of the parallelogram. \([5]\)

\(*\) This question is based on one of the optional topics in the syllabus. \([2]\)