The University of the State of New York

265th High School Examination

INTERMEDIATE ALGEBRA

Wednesday, January 22, 1936 — 9.15 a.m. to 12.15 p.m., only

Instructions

Do not open this sheet until the signal is given.

Group I

This group is to be done first and the maximum time allowed for it is one and one half hours.

If you finish group I before the signal to stop is given you may begin group II. However, it is advisable to look your work over carefully before proceeding, since no credit will be given any answer in group I which is not correct and in its simplest form.

When the signal to stop is given at the close of the one and one half hour period, work on group I must cease and this sheet of the question paper must be detached. The sheets will then be collected and you should continue with the remainder of the examination.

Group II

Write at top of first page of answer paper to group II (a) name of school where you have studied, (b) number of weeks and recitations a week in intermediate algebra.

The minimum time requirement is five recitations a week for half a school year after the completion of elementary algebra.

The use of the slide rule will be allowed for checking but all computations with tables must be shown on the answer paper.
Fill in the following lines:

Name of school...........................................Name of pupil...........................................

Detach this sheet and hand it in at the close of the one and one half hour period.

Answer all questions in this group. Each correct answer will receive 2½ credits. No partial
credit will be allowed. Each answer must be reduced to its simplest form.

Group I

1. Find the logarithm of 83.24

2. Find the 4-place decimal whose logarithm is 9.7329 — 10

3. In the triangle \(ABC\), \(AB = 15\), \(AC = 13\) and \(C = 90^\circ\); find angle \(A\)
correct to the nearest degree.

4. Factor \(x^2 - x - 6\)

5. Find the value of \(8^{\frac{1}{3}} \times 2^2 \times 2^{-3}\)

6. Find a fraction equal to the fraction \(\frac{2 - \sqrt{3}}{5 - \sqrt{3}}\) and having a rational
denominator.

7. Express \(\sqrt{-8}\) in terms of the imaginary unit \(i\).

8. Write the quadratic equation the sum of whose roots is 4 and the
   product of whose roots is 5.

9. Solve the following equation:
   \[1 + \frac{x - 3}{\sqrt{x - 1}} = \sqrt{x - 1}\]

10. The area \(A\) of a circular ring whose inner radius is \(r\) and whose
    width is \(w\) is given by the formula \(A = \pi w(2r + w)\). Find \(r\) if \(w = 3\)
    and \(A = 66\) [Use \(\pi = \frac{22}{7}\)].

11. Solve the following formula for \(m\):
    \[V = \frac{h}{6} (b + b' + 4m)\]

12. Write the equation of the straight line which has a slope of \(\frac{2}{3}\) and a
    \(y\)-intercept of 4.

13. Write the equation of the straight line passing through the points
    whose coordinates are given in the following table:
    \[
    \begin{array}{c|cccccc}
    x & 0 & 2 & 4 & 6 & \ldots \\
    \hline
    y & -1 & 3 & 7 & 11 & \ldots \\
    \end{array}
    \]

14. Express as a single fraction in its lowest terms
    \[
    \frac{2x}{x^2 - 1} + \frac{1}{1 - x}
    \]

15. Complete the following statement: In order that the roots of the
equation \(x^2 - 4x + k = 0\) shall be imaginary, the value of \(k\) must be a
number greater than . . . .

16. Find the abscissa of the lowest point of the parabola \(y = x^2 - 4\)

[OVER]
17 Write in simplest form the first two terms of the expansion \((2x^2 - 1)^5\).

18 Insert three arithmetic means between 6 and 22.

19 Find the sum of the infinite number of terms of the series \(2 + \frac{3}{2} + \frac{4}{3} + \frac{5}{4} + \ldots\).

20 One side of a rectangle is \(s\) and the perimeter is 8. Express the area \(k\) as a function of \(s\); that is, express \(k\) in terms of \(s\).
Answer five questions from this group. Full credit will not be granted unless all operations (except mental ones) necessary to find results are given; simply indicating the operations is not sufficient. Each answer should be reduced to its simplest form. Purely arithmetical solutions for problems will not be accepted.

21 Find, correct to the nearest tenth, the roots of the equation \(2x^2 - 7x - 3 = 0\) \[10\]

22 Solve the following pair of simultaneous equations, correctly group your answers and check one set:

\[
\begin{align*}
x^2 - 6x + y^2 &= 0 \\
x + y &= 6
\end{align*}
\]

\[7, 2, 1\]

23 The area \(k\) of a regular pentagon in terms of its side \(s\) is given by the formula \(k = \frac{5s^2 \tan 54^\circ}{4}\); using logarithms, find, correct to the nearest tenth of an inch, the side of a regular pentagon whose area is 25 square inches. \[10\]

24 How many ounces of alcohol 90% pure must be added to 12 ounces 50% pure to make a mixture 75% pure? \[7, 3\]

25 Two machines working together can do a piece of work in 1½ hours. To do the same piece of work alone, one machine requires 1 hour longer than the other. How long does it take the faster machine to do the work alone? \[7, 3\]

26 Write the equations that would be used in solving any two of the following problems; in each case state what the unknown letter or letters represent. [Solution of the equations is not required.]

- A square and a rectangle have equal areas. The length of the rectangle is greater by 3 inches and the width less by 2 inches than a side of the square. Find the dimensions of the rectangle. \[5\]
- The sum of the cubes of two numbers is 133. The sum of the squares of the two numbers exceeds the product of the two numbers by 19. Find the numbers. \[5\]
- Three numbers, of which the second is 9, are in geometric progression. If 2 is added to the first number, 3 added to the second number and 8 subtracted from the third number, the resulting numbers will be in arithmetic progression. Find the numbers. \[5\]

27 a Draw the graph of the equation \(x^2 + 3 = y\) from \(x = -3\) to \(x = +3\) inclusive. \[6\]

b Using the same set of axes as in a, draw a circle whose center is the origin and which is tangent to the curve drawn in answer to a. \[2\]

c Write the equation of the circle drawn in answer to b. \[2\]

*28 Two automobiles are traveling over the same route and in the same direction, one at an average rate of 30 miles per hour, the other at 45 miles per hour. At noon the slower automobile is 50 miles ahead of the faster.

a On the same set of axes represent graphically these facts as they occurred from noon until 6 p.m. inclusive. \[6\]

b From the graph made in answer to a determine the following:

(1) At what time will the faster automobile overtake the slower? \[2\]

(2) How far apart are the automobiles at 5 p.m.? \[2\]

* This question is based on one of the optional topics in the syllabus.