The University of the State of New York
REGENTS HIGH SCHOOL EXAMINATION

INTEGRATED ALGEBRA

Thursday, January 30, 2014 — 9:15 a.m. to 12:15 p.m., only

Student Name: \\

School Name: \\

The possession or use of any communications device is strictly prohibited when taking this examination. If you have or use any communications device, no matter how briefly, your examination will be invalidated and no score will be calculated for you.

Print your name and the name of your school on the lines above.

A separate answer sheet for Part I has been provided to you. Follow the instructions from the proctor for completing the student information on your answer sheet.

This examination has four parts, with a total of 39 questions. You must answer all questions in this examination. Record your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in Parts II, III, and IV directly in this booklet. All work should be written in pen, except graphs and drawings, which should be done in pencil. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. The formulas that you may need to answer some questions in this examination are found at the end of the examination. This sheet is perforated so you may remove it from this booklet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. You may remove this sheet from this booklet. Any work done on this sheet of scrap graph paper will not be scored.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

Notice...
A graphing calculator and a straightedge (ruler) must be available for you to use while taking this examination.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.
Part I

Answer all 30 questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. For each question, write on the separate answer sheet the numeral preceding the word or expression that best completes the statement or answers the question. [60]

1 An example of an equation is

(1) $2x^2 - 4x + 12$
(2) $|x - 6|$

No equal signs

2 The greatest common factor of $3m^2n + 12mn^2$ is

(1) $3n$
(2) $3m$
(3) $3mn$
(4) $3mn^2$

An equation is two expressions with an equal sign between them.

3 Jeremy is hosting a Halloween party for 80 children. He will give each child at least one candy bar. If each bag of candy contains 18 candy bars, which inequality can be used to determine how many bags, $c$, Jeremy will need to buy?

(1) $18c \geq 80$
(2) $18c \leq 80$
(3) $\frac{c}{18} \geq 80$
(4) $\frac{c}{18} \leq 80$

Let $c$ represent # of bags

$18c \geq 80$

Check for 5 bags ($\frac{18(5)}{80} \geq 80$)

4 Which statement regarding biased sampling is false?

(1) Online sampling is biased because only the people who happen to visit the web site will take the survey.

(2) A radio call-in survey is biased because only people who feel strongly about the topic will respond.

(3) A survey handed to every third person leaving a library is biased because everyone leaving the library was not asked to participate.

(4) Asking for experts to take a survey is biased because they may have particular knowledge of the topic.
5 Which relation is not a function?

(1) \(\{(2,4), (1,2), (0,0), (-1,2), (-2,4)\}\)
(2) \(\{(2,4), (1,1), (0,0), (-1,1), (-2,4)\}\)
(3) \(\{(2,2), (1,1), (0,0), (-1,1), (-2,2)\}\)
(4) \(\{(2,2), (1,1), (0,0), (1,-1), (2,-2)\}\)

When \(x = 2\), there is more than one value for \(y\).

6 What is an equation of the line that passes through the point \((-2, -8)\) and has a slope of 3?

\[ y = 3x - 2 \]

(2) \(y = 3x + 2\)  
(3) \(y = 3x - 2\)  
(4) \(y = 3x + 2\)

7 A figure consists of a square and a semicircle, as shown in the diagram below.

If the length of a side of the square is 6, what is the area of the shaded region?

(1) \(36 - 3\pi\)  
(2) \(36 - 4.5\pi\)  
(3) \(36 - 6\pi\)  
(4) \(36 - 9\pi\)

Integrated Algebra – January ’14
8. The box-and-whisker plot shown below represents the number of magazine subscriptions sold by members of a club.

Which statistical measures do points B, D, and E represent, respectively, in order?

1. minimum, median, maximum
2. first quartile, median, third quartile
3. first quartile, third quartile, maximum
4. median, third quartile, maximum

9. What is the slope of a line represented by the equation \(2y = x - 4\)?

1. 1
2. \(\frac{1}{2}\)
3. -1
4. \(-\frac{1}{2}\)

10. What is the solution of the system of equations below?

\[
\begin{align*}
2x + 3y &= 7 \\
x + y &= 3
\end{align*}
\]

1. (1, 2)
2. (2, 1)
3. (4, -1)
4. (4, 1)

\[
\begin{align*}
\text{Eq. 1: } & \quad 2x + 3y = 7 \\
\text{Eq. 2: } & \quad x + y = 3
\end{align*}
\]
11 The graph below illustrates the number of acres used for farming in Smalltown, New York, over several years.

Using a line of best fit, approximately how many acres will be used for farming in the 5th year?

(1) 0  (3) 300
(2) 200  (4) 400

12 When \(16x^3 - 12x^2 + 4x\) is divided by \(4x\), the quotient is

(1) \(12x^2 - 8x\)  (3) \(4x^2 - 3x\)
(2) \(12x^2 - 8x + 1\)  (4) \(4x^2 - 3x + 1\)

13 The width of a rectangle is 4 less than half the length. If \(l\) represents the length, which equation could be used to find the width, \(w\)?

(1) \(w = \frac{1}{2}(4 - l)\)  (2) \(w = \frac{1}{2}l - 4\)
(3) \(w = \frac{1}{2}(l - 4)\)  (4) \(w = 4 - \frac{1}{2}l\)
14 Which data can be classified as quantitative?

(1) favorite stores at which you shop  
(2) U.S. Representatives and their home states  
(3) sales tax rate in each New York county  
(4) opinion of a freshman on the color of Paul's shirt

15 Two cubes with sides numbered 1 through 6 were rolled 20 times. Their sums are recorded in the table below.

<table>
<thead>
<tr>
<th>4</th>
<th>9</th>
<th>8</th>
<th>9</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>4</td>
<td>6</td>
<td>12</td>
<td>10</td>
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<tr>
<td>8</td>
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<td>9</td>
<td>11</td>
<td>10</td>
</tr>
<tr>
<td>8</td>
<td>7</td>
<td>9</td>
<td>3</td>
<td>5</td>
</tr>
</tbody>
</table>

What is the empirical probability of rolling a sum of 9?  
(1) \( \frac{4}{20} \)  
(2) \( \frac{5}{20} \)  
(3) \( \frac{4}{36} \)  
(4) \( \frac{5}{36} \)

16 What is the vertex of the graph of the equation \( y = 3x^2 + 6x + 1 \)?

(1) \((-1,-2)\)  
(2) \((-1,10)\)  
(3) \((1,-2)\)  
(4) \((1,10)\)

17 The length and width of a rectangle are 48 inches and 40 inches. To the nearest inch, what is the length of its diagonal?  

(1) 27  
(2) 62  
(3) 88  
(4) 90

\[ a^2 + b^2 = c^2 \]
\[ 48^2 + 40^2 = c^2 \]
\[ 2304 + 1600 = c^2 \]
\[ 3904 = c^2 \]
\[ \sqrt{3904} = c \]
\[ 62.48199741 \]

\text{round to nearest inch = 62 inches}
18 Which graph represents the solution set of $2x - 5 < 3$?

- Open circle means 4 is not included.
- Closed circle means 4 is included.

$2x - 5 < 3$

$\frac{2x}{2} < \frac{3}{2}$

$x < 4$

19 Jonathan drove to the airport to pick up his friend. A rainstorm forced him to drive at an average speed of 45 mph, reaching the airport in $\frac{3}{4}$ hours. He drove back home at an average speed of 55 mph. How long, to the nearest tenth of an hour, did the trip home take him?

(1) 2.0 hours  (3) 2.8 hours
(2) 2.5 hours  (4) 3.7 hours

20 The expression $\frac{2n}{5} + \frac{3n}{2}$ is equivalent to

(1) $\frac{5n}{7}$  
(2) $\frac{6n^2}{10}$  
(3) $\frac{19n}{10}$  
(4) $\frac{7n}{10}$

\[
\left(\frac{2}{2}\right) \frac{2n}{5} + \left(\frac{5}{2}\right) \frac{3n}{5} = \frac{4n}{10} + \frac{15n}{10} = \frac{19n}{10}
\]
21 When \( x = 4 \), the value of \( 2x^0 + x! \) is

(1) 24  
(2) 25  
(3) 26  
(4) 28

Use this space for computations.

\[
\begin{align*}
\text{Factorial:} & \quad 4! = 4 \times 3 \times 2 \times 1 \\
& \quad 4! = 24 \\
\end{align*}
\]

22 Which graph represents the solution of \( 2y + 6 > 4x \)?

\[y = \frac{2x - 3}{2}\]

Check easy point in solution area. \((0, 0)\)

\[2y + 6 > 4x\]

\[2(0) + 6 > 4(0)\]

\[6 > 0 \quad \checkmark\]
23 Which graph represents the exponential decay of a radioactive element?

(1) Looks like a Quadratic
(2) Increases Exponentially
(3) Linear
(4) Decreases Exponentially

24 Which fraction represents \( \frac{x^2 - 25}{x^2 - x - 20} \) expressed in simplest form? (Differences of perfect squares: \( a^2 - b^2 = (a+b)(a-b) \))

(1) \( \frac{5}{4} \)
(2) \( \frac{x-5}{x-4} \)
(3) \( \frac{x+5}{x+4} \)
(4) \( \frac{25}{x+20} \)

25 If \( abx - 5 = 0 \), what is \( x \) in terms of \( a \) and \( b \)?

(1) \( x = \frac{5}{ab} \)
(2) \( x = -\frac{5}{ab} \)
(3) \( x = 5 - ab \)
(4) \( x = ab - 5 \)
26 Given:

\[ U = \{ x \mid 0 < x < 10 \text{ and } x \text{ is an integer} \} \]

\[ S = \{ x \mid 0 < x < 10 \text{ and } x \text{ is an odd integer} \} \]

The complement of set \( S \) within the universal set \( U \) is

(1) \{0, 2, 4, 6, 8, 10\}  
(2) \{2, 4, 6, 8, 10\}  
(3) \{0, 2, 4, 6\}  
(4) \{2, 4, 6, 8\}

27 The roots of the equation \( 2x^2 - 8x = 0 \) are

(1) \(-2\) and \(2\)  
(2) \(0, -2, \) and \(2\)  
(3) \(0\) and \(-4\)  
(4) \(0, 4, \) and \(-4\)

28 Which equation illustrates the multiplicative inverse property?

(1) \( a \cdot 1 = a \)  
(2) \( a \cdot 0 = 0 \)  
(3) \( \frac{1}{a} \cdot a = 1 \)  
(4) \( -a \cdot \frac{-1}{a} = a^2 \)

29 What is the result when \( 4x^2 - 17x + 36 \) is subtracted from \( 2x^2 - 5x + 25 \)?

(1) \( 6x^2 - 22x + 61 \)  
(2) \( 2x^2 - 12x + 11 \)  
(3) \( -2x^2 - 22x + 61 \)  
(4) \( -2x^2 + 12x + 11 \)

30 Julie has three children whose ages are consecutive odd integers. If \( x \) represents the youngest child's age, which expression represents the sum of her children's ages?

(1) \( 3x + 3 \)  
(2) \( 3x + 4 \)  
(3) \( 3x + 5 \)  
(4) \( 3x + 6 \)
Express \( \frac{\sqrt{84}}{2\sqrt{3}} \) in simplest radical form.

\[
\frac{\sqrt{84}}{2\sqrt{3}} \quad \Rightarrow \quad \frac{\sqrt{4} \cdot \sqrt{21}}{2\sqrt{3}} \quad \Rightarrow \quad \frac{2 \cdot \sqrt{21}}{2\sqrt{3}} \quad \Rightarrow \quad \frac{\sqrt{21}}{\sqrt{3}}
\]

\[
\Rightarrow \quad \sqrt{\frac{21}{3}} = \sqrt{7}
\]
The cumulative frequency table below shows the number of minutes 31 students spent text messaging on a weekend.

<table>
<thead>
<tr>
<th>Text-Use Interval (minutes)</th>
<th>Cumulative Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>41-50</td>
<td>2</td>
</tr>
<tr>
<td>51-60</td>
<td>5</td>
</tr>
<tr>
<td>61-70</td>
<td>10</td>
</tr>
<tr>
<td>71-80</td>
<td>19</td>
</tr>
<tr>
<td>81-90</td>
<td>31</td>
</tr>
</tbody>
</table>

Determine which 10-minute interval contains the median. Justify your choice.

The median is in the 71-80 interval.

There are 31 students. The middle student is #16

→ 15 students < middle student + 15 students ≥ #16

Student #16 cannot be in the first three intervals, because these three intervals have a total of only 10 students.

Student #16 cannot be in the last interval because the last interval only has 12 students.

Therefore, student #16 must be in the 71-80 interval.

10 students: 11, 12, 13, 14, 15, 16, 17, 18, 19, 20

12 students: Median
Kirsten invested $1000 in an account at an annual interest rate of 3%. She made no deposits or withdrawals on the account for 5 years. The interest was compounded annually. Find the balance in the account, to the nearest cent, at the end of 5 years.

**Easy Way**

Initial Investment: $1000

Return of Principal + Interest:

\[
1000 \times 1.03^5 = 1159.274074
\]

Round to nearest $: $1159.27

**Harder Way**

First year interest: $1000 \times 0.03 = 30

End of first year: $1000 + 30 = 1030$

Second year interest: $1030 \times 0.03 = 30.90$

End of second year: $1030 + 30.90 = 1060.90$

Third year interest: $1060.90 \times 0.03 = 31.827$

End of third year: $1060.90 + 31.827 = 1092.727$

Fourth year interest: $1092.727 \times 0.03 = 32.78181$

End of fourth year: $1092.727 + 32.78181 = 1125.50881$

Fifth year interest: $1125.50881 \times 0.03 = 33.7652643$

End of fifth year: $1125.50881 + 33.7652643 = 1159.274074$

Round to nearest cent: $1159.27
34 Graph and label the functions \( y = |x| \) and \( y = |2x| \) on the set of axes below.

Explain how increasing the coefficient of \( x \) affects the graph of \( y = |x| \).

Increasing the coefficient of \( x \) will cause the graph to become narrower.
Terry estimated the length of the edge of a cube to be 5 cm. The actual length of the side is 5.2 cm. Find the relative error of the surface area of the cube, to the nearest thousandth.

\[
\text{Difference between actual and measured} = \frac{\text{Error}}{\text{Actual}}
\]

Actual

The area of one side is \(5^2\)
The surface area of a cube is six times the area of one side.

\[
6 \times 5^2 = \text{Total Surface Area}
\]

Actual Surface Area
\[
6(5.2)^2 = \text{Actual}
\]
\[
162.24 = \text{Actual}
\]

Measured Surface Area
\[
6(5)^2 = \text{Measured}
\]
\[
150 = \text{Measured}
\]

\[
\frac{162.24 - 150}{162.24} = \frac{12.24}{162.24} = 0.0754437
\]

Round to nearest thousandth

0.075
From the top of an apartment building, the angle of depression to a car parked on the street below is 38 degrees, as shown in the diagram below. The car is parked 80 feet from the base of the building. Find the height of the building, to the nearest tenth of a foot.

\[
\tan 38^\circ = \frac{\text{opposite}}{80}
\]

\[80 \times \tan 38^\circ = \text{opposite}
\]

\[62.50285012 = \text{opposite}
\]

Round to nearest tenth

62.5 feet
Part IV

Answer all 3 questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [12]

37 On the set of axes below, solve the following system of equations graphically for all values of x and y. State the coordinates of all the solutions.

\[ y = x^2 + 4x - 5 \]
\[ y = 2x + 3 \]

Strategy: Put in graphing calculator
38 Solve algebraically for all values of $x$: \[ \frac{3}{x+5} = \frac{2x}{x^2 - 8} \]

\[ 3(x^2 - 8) = (x+5)(2x) \]

\[ 3x^2 - 24 = 2x^2 + 10x \]

\[ -2x^2 - 2x^2 \]

\[ x^2 - 24 = 10x \]

\[ -10x \]

\[ x^2 - 10x - 24 = 0 \]

\[ (x + \_)(x - \_) = 0 \]

\[ (x + 2)(x - 12) = 0 \]

\[ x + 2 = 0 \quad x - 12 = 0 \]

\[ x = -2 \quad x = 12 \]

Check $(-2)$

\[ \frac{3}{-2+5} = \frac{2(-2)}{(-2)^2 - 8} \]

\[ \frac{3}{3} = \frac{-4}{-4} \]

\[ 1 = 1 \quad \checkmark \]

Check $12$

\[ \frac{3}{12+5} = \frac{2(12)}{(12)^2 - 8} \]

\[ \frac{3}{17} = \frac{24}{136} \]

\[ 3(136) = 17(24) \]

\[ 408 = 408 \quad \checkmark \]
Doug has four baseball caps: one tan, one blue, one red, and one green. He also has three jackets: one blue, one red, and one white. Draw a tree diagram or list a sample space to show all possible outfits consisting of one baseball cap and one jacket.

Sample Space

<table>
<thead>
<tr>
<th>Cap</th>
<th>Jacket</th>
</tr>
</thead>
<tbody>
<tr>
<td>tan</td>
<td>blue</td>
</tr>
<tr>
<td>tan</td>
<td>red</td>
</tr>
<tr>
<td>tan</td>
<td>white</td>
</tr>
<tr>
<td>blue</td>
<td>blue</td>
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<tr>
<td>green</td>
<td>red</td>
</tr>
<tr>
<td>green</td>
<td>white</td>
</tr>
</tbody>
</table>

Find the number of Doug's outfits that consist of a cap and a jacket that are different colors.

\[
\text{Ten}
\]

\[
12 - 2 = 10
\]

On Spirit Day, Doug wants to wear either green or white, his school's colors. Find the number of his outfits from which he can choose.

\[
\text{Six}
\]