INTEGRATED ALGEBRA

The University of the State of New York
REGENTS HIGH SCHOOL EXAMINATION

INTEGRATED ALGEBRA

Thursday, June 16, 2011—1:15 to 4:15 p.m., only

Student Name: Steve Watson

School Name: IHS@PH

Print your name and the name of your school on the lines above.

A separate answer sheet for Part I has been provided to you. Follow the instructions from the proctor for completing the student information on your answer sheet.

This examination has four parts, with a total of 39 questions. You must answer all questions in this examination. Record your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in Parts II, III, and IV directly in this booklet. All work should be written in pen, except graphs and drawings, which should be done in pencil. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. The formulas that you may need to answer some questions in this examination are found at the end of the examination. This sheet is perforated so you may remove it from this booklet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. You may remove this sheet from this booklet. Any work done on this sheet of scrap graph paper will not be scored.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

Notice...
A graphing calculator and a straightedge (ruler) must be available for you to use while taking this examination.

The use of any communications device is strictly prohibited when taking this examination. If you use any communications device, no matter how briefly, your examination will be invalidated and no score will be calculated for you.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.
Part I

Answer all 30 questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. For each question, write on the separate answer sheet the numeral preceding the word or expression that best completes the statement or answers the question. [60]

1. The expression \( x^2 - 36y^2 \) is equivalent to

   (1) \( (x - 6y)(x - 6y) \)  
   (2) \( (x - 18y)(x - 18y) \)  
   (3) \( (x + 6y)(x - 6y) \)  
   (4) \( (x + 18y)(x - 18y) \)

Use this space for computations.

2. The legs of an isosceles right triangle each measure 10 inches. What is the length of the hypotenuse of this triangle, to the nearest tenth of an inch?

   (1) 6.3  
   (2) 7.1  
   (3) 14.1  
   (4) 17.1

3. The expression \( \frac{12w^3y^3}{-3w^3y^3} \) is equivalent to

   (1) \(-4w^6\)  
   (2) \(-4w^3y\)  
   (3) \(9w^6\)  
   (4) \(9w^3y\)

\[ \sqrt{200} = \frac{14.14213562 \approx C} \]

Integrated Algebra – June ’11 [2]
4 The spinner shown in the diagram below is divided into six equal sections.

![Diagram of a spinner divided into six sections with numbers 1, 2, 3, 4, 5, 6]

Probability = \( \frac{\text{# desired outcomes}}{\text{# total outcomes}} \)

Which outcome is least likely to occur on a single spin?

- (1) an odd number \( \frac{3}{6} \)
- (2) a prime number \( \frac{3}{6} \)
- (3) a perfect square \( \frac{2}{6} \)
- (4) a number divisible by 2 \( \frac{3}{6} \)

5 What are the factors of the expression \( x^2 + x - 20 \)?

- (1) \( (x + 5) \) and \( (x + 4) \)
- (2) \( (x + 5) \) and \( (x - 4) \)
- (3) \( (x - 5) \) and \( (x + 4) \)
- (4) \( (x - 5) \) and \( (x - 4) \)

Check: \( (x + 5)(x - 4) \)

\[ x^2 + 5x - 4x - 20 \]

\[ x^2 + x - 20 \]

6 What is \( 3\sqrt{250} \) expressed in simplest radical form?

- (1) \( 5\sqrt{10} \)
- (2) \( 8\sqrt{10} \)
- (3) \( 15\sqrt{10} \)
- (4) \( 75\sqrt{10} \)

Check: \( 3\sqrt{250} = 47.4341649 \)

\( 3\sqrt{250} = 47.4341649 \)
7 A survey is being conducted to determine which school board candidate would best serve the Yonkers community. Which group, when randomly surveyed, would likely produce the most bias?

- 15 employees of the Yonkers school district
- 25 people driving past Yonkers High School
- 75 people who enter a Yonkers grocery store
- 100 people who visit the local Yonkers shopping mall

8 An 8-foot rope is tied from the top of a pole to a stake in the ground, as shown in the diagram below.

If the rope forms a 57° angle with the ground, what is the height of the pole, to the nearest tenth of a foot?

- 4.4
- 6.7
- 9.5
- 12.3

9 How many different ways can five books be arranged on a shelf?

- 5
- 15
- 25
- 120

Choose 5 places for the books:

1st: 5 choices
2nd: 4 choices
3rd: 3 choices
4th: 2 choices
5th: 1 choice

Total: 5 x 4 x 3 x 2 x 1 = 120
10 What is the slope of the line passing through the points (-2,4) and (3,6)?

(1) \(-\frac{5}{2}\)  
(2) \(-\frac{3}{5}\)  
(3) \(\frac{5}{2}\)  
(4) \(\frac{3}{2}\)

(See Scrap Graph Paper for a second solution)

\[ m = \frac{y_2 - y_1}{x_2 - x_1} \]

\( m = \frac{6 - 4}{3 - (-2)} = \frac{2}{5} \)  

11 Which type of function is represented by the graph shown below?

(1) absolute value  
(2) exponential  
(3) linear  
(4) quadratic

12 Which equation represents a line parallel to the y-axis?

(1) \(y = x\)  
(2) \(y = 3\)  
(3) \(x = -y\)  
(4) \(x = -4\)
13 Melissa graphed the equation $y = x^2$ and Dave graphed the equation $y = -3x^2$ on the same coordinate grid. What is the relationship between the graphs that Melissa and Dave drew?

- Dave’s graph is wider and opens in the opposite direction from Melissa’s graph.
- Dave’s graph is narrower and opens in the opposite direction from Melissa’s graph.
- Dave’s graph is wider and is three units below Melissa’s graph.
- Dave’s graph is narrower and is three units to the left of Melissa’s graph.

The bigger the absolute value of $a$, the more narrow the parabola.
Negative values of $a$ open down. Positive open up.

14 In right triangle $ABC$ shown below, $AB = 18.3$ and $BC = 11.2$.

What is the measure of $\angle A$, to the nearest tenth of a degree?

- 31.5
- 37.7
- 52.3
- 58.5

Remember: Set calculator to degree mode.
15 The maximum height and speed of various roller coasters in North America are shown in the table below.

<table>
<thead>
<tr>
<th>Maximum Speed, in mph, ( (x) )</th>
<th>45</th>
<th>50</th>
<th>54</th>
<th>60</th>
<th>65</th>
<th>70</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum Height, in feet, ( (y) )</td>
<td>63</td>
<td>80</td>
<td>105</td>
<td>118</td>
<td>141</td>
<td>107</td>
</tr>
</tbody>
</table>

Which graph represents a correct scatter plot of the data?

16 Which set of ordered pairs represents a function?

1. \( \{(0,4), (2,4), (2,5)\} \)
2. \( \{(6,0), (5,0), (4,0)\} \)
3. \( \{(4,1), (6,2), (6,3), (5,0)\} \)
4. \( \{(0,4), (1,4), (0,5), (1,5)\} \)

A function has one and only one value of \( y \) for each value of \( x \).
17 A hiker walked 12.8 miles from 9:00 a.m. to noon. He walked an additional 17.2 miles from 1:00 p.m. to 6:00 p.m. What is his average rate for the entire walk, in miles per hour?

(1) 3.75  
(2) 3.86  
(3) 4.27  
(4) 7.71

\[ \frac{12.8 \text{ miles}}{3 \text{ hours}} + \frac{17.2 \text{ miles}}{5 \text{ hours}} = \frac{30 \text{ miles}}{8 \text{ hours}} \]

\[ \frac{30 \text{ miles}}{8 \text{ hours}} = 3.75 \text{ miles/hour} \]

18 Which ordered pair is a solution of the system of equations \( y = x + 3 \) and \( y = x^2 - x \)?

(1) (6,9)  
(2) (3,1)  
(3) (3,-1)  
(4) (2,5)

\[ x + 3 = x^2 - x \]
\[ 0 = x^2 - 2x - 3 \]
\[ 0 = (x+1)(x-3) \]
\[ x = -1 \]
\[ x = 3 \]

19 Which verbal expression can be represented by \( 2(x - 5) \)?

(1) 5 less than 2 times \( x \)  
(2) 2 multiplied by \( x \) less than 5  
(3) twice the difference of \( x \) and 5  
(4) the product of 2 and \( x \), decreased by 5

\[ 2(x - 5) = 2x - 10 \]

20 The dimensions of a rectangle are measured to be 12.2 inches by 11.8 inches. The actual dimensions are 12.3 inches by 11.9 inches. What is the relative error, to the nearest ten-thousandth, in calculating the area of the rectangle?

(1) 0.0168  
(2) 0.0167  
(3) 0.0165  
(4) 0.0164

\[ \text{Relative Error} = \frac{\text{Measured} - \text{Actual}}{\text{Actual}} \]
\[ = \frac{143.96 - 146.37}{146.37} = -0.0164 \]

\[ \text{Answer: } 0.0164 \]
21 An example of an algebraic expression is
- \( y = mx + b \)
- \( 3x + 4y - 7 \)
- \( 2x + 3y \leq 18 \)
- \( (x + y)(x - y) = 25 \)

22 A study showed that a decrease in the cost of carrots led to an increase in the number of carrots sold. Which statement best describes this relationship?
- (1) positive correlation and a causal relationship
- (2) negative correlation and a causal relationship
- (3) positive correlation and not a causal relationship
- (4) negative correlation and not a causal relationship

23 Given: \( A = \{3, 6, 9, 12, 15\} \)
\( B = \{2, 4, 6, 8, 10, 12\} \)
What is the union of sets \( A \) and \( B \)?
- (1) \( \{6\} \)
- (2) \( \{6, 12\} \)
- (3) \( \{2, 3, 4, 6, 8, 9, 10, 12, 15\} \)
- (4) \( \{2, 3, 4, 6, 8, 9, 10, 12, 15\} \)

24 The value of a car purchased for $20,000 decreases at a rate of 12% per year. What will be the value of the car after 3 years?
- (1) $12,800.00
- (2) $13,629.44
- (3) $17,600.00
- (4) $28,098.56

\[ \text{Initial Value} \times (100\% - 12\%)^3 = \text{value} \]
\[ 20,000 \times (0.88)^3 = \text{value} \]
\[ \#13,629.44 = \text{value} \]
25 For which set of values of $x$ is the algebraic expression $\frac{x^2 - 16}{x^2 - 4x - 12}$ undefined?

(1) $\{-6, 2\}$  (3) $\{-4, 4\}$
(2) $\{-4, 3\}$  (4) $\{-2, 6\}$

$$x^2 - 4x - 12 = 0$$

$$(x + 2)(x - 6) = 0$$

$x + 2 = 0$  $x - 6 = 0$

$x = -2$  $x = 6$

26 Michael is 25 years younger than his father. The sum of their ages is 53. What is Michael’s age?

$\bullet$ 14  (3) 28
(2) 25  (4) 39

$M + 25 = F \Rightarrow M - F = -25$
$M + F = 53 \Rightarrow M + F = 53$

$2M = 28$

$M = 14$

27 What is the product of $(6 \times 10^3)$, $(4.6 \times 10^5)$, and $(2 \times 10^{-2})$ expressed in scientific notation?

(1) $55.2 \times 10^6$  (3) $55.2 \times 10^7$
$\bullet$ $5.52 \times 10^7$  (4) $5.52 \times 10^{10}$

$6 \times 10^3$
$4.6 \times 10^5$
$2 \times 10^{-2}$

$\frac{(6 \times 4.6 \times 2) \times 10^{(5+5-2)}}{55.2 \times 10^6}$

$5.52 \times 10^7$

28 Which notation describes $\{1, 2, 3\}$?

(1) $\{x | 1 \leq x < 3, \text{ where } x \text{ is an integer} \}$  $\Rightarrow$ 3

$\bullet$ $\{x | 0 < x \leq 3, \text{ where } x \text{ is an integer} \}$

(3) $\{x | 1 < x < 3, \text{ where } x \text{ is an integer} \}$  $\Rightarrow$ 3

(4) $\{x | 0 \leq x \leq 3, \text{ where } x \text{ is an integer} \}$  includes 0
29 What is \( \frac{7}{12x} - \frac{y}{6x^2} \) expressed in simplest form?

(1) \( \frac{7 - y}{6x} \)

(2) \( \frac{7 - y}{12x - 6x^2} \)

(3) \( -\frac{7y}{12x^2} \)

\( \frac{7}{12x} - \frac{y}{6x^2} \)

(30) When \( 5x + 4y \) is subtracted from \( 5x - 4y \), the difference is

(1) 0

(2) 10x

(3) 8y

(\( 5x - 4y \))

\( -(5x + 4y) \)
Part II

Answer all 3 questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [6]

31 The area of a rectangle is represented by $x^2 - 5x - 24$. If the width of the rectangle is represented by $x - 8$, express the length of the rectangle as a binomial.

Factors multiply to -24
Factors sum to -5

$x^2 - 5x - 24$

$\checkmark$

Check

$$(x+3)(x-8)$$

$x^2 - 8x + 3x - 24$

$x^2 - 5x - 24 \checkmark$
A method for solving $5(x - 2) - 2(x - 5) = 9$ is shown below. Identify the property used to obtain each of the two indicated steps.

$$5(x - 2) - 2(x - 5) = 9$$

(1) $5x - 10 - 2x + 10 = 9$  
(1) **Distributive**

(2) $5x - 2x - 10 + 10 = 9$  
(2) **Commutative**

$$3x + 0 = 9$$

$$3x = 9$$

$$x = 3$$
33 State the equation of the axis of symmetry and the coordinates of the vertex of the parabola graphed below.
Part III

Answer all 3 questions in this part. Each correct answer will receive 3 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [9]

34 Given the following list of students' scores on a quiz: 5, 12, 7, 15, 20, 14, 7, 12, 14, 15, 20

With three more scores: 8, 10, 10, 15, 17, 18, 23

Determine the median of these scores.

Median = 12

Determine the mode of these scores.

Mode = 7

The teacher decides to adjust these scores by adding three points to each score. Explain the effect, if any, that this will have on the median and mode of these scores.

Both the median and the mode will increase by 3 points.

New median = 15
New mode = 10
Chelsea has $45 to spend at the fair. She spends $20 on admission and $15 on snacks. She wants to play a game that costs $0.65 per game. Write an inequality to find the maximum number of times, $x$, Chelsea can play the game.

\[
\frac{45 - (20 + 15)}{0.65} \geq x
\]

Using this inequality, determine the maximum number of times she can play the game.

\[
\frac{45 - (20 + 15)}{0.65} \geq x
\]

\[
\frac{45 - 35}{0.65} \geq x
\]

\[
\frac{10}{0.65} \geq x
\]

\[
15.3846 \geq x
\]

\[15\] is the maximum number of times she can play the game.
A plastic storage box in the shape of a rectangular prism has a length of \( x + 3 \), a width of \( x - 4 \), and a height of 5.

Represent the surface area of the box as a trinomial in terms of \( x \).

Surface area equal 2 times each side

\[
S^2 = \left[ 2(x+3)(x-4) \right] + \left[ 2(5)(x+3) \right] + \left[ 2(5)(x-4) \right]
\]

\[
= 2(x^2 - 4x + 3x - 12) + 10(x+3) + 10(x-4)
\]

\[
= 2x^2 - 8x + 6x - 24 + 10x + 30 + 10x - 40
\]

\[
= 2x^2 - 2x - 24 + 10x + 30 + 10x - 40
\]

\[
= 2x^2 + 18x - 34
\]
37 Solve algebraically for \( x \):

\[
\frac{3}{4} = \frac{-x - 11}{4x} + \frac{1}{2x}
\]

\[
\frac{3}{4} = \frac{-x - 11 + 2}{4x} = \frac{-x - 9}{4x}
\]

\[
12x = -4x - 36
\]

\[
16x = -36
\]

\[
x = \frac{-36}{16} = \frac{-9}{4}
\]
An outfit Jennifer wears to school consists of a top, a bottom, and shoes. Possible choices are listed below.

- **Tops:** T-shirt, blouse, sweater
- **Bottoms:** jeans, skirt, capris
- **Shoes:** flip-flops, sneakers

List the sample space or draw a tree diagram to represent all possible outfits consisting of one type of top, one type of bottom, and one pair of shoes.

![Tree Diagram]

Determine how many different outfits contain jeans and flip-flops.

3

Determine how many different outfits do not include a sweater.

12
Solve the following system of inequalities graphically on the set of axes below.

\[ 3x + y < 7 \]
\[ y \geq \frac{2}{3}x - 4 \]

State the coordinates of a point in the solution set.

\(0, 0\)

Check:

\[3x + y \leq 7\]
\[0 + 0 \leq 7\]
\[y \geq \frac{2}{3}x - 4\]
\[0 \geq \frac{2}{3}(0) - 4\]
\[0 \geq -4\]
Scrap Graph Paper — This sheet will not be scored.

#10

\[ (3, 6) \]

\[ (-2, 4) \]

\[ \frac{-1}{2} \]

\[ \frac{1}{5} \]

\[ \frac{2}{3} \]

\[ \frac{1}{6} \]

\[ \frac{1}{5} \]