Write at top of first page of answer paper (a) name of school where you have studied, (b) number of weeks and recitations a week in (1) elementary algebra, (2) intermediate algebra.

The minimum time requirement is five recitations a week for half a school year, or the equivalent, after the completion of elementary algebra.

Answer eight questions. Full credit will not be granted unless all operations (except mental ones) necessary to find results are given; simply indicating the operations is not sufficient. Each answer should be reduced to its simplest form.

In the examination in intermediate algebra the use of the slide rule will be allowed for checking, provided all computations with tables are shown on the answer paper.

1 Find the prime factors of each of the following:
   \[ axy - cy^2 - a^2x^2 + c^2xy \]
   \[ x^2 - 7x + 6 \]
   \[ x^2 - .5x - .84 \]
   \[ 6x^2 + 33xy - 153y^2 \]
   \[ x^{10} - 243 \]

2 a Rationalize the denominator of \( \frac{3 \sqrt{3} + 2 \sqrt{5}}{3 \sqrt{3} - 2 \sqrt{5}} \) \[ \text{[7]} \]

b Find the value of the result obtained in a correct to two decimal places. \[ \text{[5.4]} \]

3 Insert between 1 and 36 such a number of arithmetic means that the sum of the resulting arithmetic progression shall be 148. \[ \text{[12.4]} \]

4 a For what values of \( m \) will the roots of \( 2x^2 + mx + 2 = 0 \) be real and equal? \[ \text{[5]} \]

b For what values of \( t \) will the roots of \( tx^2 + 4x + 1 = 0 \) be real and not equal? \[ \text{[2.5]} \]

c For what value of \( n \) will one root of \( 2x^2 - 27x + n = 0 \) be twice the other? \[ \text{[5]} \]

5 a Divide
   \[ x^{-1} + 3x^{-4} + 2x^{-1} + 4 + x^4 + x^8 \] by \( x^{-1} + x^4 \) \[ \text{[8.5]} \]

b Find the value of the dividend when \( x = 8 \). \[ \text{[4]} \]

6 a Find by the use of logarithms the principal which will amount in 8 years to \( $11,290 \) if the interest is compounded annually at 4\%, using the formula \( A = P \left(1 + r\right)^n \), where \( A \) is the amount, \( P \) the principal, \( r \) the rate per cent and \( n \) the number of years required. \[ \text{[64]} \]

b Find by the use of logarithms the value of the following expression: \[ \frac{(3.92)^5 \times 72.16}{\sqrt{654}} \] \[ \text{[6]} \]

7 If the product of three consecutive numbers is divided by each of the numbers in turn, the sum of the three quotients thus obtained is 107. Find the numbers. \[ \text{[8, 42]} \]

8 Solve the following set of equations for \( x \) and \( y \) and correctly group your answers:
   \[ x^2 - xy = 8 \]
   \[ xy + y^2 = 12 \] \[ \text{[10, 24]} \]

9 A boat can travel down a stream 15 miles and back again in 1 hour and 36 minutes. If it can travel 5 miles downstream in the same time that it requires to travel 3 miles upstream, find the rate of the boat in still water. \[ \text{[8, 42]} \]

10 The third term of a geometric progression is 6, and the fifth term is 81 times the first term. Write the first five terms of the progression. \[ \text{[12.4]} \]

11 Solve for \( x \) and check one of the results obtained:
   \[ \sqrt{x^2 - \sqrt{2x + 1}} = 2 - x \] \[ \text{[10, 24]} \]

12 Plot on the same set of axes the graph of each of the following equations and from the graphs determine the common solutions:
   \[ x^2 + y^2 = 50 \]
   \[ 2x - 3y = -5 \] \[ \text{[5.4, 5, 2]} \]