

The University of the State of New York
REGENTS HIGH SCHOOL EXAMINATION

GEOMETRY

Thursday, August 13, 2015 — 8:30 to 11:30 a.m., only

Student Name: Mr. Sibol

School Name: JMAP

The possession or use of any communications device is strictly prohibited when taking this examination. If you have or use any communications device, no matter how briefly, your examination will be invalidated and no score will be calculated for you.

Print your name and the name of your school on the lines above.

A separate answer sheet for Part I has been provided to you. Follow the instructions from the proctor for completing the student information on your answer sheet.

This examination has four parts, with a total of 38 questions. You must answer all questions in this examination. Record your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in Parts II, III, and IV directly in this booklet. All work should be written in pen, except for graphs and drawings, which should be done in pencil. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc.

The formulas that you may need to answer some questions in this examination are found at the end of the examination. This sheet is perforated so you may remove it from this booklet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. You may remove this sheet from this booklet. Any work done on this sheet of scrap graph paper will *not* be scored.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

Notice...

A graphing calculator, a straightedge (ruler), and a compass must be available for you to use while taking this examination.

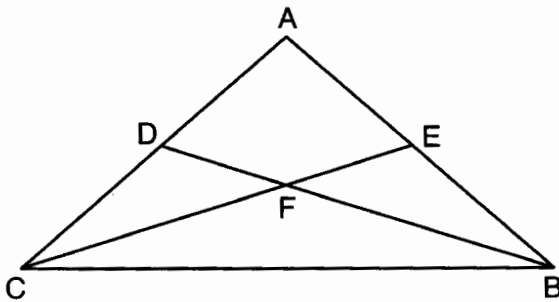
DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.

Part I

Answer all 28 questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. For each statement or question, choose the word or expression that, of those given, best completes the statement or answers the question. Record your answers on your separate answer sheet. [56]

Use this space for computations.

- 1 In $\triangle ABC$ shown below with \overline{ADC} , \overline{AEB} , \overline{CFE} , and \overline{BFD} , $\triangle ACE \cong \triangle ABD$.

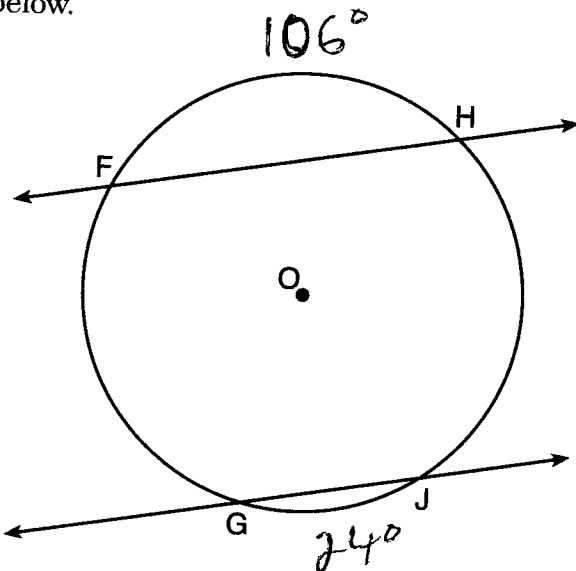


Which statement must be true?

- (1) $\angle ACF \cong \angle BCF$ (3) $\angle BCD \cong \angle ABD$
(2) $\angle DAE \cong \angle DFE$ (4) $\angle AEF \cong \angle ADF$
- 2 In a circle whose equation is $(x - 1)^2 + (y + 3)^2 = 9$, the coordinates of the center and length of its radius are
- (1) $(1, -3)$ and $r = 81$ (3) $(1, -3)$ and $r = 3$
(2) $(-1, 3)$ and $r = 81$ (4) $(-1, 3)$ and $r = 3$

3 Parallel secants \overleftrightarrow{FH} and \overleftrightarrow{GJ} intersect circle O , as shown in the diagram below.

Use this space for computations.



$$\frac{360 - (106 + 24)}{2} = \frac{230}{2}$$

If $m\widehat{FH} = 106$ and $m\widehat{GJ} = 24$, then $m\widehat{FG}$ equals

- (1) 106
 (2) 115
 (3) 130
 (4) 156

4 What are the coordinates of P' , the image of point $P(x,y)$ after translation $T_{4,4}$?

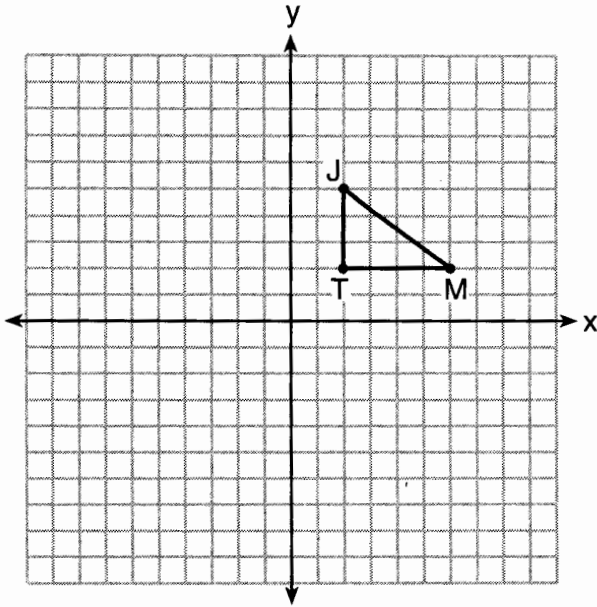
- (1) $(x - 4, y - 4)$
 (2) $(x + 4, y + 4)$
 (3) $(4x, 4y)$
 (4) $(4, 4)$

5 The statement " $x > 5$ or $x < 3$ " is false when x is equal to

- (1) 1
 (2) 2
 (3) 7
 (4) 4

Use this space for computations.

6 Triangle JTM is shown on the graph below.



Which transformation would result in an image that is *not* congruent to $\triangle JTM$?

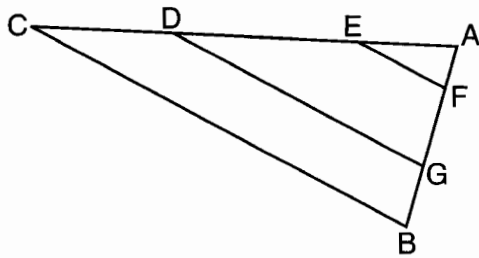
(1) $r_{y=x}$

(3) $T_{0,-3}$

(2) R_{90°

(4) D_2

7 In the diagram below of $\triangle ABC$, with \overline{CDEA} and \overline{BGFA} , $\overline{EF} \parallel \overline{DG} \parallel \overline{CB}$.



Which statement is *false*?

(1) $\frac{AC}{AD} = \frac{AB}{AG}$

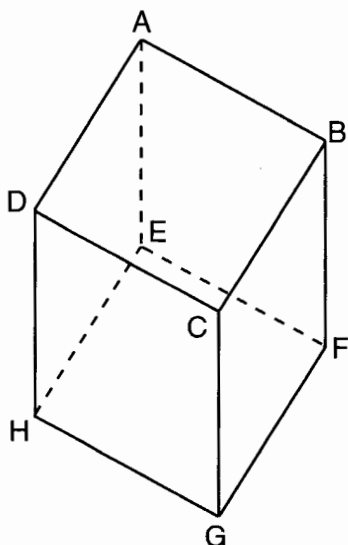
(3) $\frac{AE}{AD} = \frac{EC}{AC}$

(2) $\frac{AE}{AF} = \frac{AC}{AB}$

(4) $\frac{BG}{BA} = \frac{CD}{CA}$

8 Which pair of edges is *not* coplanar in the cube shown below?

Use this space for computations.



- (1) \overline{EH} and \overline{CD} (3) \overline{DH} and \overline{AE}
 (2) \overline{AD} and \overline{FG} (4) \overline{AB} and \overline{EF}

9 What is an equation of the line that passes through the point $(-2, 1)$ and is parallel to the line whose equation is $4x - 2y = 8$?

(1) $y = \frac{1}{2}x + 2$

(3) $y = 2x + 5$

(2) $y = \frac{1}{2}x - 2$

(4) $y = 2x - 5$

$\frac{-A}{B} = \frac{-4}{-2} = 2$

$1 = 2(-2) + b$
 $5 = b$

10 In $\triangle JKL$, $\overline{JL} \cong \overline{KL}$. If $m\angle J = 58$, then $m\angle L$ is

(1) 61

(3) 116

(2) 64

(4) 122

$180 - 2(58) = 64$

11 The corresponding medians of two similar triangles are 8 and 20. If the perimeter of the larger triangle is 45, what is the perimeter of the *smaller* triangle?

(1) 14

(3) 33

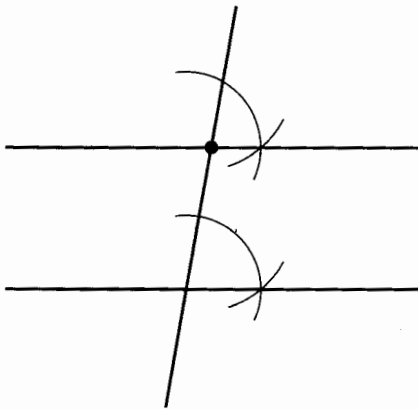
(2) 18

(4) 37

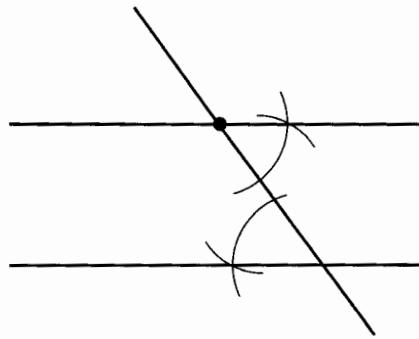
$45 \cdot \frac{8}{20} = 18$

Use this space for
computations.

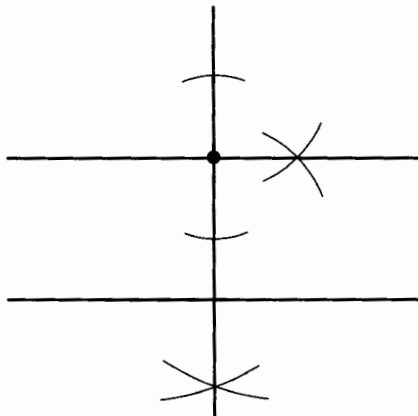
- 12 Which construction of parallel lines is justified by the theorem “If two lines are cut by a transversal to form congruent alternate interior angles, then the lines are parallel”?



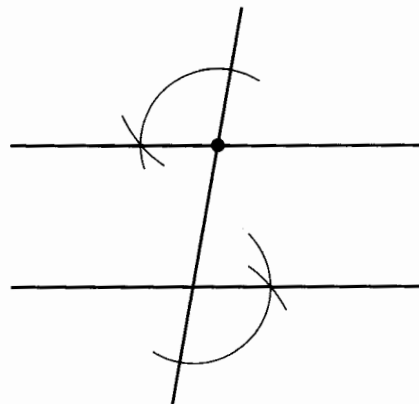
(1)



(3)



(2)



(4)

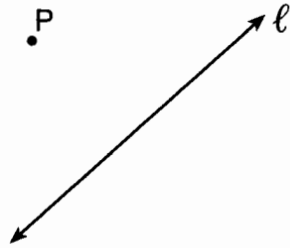
- 13 Given: “If a polygon is a triangle, then the sum of its interior angles is 180° .”

What is the contrapositive of this statement?

- (1) “If the sum of the interior angles of a polygon is not 180° , then it is not a triangle.”
- (2) “A polygon is a triangle if and only if the sum of its interior angles is 180° .”
- (3) “If a polygon is not a triangle, then the sum of the interior angles is not 180° .”
- (4) “If the sum of the interior angles of a polygon is 180° , then it is a triangle.”

Use this space for computations.

14 In the diagram below, point P is not on line ℓ .



How many distinct planes that contain point P are also perpendicular to line ℓ ?

- (1) 1
(2) 2
(3) 0
(4) an infinite amount

15 The image of $\triangle ABC$ after the transformation $r_{y\text{-axis}}$ is $\triangle A'B'C'$. Which property is *not* preserved?

- (1) distance
(2) orientation
(3) collinearity
(4) angle measure

16 The equations $y = 2x + 3$ and $y = -x^2 - x + 1$ are graphed on the same set of axes. The coordinates of a point in the solution of this system of equations are

- (1) (0,1)
(2) (1,5)
(3) (-1,-2)
(4) (-2,-1)

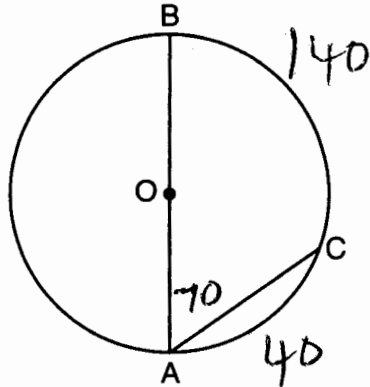
$$2x + 3 = -x^2 - x + 1$$
$$x^2 + 3x + 2 = 0$$
$$(x+2)(x+1) = 0$$
$$x = -2 \quad -1$$
$$y = 2(-2) + 3$$
$$= -4 + 3$$
$$= -1$$

17 Which quadrilateral has diagonals that are always perpendicular bisectors of each other?

- (1) square
(2) rectangle
(3) trapezoid
(4) parallelogram

Use this space for
computations.

- 18 As shown in the diagram below, \overline{AB} is a diameter of circle O , and chord \overline{AC} is drawn.



If $m\angle BAC = 70$, then $m\widehat{AC}$ is

- (1) 40
(2) 70
(3) 110
(4) 140

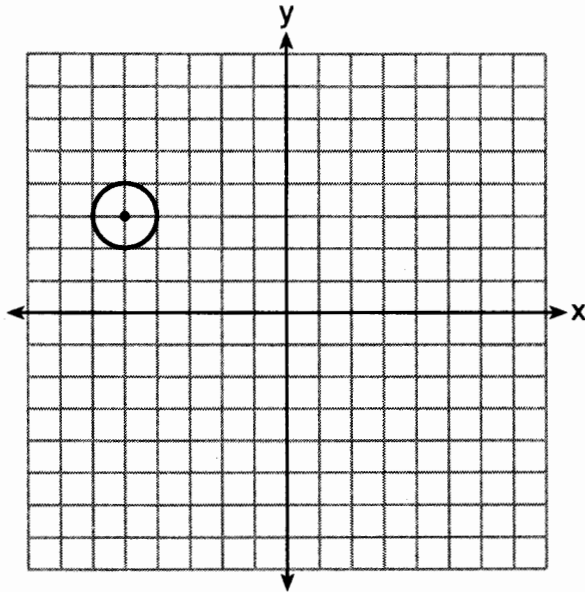
- 19 In parallelogram $JKLM$, $m\angle L$ exceeds $m\angle M$ by 30 degrees. What is the measure of $\angle J$?

- (1) 75°
(2) 105°
(3) 165°
(4) 195°

$$\begin{aligned}L + L - 30 &= 180 \\2L &= 210 \\L &= 105\end{aligned}$$

Use this space for
computations.

20 Which equation represents the circle shown in the graph below?



- (1) $(x - 5)^2 + (y + 3)^2 = 1$
- (2) $(x + 5)^2 + (y - 3)^2 = 1$
- (3) $(x - 5)^2 + (y + 3)^2 = 2$
- (4) $(x + 5)^2 + (y - 3)^2 = 2$

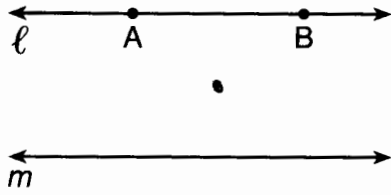
21 What is the measure of each interior angle in a regular octagon?

- (1) 108°
- (2) 135°
- (3) 144°
- (4) 1080°

$$\frac{(n-2)180}{n}$$
$$\frac{(8-2)(180)}{8}$$
$$135$$

Use this space for
computations.

- 22 Points A and B are on line ℓ , and line ℓ is parallel to line m , as shown in the diagram below.



How many points are in the same plane as ℓ and m and equidistant from ℓ and m , and also equidistant from A and B ?

- (1) 1 (3) 3
(2) 2 (4) 0
- 23 A carpenter made a storage container in the shape of a rectangular prism. It is 5 feet high and has a volume of 720 cubic feet. He wants to make a second container with the same height and volume as the first one, but in the shape of a triangular prism. What will be the number of square feet in the area of the base of the new container?

- (1) 36 (3) 144
(2) 72 (4) 288

$$V = Bh$$
$$720 = 5B$$
$$144 = B$$

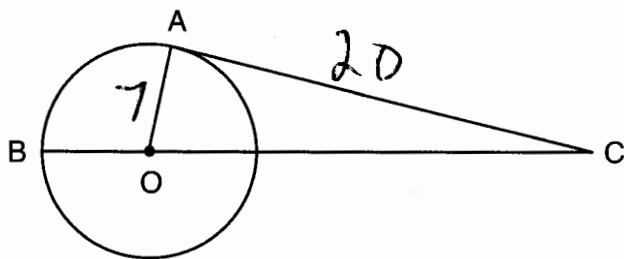
$$AC < BC < AB$$

- 24 In $\triangle ABC$, $m\angle B < m\angle A < m\angle C$. Which statement is *false*?

- (1) $AC > BC$ (3) $AC < AB$
(2) $BC > AC$ (4) $BC < AB$

Use this space for computations.

- 25 In the diagram below of circle O with radius \overline{OA} , tangent \overline{CA} and secant \overline{COB} are drawn.



(Not drawn to scale)

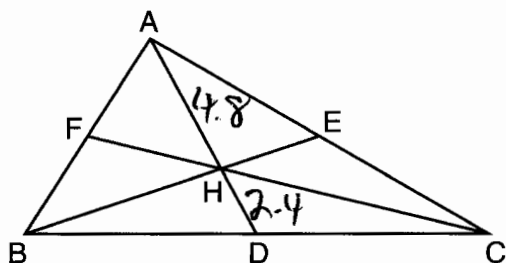
If $AC = 20$ cm and $OA = 7$ cm, what is the length of \overline{OC} , to the nearest centimeter?

- (1) 19
(2) 20

- (3) 21
(4) 27

$$\sqrt{20^2 + 7^2} = \sqrt{449} \approx 21$$

- 26 In the diagram below of $\triangle ABC$, point H is the intersection of the three medians.



If \overline{DH} measures 2.4 centimeters, what is the length, in centimeters, of \overline{AD} ?

- (1) 3.6
(2) 4.8

- (3) 7.2
(4) 9.6

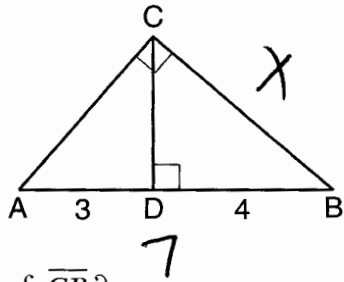
$$4.8 + 2.4 = 7.2$$

Use this space for computations.

27 Which set of numbers could be the lengths of the sides of an isosceles triangle?

- (1) {1, 1, 2} not a Δ (3) {3, 4, 5} not isosceles
(2) {3, 3, 5} (4) {4, 4, 9} not a Δ

28 In the diagram below of right triangle ABC , \overline{CD} is the altitude to hypotenuse \overline{AB} , $AD = 3$, and $DB = 4$.



What is the length of \overline{CB} ?

- (1) $2\sqrt{3}$ (3) $2\sqrt{7}$
(2) $\sqrt{21}$ (4) $4\sqrt{3}$

$$x^2 = 4 \cdot 7$$
$$x = \sqrt{28}$$
$$x = 2\sqrt{7}$$

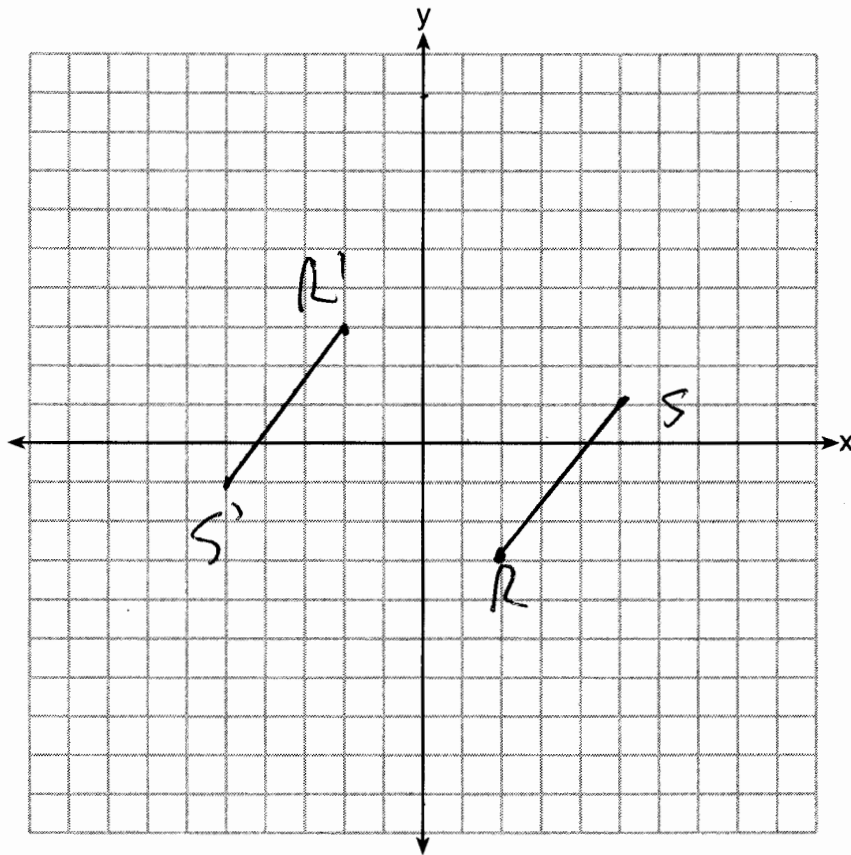
Part II

Answer all 6 questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [12]

29 The image of \overline{RS} after a reflection through the origin is $\overline{R'S'}$. If the coordinates of the endpoints of \overline{RS} are $R(2,-3)$ and $S(5,1)$, state and label the coordinates of R' and S' .

[The use of the set of axes below is optional.]

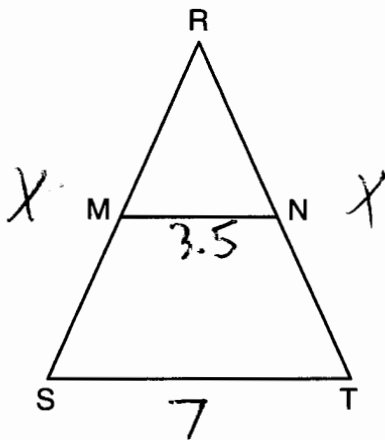
$R'(-2,3)$ $S'(-5,-1)$



- 30 A paper container in the shape of a right circular cone has a radius of 3 inches and a height of 8 inches. Determine and state the number of cubic inches in the volume of the cone, in terms of π .

$$\begin{aligned} V &= \frac{1}{3} \pi \cdot 3^2 \cdot 8 \\ &= 24\pi \end{aligned}$$

- 31 In isosceles triangle RST shown below, $\overline{RS} \cong \overline{RT}$, M and N are midpoints of \overline{RS} and \overline{RT} , respectively, and \overline{MN} is drawn. If $MN = 3.5$ and the perimeter of $\triangle RST$ is 25, determine and state the length of \overline{NT} .



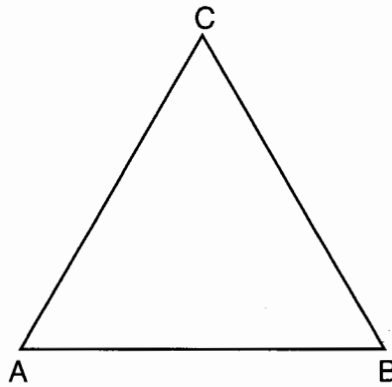
$$2x + 7 = 25$$

$$2x = 18$$

$$x = 9$$

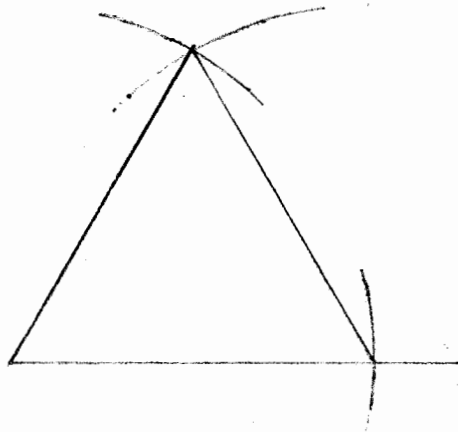
$$\overline{NT} = 4.5$$

32 In the diagram below, $\triangle ABC$ is equilateral.



Using a compass and straightedge, construct a new equilateral triangle congruent to $\triangle ABC$ in the space below.

[Leave all construction marks.]



33 Write an equation of the line that is perpendicular to the line whose equation is $\frac{2y}{2} = \frac{3x}{2} + \frac{12}{2}$ and that passes through the origin.

$$M = \frac{3}{2}$$

$$M_{\perp} = -\frac{2}{3}$$

$$y = -\frac{2}{3}x$$

34 Rectangle $KLMN$ has vertices $K(0,4)$, $L(4,2)$, $M(1,-4)$, and $N(-3,-2)$. Determine and state the coordinates of the point of intersection of the diagonals.

$$\left(\frac{0+1}{2}, \frac{4+(-4)}{2} \right)$$

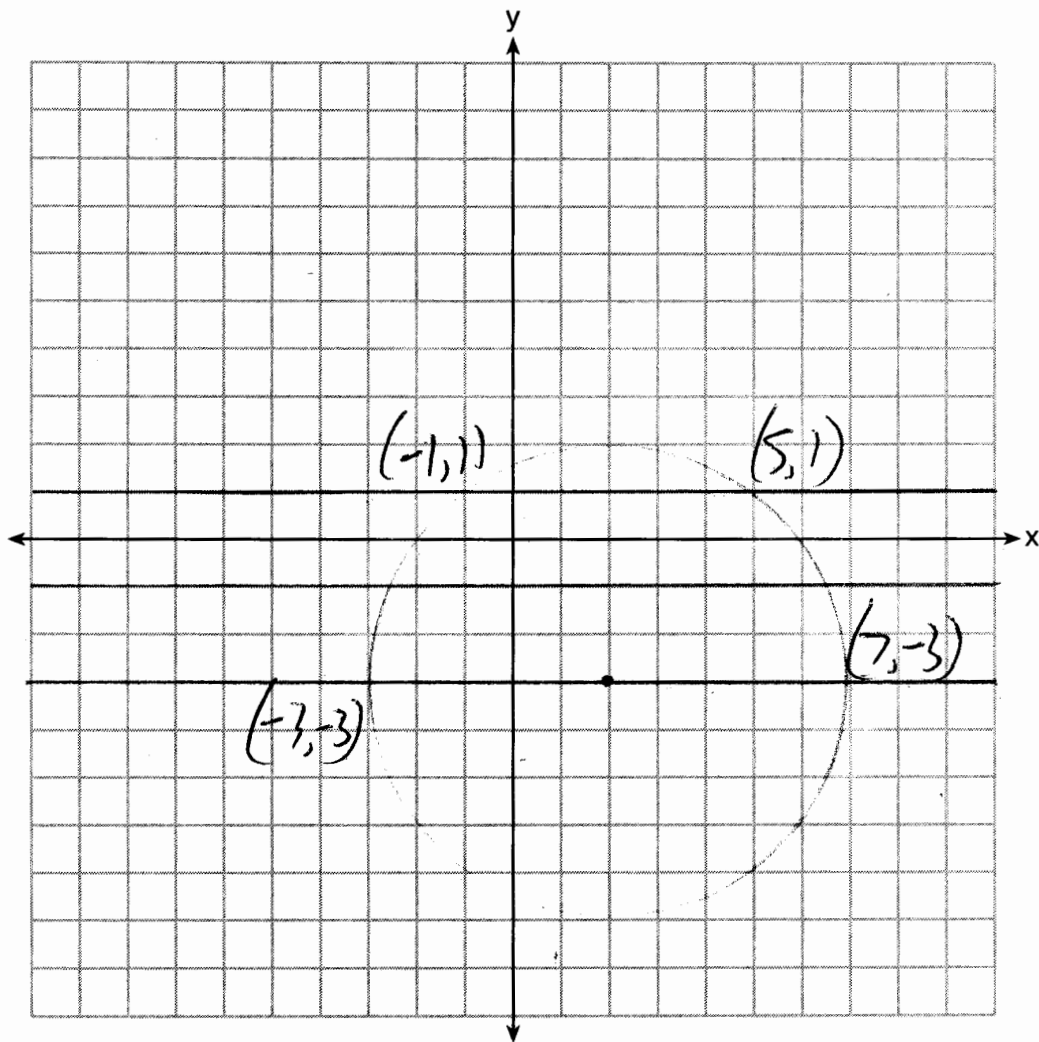
$$\left(\frac{1}{2}, 0 \right)$$

Part III

Answer all 3 questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [12]

35 On the set of axes below, graph the locus of points 5 units from the point $(2, -3)$ and the locus of points 2 units from the line whose equation is $y = -1$.

State the coordinates of all points that satisfy *both* conditions.



36 If \overline{AB} is defined by the endpoints $A(4,2)$ and $B(8,6)$, write an equation of the line that is the perpendicular bisector of \overline{AB} .

$$M\left(\frac{4+8}{2}, \frac{2+6}{2}\right)$$

$$M(6, 4)$$

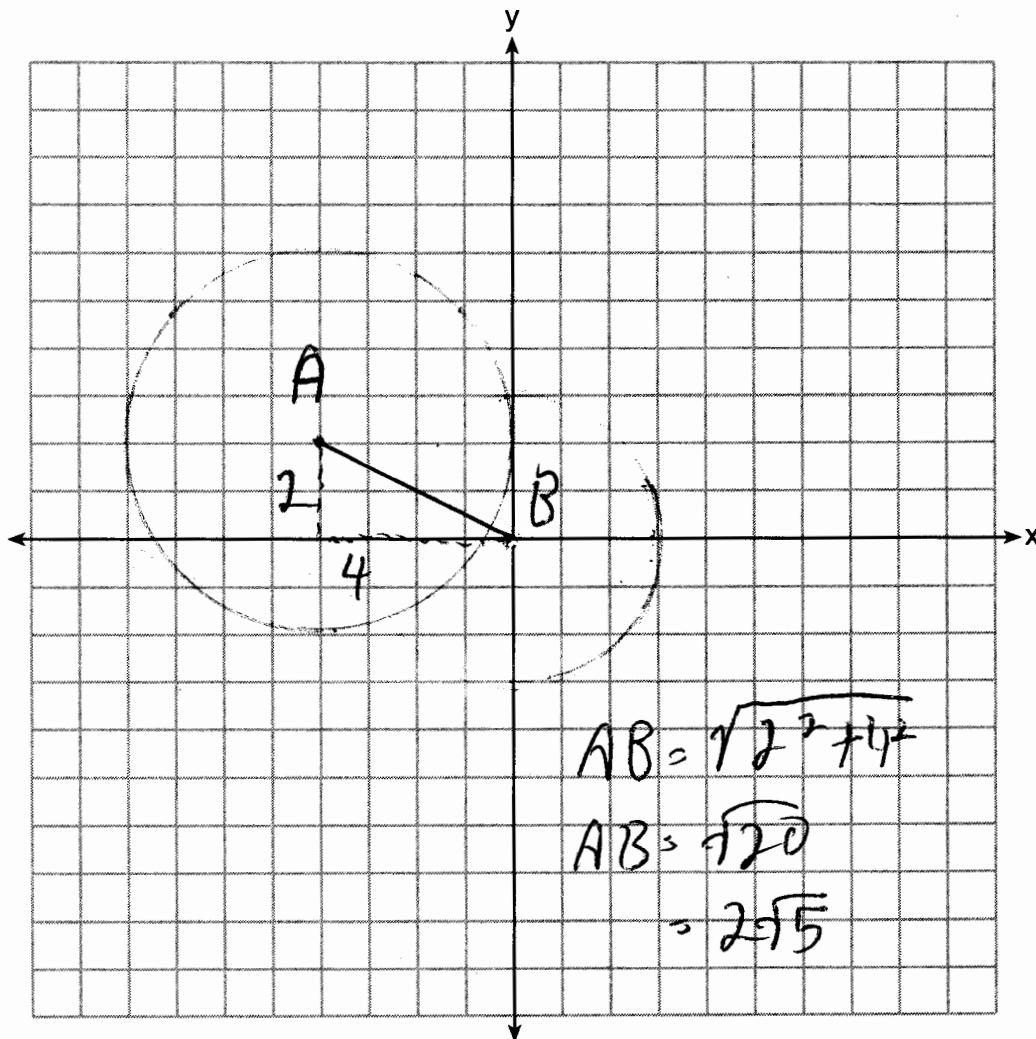
$$m = \frac{6-2}{8-4} = \frac{4}{4} = 1$$

$$m_{\perp} = -1$$

$$y-4 = -(x-6)$$

- 37 On the set of axes below, graph and label circle A whose equation is $(x + 4)^2 + (y - 2)^2 = 16$ and circle B whose equation is $x^2 + y^2 = 9$.

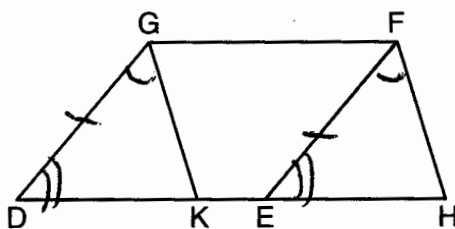
Determine, in simplest radical form, the length of the line segment with endpoints at the centers of circles A and B.



Part IV

Answer the question in this part. A correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. A correct numerical answer with no work shown will receive only 1 credit. The answer should be written in pen, except for graphs and drawings, which should be done in pencil. [6]

38 Given: Parallelogram $DEFG$, K and H are points on \overline{DE} such that $\angle DGK \cong \angle EFH$ and \overline{GK} and \overline{FH} are drawn



Prove: $\overline{DK} \cong \overline{EH}$

Statement	Reason
① Parallelogram $DEFG$, K & H are points on \overline{DE} such that $\angle DGK \cong \angle EFH$ & \overline{GK} & \overline{FH} are drawn	① Given
② $\overline{DG} \cong \overline{EF}$; $\overline{DG} \parallel \overline{EF}$	② Opposite sides of a parallelogram are congruent & parallel
③ $\angle D \cong \angle FEH$	③ Corresponding angles formed by parallel lines & a transversal are congruent
④ $\triangle DGK \cong \triangle EFH$	④ ASA
⑤ $\overline{DK} \cong \overline{EH}$	⑤ CPCTC