INTEGRATED ALGEBRA

The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

INTEGRATED ALGEBRA

Tuesday, June 17, 2008 – 9:15 a.m. to 12:15 p.m., only

Print Your Name:  

Print Your School’s Name:  

Print your name and the name of your school in the boxes above.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. You may remove this sheet from this booklet. Any work done on this sheet of scrap graph paper will not be scored.

The formulas that you may need to answer some questions in this examination are found at the end of the examination. This sheet is perforated so you may remove it from this booklet.

This examination has four parts, with a total of 39 questions. You must answer all questions in this examination. Record your answers to the Part I multiple-choice questions, using a #2 pencil on the separate answer sheet provided to you. Write your answers to the questions in Parts II, III, and IV directly in this test booklet. All work for Parts II, III, and IV should be written in pen, except graphs and drawings, which should be done in pencil. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

Notice:
A graphing calculator and a straightedge (ruler) must be available for you to use while taking this examination.

The use of any communications device is strictly prohibited when taking this examination. If you use any communications device, no matter how briefly, your examination will be invalidated and no score will be calculated for you.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.
Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. For each question, record your answer using a #2 pencil on the separate answer sheet provided to you. [60]

1. Which graph represents a linear function?

   ![Graphs]

   - (1) Line: Yes, Function: Yes
   - (2) Line: No, Function: Yes
   - (3) Line: No, Function: No
   - (4) Line: Yes, Function: Yes

Use this space for computations.
2 A spinner is divided into eight equal regions as shown in the diagram below.

Which event is most likely to occur in one spin?
(1) The arrow will land in a green or white area.
(2) The arrow will land in a green or black area.
(3) The arrow will land in a yellow or black area.
(4) The arrow will land in a yellow or green area.

\[
P(\text{event}) = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}}
\]

\[
P(\text{white}) = \frac{2}{8} = \frac{1}{4}
\]

\[
P(\text{black}) = \frac{1}{8}
\]

\[
P(\text{green}) = \frac{3}{8}
\]

\[
P(\text{yellow}) = \frac{3}{8}
\]

Total \(\frac{8}{8} = 1\) for mutually exclusive events

Green or White = \(\frac{3}{8} + \frac{3}{8} = \frac{6}{8} = \frac{3}{4}\)

Green or Black = \(\frac{3}{8} + \frac{1}{8} = \frac{4}{8} = \frac{1}{2}\)

Yellow or Black = \(\frac{3}{8} + \frac{1}{8} = \frac{4}{8} = \frac{1}{2}\)

Yellow or Green = \(\frac{3}{8} + \frac{3}{8} = \frac{6}{8} = \frac{3}{4}\)

3 A school wants to add a coed soccer program. To determine student interest in the program, a survey will be taken. In order to get an unbiased sample, which group should the school survey?
(1) every third student entering the building ⇒ best choice
(2) every member of the varsity football team ⇒ biased sample
(3) every member in Ms. Zimmer’s drama classes ⇒ biased sample
(4) every student having a second-period French class ⇒ biased sample

4 Factored, the expression \(16x^2 - 25y^2\) is equivalent to
(1) \((4x - 5y)(4x + 5y)\)
(2) \((4x - 5y)(4x - 5y)\)
(3) \((8x - 5y)(8x + 5y)\)
(4) \((8x - 5y)(8x - 5y)\)
5 There is a negative correlation between the number of hours a student watches television and his or her social studies test score. Which scatter plot below displays this correlation?

![Scatter plots](image)

6 Jack bought 3 slices of cheese pizza and 4 slices of mushroom pizza for a total cost of $12.50. Grace bought 3 slices of cheese pizza and 2 slices of mushroom pizza for a total cost of $8.50. What is the cost of one slice of mushroom pizza?

(1) $1.50  
(2) $2.00  
(3) $3.00  
(4) $3.50

\[
3C + 4M = 12.50 \\
3C + 2M = 8.50
\]

(Subtract) \[
3C + 4M = 12.50 \\
3C + 2M = 8.50 \]

\[
2M = 4.00
\]

(Divide by 2) \[
\frac{2M}{2} = \frac{4}{2} \]

\[
M = 2.00
\]
7 What is the product of \(-3x^2y\) and \((5xy^2 + xy)\)?

\[
\begin{array}{ll}
(1) & -15x^3y^3 - 3x^2y^2 \\
(2) & -15x^3y^3 - 3x^2y \\
(3) & -15x^2y^2 - 3x^3y \\
(4) & -15x^2y^2 + xy
\end{array}
\]

8 The bowling team at Lincoln High School must choose a president, vice president, and secretary. If the team has 10 members, which expression could be used to determine the number of ways the officers could be chosen?

\[
\begin{array}{ll}
(1) & _3P_{10} \\
(2) & _7P_3 \\
(3) & _{10}P_3 \\
(4) & _{10}P_7
\end{array}
\]

9 Lenny made a cube in technology class. Each edge measured 1.5 cm. What is the volume of the cube in cubic centimeters?

\[
\begin{array}{ll}
(1) & 2.25 \\
(2) & 3.375 \\
(3) & 9.0 \\
(4) & 13.5
\end{array}
\]

10 Which ordered pair is a solution to the system of equations \(y = x\) and \(y = x^2 - 2\)?

\[
\begin{array}{ll}
(1) & (-2, -2) \\
(2) & (-1, 1) \\
(3) & (0, 0) \\
(4) & (2, 2)
\end{array}
\]
11 What are the vertex and the axis of symmetry of the parabola shown in the diagram below?

(1) The vertex is \((-2,-3)\), and the axis of symmetry is \(x = -2\).

(2) The vertex is \((-2,-3)\), and the axis of symmetry is \(y = -2\).

(3) The vertex is \((-3,-2)\), and the axis of symmetry is \(y = -2\).

(4) The vertex is \((-3,-2)\), and the axis of symmetry is \(x = -2\).

12 Pam is playing with red and black marbles. The number of red marbles she has is three more than twice the number of black marbles she has. She has 42 marbles in all. How many red marbles does Pam have?

(1) 13
(2) 15
(3) 29
(4) 33

\[ R = 2B + 3 \]
\[ R + B = 42 \]
\[ 2B + 3 + B = 42 \]
\[ 3B + 3 = 42 \]
\[ 3B = 39 \]
\[ B = 13 \]
\[ R = 2B + 3 \]
\[ R = 2(13) + 3 = 29 \]
13 What is half of $2^6$?

(1) $1^3$
(2) $1^6$

$$\frac{2^6}{2} = 2^{6-1} = 2^5$$

14 Which equation represents a line that is parallel to the line $y = -4x + 5$?

(1) $y = -4x + 3$
(2) $y = -\frac{1}{4}x + 5$
(3) $y = \frac{1}{4}x + 3$
(4) $y = 4x + 5$

Parallel lines have same slope and different y-intercept.

15 What is the product of $\frac{x^2 - 1}{x + 1}$ and $\frac{x + 3}{3x - 3}$ expressed in simplest form?

(1) $x$
(2) $\frac{x}{3}$
(3) $x + 3$
(4) $\frac{x + 3}{3}$

$$\left(\frac{x^2 - 1}{x + 1}\right) \left(\frac{x + 3}{3x - 3}\right) \Rightarrow \left(\frac{(x+1)(x-1)}{x+1}\right) \left(\frac{x+3}{3(x-1)}\right) = \frac{x+3}{3}$$
16. The center pole of a tent is 8 feet long, and a side of the tent is 12 feet long as shown in the diagram below.

If a right angle is formed where the center pole meets the ground, what is the measure of angle A to the nearest degree?

(1) 34  (2) 42  (3) 48  (4) 56

17. Which value of $x$ makes the expression $\frac{x+4}{x-3}$ undefined?

(1) -4  (2) -3  (3) 3  (4) 0

18. Consider the set of integers greater than -2 and less than 6. A subset of this set is the positive factors of 5. What is the complement of this subset?

(1) \{0, 2, 3, 4\}  (2) \{-1, 0, 2, 3, 4\}  (3) \{-2, -1, 0, 2, 3, 4, 6\}  (4) \{-2, -1, 0, 1, 2, 3, 4, 5, 6\}
19 Which data set describes a situation that could be classified as qualitative?

(1) the elevations of the five highest mountains in the world
(2) the ages of presidents at the time of their inauguration
(3) the opinions of students regarding school lunches
(4) the shoe sizes of players on the basketball team

Use this space for computations.

20 What is the slope of the line that passes through the points (-6,1) and (4,-4)?

(1) -2
(2) 2
(3) \(-\frac{1}{2}\)
(4) \(\frac{1}{2}\)

Use Reference Sheet

\[ m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} \]

21 Students in a ninth grade class measured their heights, \(h\), in centimeters. The height of the shortest student was 155 cm, and the height of the tallest student was 190 cm. Which inequality represents the range of heights?

(1) \(155 < h < 190\)
(2) \(155 \leq h \leq 190\)
(3) \(h \geq 155\) or \(h \leq 190\)
(4) \(h > 155\) or \(h < 190\)

\[ 155 \leq \text{height} \leq 190 \]

height of shortest student is included

height of tallest student is included.
The table below shows a cumulative frequency distribution of runners' ages.

<table>
<thead>
<tr>
<th>Cumulative Frequency Distribution of Runners' Ages</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age Group</td>
</tr>
<tr>
<td>-----------</td>
</tr>
<tr>
<td>Twenties</td>
</tr>
<tr>
<td>Thirties</td>
</tr>
<tr>
<td>Forties</td>
</tr>
<tr>
<td>Fifties</td>
</tr>
<tr>
<td>Sixties</td>
</tr>
</tbody>
</table>

According to the table, how many runners are in their forties?

(1) 25  (2) 10  (3) 7  (4) 6

23 Mr. Turner bought $x$ boxes of pencils. Each box holds 25 pencils. He left 3 boxes of pencils at home and took the rest to school. Which expression represents the total number of pencils he took to school?

(1) $22x$  (2) $25x - 3$  (3) $25 - 3x$  (4) $25x - 75$

24 Which expression represents $\frac{2x^2 - 12x}{x - 6}$ in simplest form?

(1) 0  (2) $2x$  (3) $4x$  (4) $2x + 2$
25 Don placed a ladder against the side of his house as shown in the diagram below.

Which equation could be used to find the distance, x, from the foot of the ladder to the base of the house?

(1) \( x = 20 - 19.5 \)
(2) \( x = 20^2 - 19.5^2 \)
(3) \( x = \sqrt{20^2 - 19.5^2} \)
(4) \( x = \sqrt{20^2 + 19.5^2} \)

Use this space for computations.

Pythagorean Theorem

\[ a^2 + b^2 = c^2 \]

\[ x^2 + (19.5)^2 = (20)^2 \]

\[ x^2 = (20)^2 - (19.5)^2 \]

\[ x = \sqrt{(20)^2 - (19.5)^2} \]

26 Which value of x is a solution of \( \frac{5}{x} = \frac{x + 13}{6} \)?

(1) -2
(2) -3
(3) -10
(4) -15

Cross Multiply

\[ 5(6) = x(x + 13) \]

\[ 30 = x^2 + 13x \]

\[ 0 = x^2 + 13x - 30 \]

Factor

\[ 0 = (x + 15)(x - 2) \]

\[ x + 15 = 0 \quad x - 2 = 0 \]

\[ x = -15 \quad x = 2 \]

27 Mrs. Ayer is painting the outside of her son’s toy box, including the top and bottom. The toy box measures 3 feet long, 1.5 feet wide, and 2 feet high. What is the total surface area she will paint?

(1) 9.0 ft\(^2\)
(2) 13.5 ft\(^2\)
(3) 22.5 ft\(^2\)
(4) 27.0 ft\(^2\)

Area of Top = 1.5 \times 3 = 4.5
Area of Bottom = 1.5 \times 3 = 4.5
Area of End \#1 = 1.5 \times 2 = 3
Area of End \#2 = 1.5 \times 2 = 3
Area of Side \#1 = 3 \times 2 = 6
Area of Side \#2 = 3 \times 2 = 6

Total Area = 27
28 What is \( \frac{\sqrt{32}}{4} \) expressed in simplest radical form?

1. \( \sqrt{2} \)
2. \( 4\sqrt{2} \)
3. \( \sqrt{8} \)
4. \( \frac{\sqrt{8}}{2} \)

Use this space for computations.

\[ \frac{\sqrt{32}}{4} = \frac{\sqrt{2} \cdot \sqrt{16}}{4} = \frac{4\sqrt{2}}{4} = \sqrt{2} \]

29 Consider the graph of the equation \( y = ax^2 + bx + c \), when \( a \neq 0 \). If \( a \) is multiplied by 3, what is true of the graph of the resulting parabola?

1. The vertex is 3 units above the vertex of the original parabola.
2. The new parabola is 3 units to the right of the original parabola.
3. The new parabola is wider than the original parabola.
4. The new parabola is narrower than the original parabola.

When the absolute value of \( a \) increases, the parabola gets more narrow.

30 Kathy plans to purchase a car that depreciates (loses value) at a rate of 14% per year. The initial cost of the car is $21,000. Which equation represents the value, \( v \), of the car after 3 years?

1. \( v = 21,000(0.14)^3 \)
2. \( v = 21,000(0.86)^3 \)
3. \( v = 21,000(1.14)^3 \)
4. \( v = 21,000(0.86)(3) \)

At the end of the first year, the car is worth 100% - 14% of its purchase price.

\[ (21,000) \times (0.86) \times \frac{0.86}{1 \text{ year}} \]

If we want the car value after 3 years, we have to use \( (0.86)(0.86)(0.86) = (0.86)^3 \)

1st year 2nd year 3rd year All 3 years

After 3 years, the car is worth \( (21,000)(0.86)^3 \)
Part II

Answer all questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [6]

31 Tom drove 290 miles from his college to home and used 23.2 gallons of gasoline. His sister, Ann, drove 225 miles from her college to home and used 15 gallons of gasoline. Whose vehicle had better gas mileage? Justify your answer.

\[
\text{Gas mileage} = \frac{\text{miles}}{\text{gallons}}
\]

Tom's Car: \(\frac{290}{23.2} = 12.5 \text{ miles/gallon}\)

Ann's Car: \(\frac{225}{15} = 15 \text{ miles/gallon}\)

Answer: Ann's car gets better mileage. See above reasoning.
A designer created the logo shown below. The logo consists of a square and four quarter-circles of equal size.

Area of Square

\[(3+3)(3+3) = 36\]

\[6 \times 6 = 36\]

Each corner = \(\frac{1}{4}\) circle

\[4 \times \frac{1}{4}\text{circle} = 1 \text{ circle} \]

Express, in terms of \(\pi\), the exact area, in square inches, of the shaded region.

\[36 - 9\pi \text{ square inches} \]
Maureen tracks the range of outdoor temperatures over three days. She records the following information.

Day 1: \(-20^\circ F\) to \(40^\circ F\)

Day 2: \(0^\circ F\) to \(50^\circ F\)

Day 3: \(-23^\circ F\) to \(45^\circ F\)

Express the intersection of the three sets as an inequality in terms of temperature, \(t\).

Answer: \(0 \leq t \leq 40\)
Part III

Answer all questions in this part. Each correct answer will receive 3 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [9]

34 Peter begins his kindergarten year able to spell 10 words. He is going to learn to spell 2 new words every day.

Write an inequality that can be used to determine how many days, $d$, it takes Peter to be able to spell at least 75 words.

\[ 10 + 2d \geq 75 \]

Use this inequality to determine the minimum number of whole days it will take for him to be able to spell at least 75 words.

\[
\begin{align*}
10 + 2d & \geq 75 \\
-10 & \quad -10 \\
2d & \geq 65 \\
\text{Divide by 2} & \quad \frac{2d}{2} \geq \frac{65}{2} \\
\end{align*}
\]

\[ d \geq 32.5 \]

Peter needs at least 33 days.

Answer
The Hudson Record Store is having a going-out-of-business sale. CDs normally sell for $18.00. During the first week of the sale, all CDs will sell for $15.00.

Written as a fraction, what is the rate of discount?

\[
\frac{\text{New Price} - \text{Original Price}}{\text{Original Price}} = \text{rate of price change}
\]

\[
\frac{15 - 18}{18} = \frac{-3}{18}
\]

The direction is negative, meaning a discount.

The rate is \( \frac{-3}{18} \)

What is this rate expressed as a percent? Round your answer to the nearest hundredth of a percent.

\[
\frac{-3}{18} = -0.1666\overline{6} = 16.66\overline{6}\% = 16.67\%
\]

Answer

During the second week of the sale, the same CDs will be on sale for 25\% off the original price. What is the price of a CD during the second week of the sale?

\[
18 \times (100\% - 75\%) = x
\]

\[
18 \times (0.75) = x
\]

\[
13.5 = x
\]

\( \#13.50 \)

Answer
36 Graph the equation \( y = x^2 - 2x - 3 \) on the accompanying set of axes.
Using the graph, determine the roots of the equation \( x^2 - 2x - 3 = 0 \).

Roots are the \( x \) values of the \( x \)-axis intercepts.

\[
\begin{array}{c|c|c}
 x & y = x^2 - 2x - 3 & y \\
-2 & y = (-2)^2 - 2(-2) - 3 & 5 \\
-1 & y = (-1)^2 - 2(-1) - 3 & 0 \\
0 & y = (0)^2 - 2(0) - 3 & -3 \\
1 & y = (1)^2 - 2(1) - 3 & -4 \\
2 & y = (2)^2 - 2(2) - 3 & -3 \\
3 & y = (3)^2 - 2(3) - 3 & 0 \\
4 & y = (4)^2 - 2(4) - 3 & 5 \\
\end{array}
\]

The roots are \(-1\) and \(3\).

\((-1,0)\) Coordinate
\((3,0)\) Coordinate

The roots are -1 and 3

Answer

Check
\[
y = x^2 - 2x - 3 \\
y = (-1)^2 - 2(-1) - 3 \Rightarrow y = 0 \checkmark
\]

\[
y = (3)^2 - 2(3) - 3 \\
y = 0 \checkmark
\]
Part IV

Answer all questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [12]

37 A contractor needs 54 square feet of brick to construct a rectangular walkway. The length of the walkway is 15 feet more than the width. \( l = w + 15 \) \( \text{Area} = 54 \)

Write an equation that could be used to determine the dimensions of the walkway. Solve this equation to find the length and width, in feet, of the walkway.

\[
\text{Area} = \text{Length} \times \text{Width} \\
54 = (w + 15) \times (w) \\
54 = w^2 + 15w \\
0 = w^2 + 15w - 54 \\
0 = (w + 18)(w - 3) \\
w + 18 = 0 \quad w - 3 = 0 \\
w = -18 \quad w = 3
\]

Factors of 54:
\[
\begin{array}{c|c}
54 & 1 \\
27 & 2 \\
18 & 3 \\
9 & 6
\end{array}
\]

The sidewalk is 3 feet wide 18 feet long. Answer

The sidewalk is 3 feet wide 18 feet long. Answer
Sophie measured a piece of paper to be 21.7 cm by 28.5 cm. The piece of paper is actually 21.6 cm by 28.4 cm.

Determine the number of square centimeters in the area of the piece of paper using Sophie's measurements.

\[
\text{Area} = \text{length} \times \text{width} = 21.7 \times 28.5 = 618.48 \text{ cm}^2
\]

Determine the number of square centimeters in the actual area of the piece of paper.

\[
\text{Area} = 21.6 \times 28.4 = 613.44 \text{ cm}^2
\]

Determine the relative error in calculating the area. Express your answer as a decimal to the nearest thousandth.

\[
\frac{\text{Measured} - \text{Actual}}{\text{Actual}} = \frac{618.48 - 613.44}{613.44} = \frac{5.04}{613.44} = 0.008
\]

Sophie does not think there is a significant amount of error. Do you agree or disagree? Justify your answer.

\[
\text{I agree with Sophie. 0.008 relative error is less than 1%, which is good enough for measuring paper. It might not be good enough for shooting rockets at the moon.}
\]
39 The prices of seven race cars sold last week are listed in the table below.

<table>
<thead>
<tr>
<th>Price per Race Car</th>
<th>Number of Race Cars</th>
</tr>
</thead>
<tbody>
<tr>
<td>$126,000</td>
<td>1</td>
</tr>
<tr>
<td>$140,000</td>
<td>2</td>
</tr>
<tr>
<td>$180,000</td>
<td>1</td>
</tr>
<tr>
<td>$400,000</td>
<td>2</td>
</tr>
<tr>
<td>$819,000</td>
<td>1</td>
</tr>
</tbody>
</table>

What is the mean value of these race cars, in dollars?

Mean = \( \text{average} \)

\[
\text{Mean} = \frac{126,000 + 140,000 + 140,000 + 180,000 + 400,000 + 400,000 + 819,000}{7} = \frac{2,205,000}{7} = \$315,000 \text{ answer}
\]

What is the median value of these race cars, in dollars?

Median = middle value in ordered array

\(126,000, 140,000, 140,000, 180,000, 400,000, 400,000, 819,000\)

\(\$180,000 \text{ answer}\)

State which of these measures of central tendency best represents the value of the seven race cars. Justify your answer.

The median is the best measure of central tendency for these 7 cars because it is not influenced by the outlier as much as the mean is influenced by the outlier.
Reference Sheet

**Trigonometric Ratios**

\[
\sin A = \frac{\text{opposite}}{\text{hypotenuse}}
\]
\[
\cos A = \frac{\text{adjacent}}{\text{hypotenuse}}
\]
\[
\tan A = \frac{\text{opposite}}{\text{adjacent}}
\]

**Area**

trapezoid \( A = \frac{1}{2}h(b_1 + b_2) \)

**Volume**

cylinder \( V = \pi r^2 h \)

**Surface Area**

rectangular prism \( SA = 2lw + 2hw + 2lh \)

cylinder \( SA = 2\pi r^2 + 2\pi rh \)

**Coordinate Geometry**

\[
m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}
\]