INTEGRATED ALGEBRA

The University of the State of New York
REGENTS HIGH SCHOOL EXAMINATION

INTEGRATED ALGEBRA

Wednesday, August 13, 2008 – 8:30 to 11:30 a.m., only

Print Your Name: Steve Sibold

Print Your School’s Name: JMAP

Print your name and the name of your school in the boxes above. Then turn to the last page of this booklet, which is the answer sheet for Part I. Fold the last page along the perforations and, slowly and carefully, tear off the answer sheet. Then fill in the heading of your answer sheet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. You may remove this sheet from this booklet. Any work done on this sheet of scrap graph paper will not be scored. All work should be written in pen, except graphs and drawings, which should be done in pencil.

The formulas that you may need to answer some questions in this examination are found at the end of the examination. This sheet is perforated so you may remove it from this booklet.

This examination has four parts, with a total of 39 questions. You must answer all questions in this examination. Write your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in Parts II, III, and IV directly in this booklet. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

Notice...

A graphing calculator and a straightedge (ruler) must be available for you to use while taking this examination.

The use of any communications device is strictly prohibited when taking this examination. If you use any communications device, no matter how briefly, your examination will be invalidated and no score will be calculated for you.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.
Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. For each question, write on the separate answer sheet the numeral preceding the word or expression that best completes the statement or answers the question. [60]

1 Which value of \( p \) is the solution of \( 5p - 1 = 2p + 20? \)

(1) \( \frac{19}{7} \)  
(2) \( \frac{19}{3} \)  
(3) \( 3 \)  
(4) \( 7 \)

2 The statement \( 2 + 0 = 2 \) is an example of the use of which property of real numbers?

(1) associative  
(2) additive identity  
(3) additive inverse  
(4) distributive

3 Mrs. Smith wrote "Eight less than three times a number is greater than fifteen" on the board. If \( x \) represents the number, which inequality is a correct translation of this statement?

(1) \( 3x - 8 > 15 \)  
(2) \( 3x - 8 < 15 \)  
(3) \( 8 - 3x > 15 \)  
(4) \( 8 - 3x < 15 \)
4 Which statement is true about the data set 3, 4, 5, 6, 7, 7, 10?

(1) mean = mode          (3) mean = median
(2) mean > mode          (4) mean < median

\[ \text{mean} = \frac{42}{7} = 6 \]
\[ \text{median} = 6 \]
\[ \text{mode} = 7 \]

5 Which value of \( x \) is in the solution set of the inequality \(-4x + 2 > 10\)?

(1) -2          (3) 3
(2) 2          (4) -4

\[ -4x + 2 > 10 \]
\[ -4x > 8 \]
\[ x < -2 \]

6 Factored completely, the expression \( 2x^2 + 10x - 12 \) is equivalent to

(1) \( 2(x - 6)(x + 1) \)          (3) \( 2(x + 2)(x + 3) \)
(2) \( 2(x + 6)(x - 1) \)          (4) \( 2(x - 2)(x - 3) \)

\[ 2 \left( x^2 + 5x - 6 \right) \]
\[ 2 \left( x + 6 \right) \left( x - 1 \right) \]
7 The gas tank in a car holds a total of 16 gallons of gas. The car travels 75 miles on 4 gallons of gas. If the gas tank is full at the beginning of a trip, which graph represents the rate of change in the amount of gas in the tank?

8 If $3ax + b = c$, then $x$ equals

$$\begin{align*}
(1) & \quad c - b + 3a \\
(2) & \quad c + b - 3a \\
(3) & \quad \frac{c - b}{3a} \\
(4) & \quad \frac{b - c}{3a}
\end{align*}$$
9 The length of the hypotenuse of a right triangle is 34 inches and the length of one of its legs is 16 inches. What is the length, in inches, of the other leg of this right triangle?

(1) 16  (3) 25  
(2) 18  (4) 30

10 Which equation represents a line parallel to the x-axis?

(1) $x = 5$  (3) $x = \frac{1}{3}y$
(2) $y = 10$  (4) $y = 5x + 17$

11 Sam and Odel have been selling frozen pizzas for a class fundraiser. Sam has sold half as many pizzas as Odel. Together they have sold a total of 126 pizzas. How many pizzas did Sam sell?

(1) 21  (3) 63  
(2) 42  (4) 84

12 Which ordered pair is in the solution set of the system of equations $y = -x + 1$ and $y = x^2 + 5x + 6$?

(1) $(-5,-1)$  (3) $(5,-4)$
(2) $(-5,6)$  (4) $(5,2)$
13 A swim team member performs a dive from a 14-foot-high springboard. The parabola below shows the path of her dive.

Which equation represents the axis of symmetry?

(1) \( x = 3 \)  
(2) \( y = 3 \)  
(3) \( x = 23 \)  
(4) \( y = 23 \)

14 Nicole’s aerobics class exercises to fast-paced music. If the rate of the music is 120 beats per minute, how many beats would there be in a class that is 0.75 hour long?

(1) 90  
(2) 160  
(3) 5,400  
(4) 7,200

\[ \frac{3}{4} \text{ hours} = 45 \text{ minutes} \]

\[ 120 \times 45 = 5400 \]
15 Luis is going to paint a basketball court on his driveway, as shown in the diagram below. This basketball court consists of a rectangle and a semicircle.

Which expression represents the area of this basketball court, in square feet?

(1) 80  
(2) 80 + 8π  
(3) 80 + 16π  
(4) 80 + 64π

16 John is going to line up his four golf trophies on a shelf in his bedroom. How many different possible arrangements can he make?

(1) 24  
(2) 16  
(3) 10  
(4) 4  
\[ P_4 = 4! = 24 \]
17 A rectangle has an area of 24 square units. The width is 5 units less than the length. What is the length, in units, of the rectangle?

(1) 6  (3) 3  
(2) 8  (4) 19

18 What is the value of the third quartile shown on the box-and-whisker plot below?

(1) 6  (3) 10  
(2) 8.5  (4) 12

19 When \(3g^2 - 4g + 2\) is subtracted from \(7g^2 + 5g - 1\), the difference is

(1) \(-4g^2 - 9g + 3\)  (3) \(4g^2 + 9g - 3\)  
(2) \(4g^2 + g + 1\)  (4) \(10g^2 + g + 1\)

20 Which value of \(x\) is the solution of \(\frac{2x}{5} + \frac{1}{3} = \frac{7x - 2}{15}\)?

(1) \(\frac{3}{5}\)  (3) 3  
(2) \(\frac{31}{26}\)  (4) \(\frac{7}{6}\)

\[
W(w+5) = 24 \\
W^2 + 5w - 24 = 0 \\
(w+8)(w-3) = 0 \\
w = -8, w = 3 \\
1 = w + 5 \\
1 = 3 + 5 \\
1 = 8
\]
21 Which expression represents \( \frac{25x - 125}{x^2 - 25} \) in simplest form?

(1) \( \frac{5}{x} \)

(2) \( \frac{-5}{x} \)

(3) \( \frac{25}{x - 5} \)

(4) \( \frac{25}{x + 5} \)

22 Which equation most closely represents the line of best fit for the scatter plot below?

(1) \( y = x \)

(2) \( y = \frac{2}{3}x + 1 \)

(3) \( y = \frac{3}{2}x + 4 \)

(4) \( y = \frac{3}{2}x + 1 \)

23 In a linear equation, the independent variable increases at a constant rate while the dependent variable decreases at a constant rate. The slope of this line is

(1) zero

(2) negative

(3) positive

(4) undefined
24 Which equation could be used to find the measure of one acute angle in the right triangle shown below?

- \( \sin A = \frac{4}{5} \)
- \( \tan A = \frac{5}{4} \)
- \( \cos B = \frac{5}{4} \)
- \( \tan B = \frac{4}{5} \)

25 Which ordered pair is in the solution set of the following system of inequalities?

- \( y < \frac{1}{2} x + 4 \)
- \( y \geq -x + 1 \)

- (1) (-5,3)
- (2) (0,4)
- (3) (3,-5)
- (4) (4,0)

26 What is the product of \( \frac{4x}{x-1} \) and \( \frac{x^2 - 1}{3x + 3} \) expressed in simplest form?

- \( \frac{4x}{3} \)
- \( \frac{4x^2}{3} \)
- \( \frac{4x^2}{3(x+1)} \)
- \( \frac{4(x+1)}{3} \)
27 Which expression is equivalent to \((3x^2)^3\)?

- (1) \(9x^5\)
- (2) \(9x^6\)
- (3) \(27x^5\)
- (4) \(27x^6\)

28 Ryan estimates the measurement of the volume of a popcorn container to be 282 cubic inches. The actual volume of the popcorn container is 289 cubic inches. What is the relative error of Ryan’s measurement to the nearest thousandth?

- (1) 0.024
- (2) 0.025
- (3) 0.096
- (4) 1.025

\[
\frac{289 - 282}{289} \approx 0.024
\]

29 In the diagram of \(\triangle ABC\) shown below, \(BC = 10\) and \(AB = 16\).

\[\sin A = \frac{16}{16} = 1\]

To the nearest tenth of a degree, what is the measure of the largest acute angle in the triangle?

- (1) 32.0
- (2) 38.7
- (3) 51.3
- (4) 90.0

\[
B = 180 - 90 - 38.7 = 51.3
\]
30 The faces of a cube are numbered from 1 to 6. If the cube is tossed once, what is the probability that a prime number or a number divisible by 2 is obtained?

(1) \( \frac{6}{6} \)  
(2) \( \frac{5}{6} \)  
(3) \( \frac{4}{6} \)  
(4) \( \frac{1}{6} \)

These events are not mutually exclusive

\[
P(\text{Prime}) = \frac{3}{6}
\]

\[
P(\text{Even}) = \frac{3}{6}
\]

\[
P(\text{Prime and Even}) = \frac{1}{6}
\]

\[
P(\text{Prime or Even}) = \frac{3}{6} + \frac{3}{6} - \frac{1}{6} = \frac{5}{6}
\]
Part II

Answer all questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [6]

31 In a game of ice hockey, the hockey puck took 0.8 second to travel 89 feet to the goal line. Determine the average speed of the puck in feet per second.

\[
\frac{89 \text{ ft}}{0.8 \text{ s}} = 111.25 \text{ ft/s}
\]
Brianna is using the two spinners shown below to play her new board game. She spins the arrow on each spinner once. Brianna uses the first spinner to determine how many spaces to move. She uses the second spinner to determine whether her move from the first spinner will be forward or backward.

Find the probability that Brianna will move fewer than four spaces and backward.

\[
\frac{3}{4} \times \frac{1}{2} = \frac{3}{8}
\]
Twelve players make up a high school basketball team. The team jerseys are numbered 1 through 12. The players wearing the jerseys numbered 3, 6, 7, 8, and 11 are the only players who start a game. Using set notation, list the complement of this subset.

\[\{1, 2, 4, 5, 9, 10, 12\}\]
Part III

Answer all questions in this part. Each correct answer will receive 3 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [9]

34 Express the product of $3\sqrt{20}(2\sqrt{5} - 7)$ in simplest radical form.

\[
3\sqrt{20}(2\sqrt{5} - 7)
\]

\[
6\sqrt{100} - 21\sqrt{20}
\]

\[
60 - 21\sqrt{4}\sqrt{5}
\]

\[
60 - 42\sqrt{5}
\]
35. On the set of axes below, draw the graph of \( y = 2^x \) over the interval \(-1 \leq x \leq 3\).

Will this graph ever intersect the x-axis? Justify your answer.

\[ \text{No, because for all values of } x, \quad y > 0 \]
Write an equation that represents the line that passes through the points (5,4) and (-5,0).

\[ m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 0}{5 - (-5)} = \frac{4}{10} = \frac{2}{5} \]

\[
\begin{align*}
Y &= mx + b \\
4 &= \left(\frac{2}{5}\right)5 + b \\
4 &= 2 + b \\
2 &= b
\end{align*}
\]
Part IV

Answer all questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [12]

37 The cost of 3 markers and 2 pencils is $1.80. The cost of 4 markers and 6 pencils is $2.90. What is the cost of each item? Include appropriate units in your answer.

\[
\begin{align*}
4M + 6P &= 2.90 \\
3(3M + 2P) &= 1.80 \\
9M + 6P &= 5.40 \\
4M + 6P &= 2.90 \\
\overline{5M} &= \overline{2.50} \\
5 &= .50 \\
4M + 6P &= 2.90 \\
4(.50) + 6P &= 2.90 \\
&\quad \quad \frac{6P}{6} = .90 \\
&\quad \quad \frac{P}{6} = .15
\end{align*}
\]
Twenty students were surveyed about the number of days they played outside in one week. The results of this survey are shown below.

\{6, 5, 4, 3, 0, 7, 1, 5, 4, 4, 3, 2, 2, 3, 2, 4, 3, 4, 0, 7\}

Complete the frequency table below for these data.

<table>
<thead>
<tr>
<th>Interval</th>
<th>Tally</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-1</td>
<td>III</td>
<td>3</td>
</tr>
<tr>
<td>2-3</td>
<td>IIIII</td>
<td>7</td>
</tr>
<tr>
<td>4-5</td>
<td>IIIII</td>
<td>7</td>
</tr>
<tr>
<td>6-7</td>
<td>II</td>
<td>3</td>
</tr>
</tbody>
</table>

Complete the cumulative frequency table below using these data.

<table>
<thead>
<tr>
<th>Interval</th>
<th>Cumulative Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-1</td>
<td>3</td>
</tr>
<tr>
<td>0-3</td>
<td>10</td>
</tr>
<tr>
<td>0-5</td>
<td>17</td>
</tr>
<tr>
<td>0-7</td>
<td>20</td>
</tr>
</tbody>
</table>

This question continues on the next page.
Question 38 continued

On the grid below, create a cumulative frequency histogram based on the table you made on the previous page.
39 On the set of axes below, solve the following system of equations graphically and state the coordinates of all points in the solution set.

\[
\begin{align*}
y &= x^2 + 4x - 5 \\
y &= x - 1
\end{align*}
\]