The possession or use of any communications device is strictly prohibited when taking this examination. If you have or use any communications device, no matter how briefly, your examination will be invalidated and no score will be calculated for you.

Print your name and the name of your school on the lines above.

A separate answer sheet for Part I has been provided to you. Follow the instructions from the proctor for completing the student information on your answer sheet.

This examination has four parts, with a total of 38 questions. You must answer all questions in this examination. Record your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in Parts II, III, and IV directly in this booklet. All work should be written in pen, except for graphs and drawings, which should be done in pencil. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc.

The formulas that you may need to answer some questions in this examination are found at the end of the examination. This sheet is perforated so you may remove it from this booklet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. You may remove this sheet from this booklet. Any work done on this sheet of scrap graph paper will not be scored.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

Notice...
A graphing calculator, a straightedge (ruler), and a compass must be available for you to use while taking this examination.
Part I

Answer all 28 questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. For each statement or question, choose the word or expression that, of those given, best completes the statement or answers the question. Record your answers on your separate answer sheet. [56]

1 What is the equation of a circle with its center at (5,-2) and a radius of 3?
(1) \((x - 5)^2 + (y + 2)^2 = 3\)
(2) \((x - 5)^2 + (y + 2)^2 = 9\)
(3) \((x + 5)^2 + (y - 2)^2 = 3\)
(4) \((x + 5)^2 + (y - 2)^2 = 9\)

2 In the diagram below, \(\triangle ABC\) is inscribed in circle \(O\).

[Diagram]

The ratio of the measure of \(\angle ABC\) to the measure of \(\overarc{AC}\) is
(1) 1:1
(2) 1:2
(3) 1:3
(4) 1:4
3 In the diagram below of rectangle $RSTU$, diagonals $RT$ and $SU$ intersect at $O$.

If $RT = 6x + 4$ and $SO = 7x - 6$, what is the length of $US$?

(1) 8  (3) 16
(2) 2  (4) 32

4 How many points are 3 units from the origin and also equidistant from both the $x$-axis and $y$-axis?

(1) 1  (3) 0
(2) 2  (4) 4

5 The converse of the statement "If a triangle has one right angle, the triangle has two acute angles" is

(1) If a triangle has two acute angles, the triangle has one right angle.
(2) If a triangle has one right angle, the triangle does not have two acute angles.
(3) If a triangle does not have one right angle, the triangle does not have two acute angles.
(4) If a triangle does not have two acute angles, the triangle does not have one right angle.

6 The surface area of a sphere is $2304\pi$ square inches. The length of a radius of the sphere, in inches, is

(1) 12  (3) 288
(2) 24  (4) 576
As shown in the diagram below of $\triangle ABC$, $BC$ is extended through $D$, $m\angle A = 70$, and $m\angle ACD = 115$.

Which statement is true?

1. $AC > AB$
2. $AB > BC$
3. $BC < AC$
4. $AC < AB$

In trapezoid $LMNO$ below, median $PQ$ is drawn.

If $LM = x + 7$, $ON = 3x + 11$, and $PQ = 25$, what is the value of $x$?

1. 1.75
2. 3.5
3. 8
4. 17

Points $A$ and $B$ are on line $\ell$. How many points are 3 units from line $\ell$ and also equidistant from $A$ and $B$?

1. 1
2. 2
3. 3
4. 4
10 The lines whose equations are $2x + 3y = 4$ and $y = mx + 6$ will be perpendicular when $m$ is

(1) $\frac{-3}{2}$  
(2) $\frac{-2}{3}$  
(3) $\frac{3}{2}$  
(4) $\frac{2}{3}$

11 As shown in the diagram below, $M$, $R$, and $T$ are midpoints of the sides of $\triangle ABC$.

If $AB = 18$, $AC = 14$, and $BC = 10$, what is the perimeter of quadrilateral $ACRM$?

(1) 35  
(2) 32  
(3) 24  
(4) 21
In the diagram below, \( \overline{ABC} \parallel \overline{DEFG} \). Transversal \( \overline{BHE} \) and line segment \( HF \) are drawn.

![Diagram](image)

If \( m\angle HFG = 130 \) and \( m\angle EHF = 70 \), what is \( m\angle ABE \)?

(1) 40  
(2) 50  
(3) 60  
(4) 70

The graphs of the lines represented by the equations

\[ y = \frac{1}{3}x + 7 \]  
\[ y = -\frac{1}{3}x - 2 \]

are

(1) parallel  
(2) horizontal  
(3) perpendicular  
(4) intersecting, but not perpendicular
14 Which graph represents a circle whose equation is 
\((x + 3)^2 + (y - 1)^2 = 4^2\)?

15 In \(\triangle ABC\), \(\angle CAB = 2x\) and \(\angle ACB = x + 30\). If \(\overline{AB}\) is extended through point \(B\) to point \(D\), \(\angle CBD = 5x - 50\). What is the value of \(x\)?

(1) 25  
(2) 30  
(3) 40  
(4) 46

Use this space for computations.
16 In circle $O$ shown below, chord $AB$ and diameter $CD$ are parallel, and chords $AD$ and $BC$ intersect at point $E$.

Which statement is *false*?

1. $\overline{AC} \cong \overline{BD}$
2. $BE = CE$
3. $\triangle ABE \sim \triangle CDE$
4. $\angle B \cong \angle C$

17 When the transformation $T_{2,-1}$ is performed on point $A$, its image is point $A'(-3,4)$. What are the coordinates of $A$?

1. $(5, -5)$
2. $(-5, 5)$
3. $(-1, 3)$
4. $(-6, -4)$

18 If the sum of the interior angles of a polygon is $1440^\circ$, then the polygon must be

1. an octagon
2. a decagon
3. a hexagon
4. a nonagon
19 In \( \triangle ABC \) shown below, medians \( \overline{AD}, \overline{BE}, \) and \( \overline{CF} \) intersect at point \( R \).

If \( CR = 24 \) and \( RF = 2x - 6 \), what is the value of \( x \)?

(1) 9 \hspace{1cm} (3) 15
(2) 12 \hspace{1cm} (4) 27

20 Which equation represents a line that passes through the point \((-2,6)\) and is parallel to the line whose equation is \(3x - 4y = 6\)?

(1) \(3x + 4y = 18\) \hspace{1cm} (3) \(-3x + 4y = 30\)
(2) \(4x + 3y = 10\) \hspace{1cm} (4) \(-4x + 3y = 26\)

21 The bases of a right prism are triangles in which \( \triangle MNP \cong \triangle RST \).
If \( MP = 9, MR = 18, \) and \( MN = 12 \), what is the length of \( NS \)?

(1) 9 \hspace{1cm} (3) 15
(2) 12 \hspace{1cm} (4) 18
22 Triangle ABC has the coordinates A(3,0), B(3,8), and C(6,6). If ΔABC is reflected over the line \( y = x \), which statement is true about the image of ΔABC?

(1) One point remains fixed.
(2) The size of the triangle changes.
(3) The orientation does not change.
(4) One side of ΔABC is parallel to the line \( y = x \).

23 A right circular cone has a diameter of \( 10 \sqrt{2} \) and a height of 12. What is the volume of the cone in terms of \( \pi \)?

(1) 200π
(2) 600π
(3) 800π
(4) 2400π

24 Which statement is not always true when ΔABC \( \cong \) ΔXYZ?

(1) \( \overline{BC} \cong \overline{YZ} \)
(2) \( \overline{CA} \cong \overline{XY} \)
(3) \( \angle CAB \cong \angle ZXY \)
(4) \( \angle BCA \cong \angle YZX \)

25 If two sides of a triangle have lengths of \( \frac{1}{4} \) and \( \frac{1}{5} \), which fraction can not be the length of the third side?

(1) \( \frac{1}{9} \)
(2) \( \frac{1}{8} \)
(3) \( \frac{1}{3} \)
(4) \( \frac{1}{2} \)
26 In the diagram below of $\triangle ABC$, $\overline{CDA}$, $\overline{CEB}$, $\overline{DE} \parallel \overline{AB}$, $DE = 4$, $AB = 10$, $CD = x$, and $DA = x + 3$.

What is the value of $x$?

(1) 0.5
(2) 2
(3) 5.5
(4) 6

27 Given: $\overline{AE}$ bisects $\overline{BD}$ at $C$,
$\overline{AB}$ and $\overline{DE}$ are drawn,
$\angle ABC \cong \angle EDC$

Which statement is needed to prove $\triangle ABC \cong \triangle EDC$ using ASA?

(1) $\angle ABC$ and $\angle EDC$ are right angles.
(2) $\overline{BD}$ bisects $\overline{AE}$ at $C$.
(3) $\angle BCA \cong \angle DCE$
(4) $\angle DEC \cong \angle BAC$
In the construction shown below, \( CD \) is drawn.

In \( \triangle ABC \), \( CD \) is the:

1. perpendicular bisector of side \( AB \)
2. median to side \( AB \)
3. altitude to side \( AB \)
4. bisector of \( \angle ACB \)
Part II

Answer all 6 questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [12]

29 The sides of a triangle measure 7, 4, and 9. If the longest side of a similar triangle measures 36, determine and state the length of the shortest side of this triangle.
Triangle $ABC$ has coordinates $A(6,-4)$, $B(0,2)$, and $C(6,2)$. On the set of axes below, graph and label $\triangle A'B'C'$, the image of $\triangle ABC$ after a dilation of $\frac{1}{2}$.
31 In parallelogram $RSTU$, $m\angle R = 5x - 2$ and $m\angle S = 3x + 10$.
Determine and state the value of $x$. 

Geometry - Jan. '16 [15] [OVER]
32 Determine and state the length of a line segment whose endpoints are (6,4) and (-9,-4).
The base of a right pentagonal prism has an area of 20 square inches. If the prism has an altitude of 8 inches, determine and state the volume of the prism, in cubic inches.
Using a compass and a straightedge, construct the bisector of \( \angle CDE \).

[Leave all construction marks.]
35 The coordinates of \( \triangle ABC \), shown on the graph below, are \( A(2, 5) \), \( B(5, 7) \), and \( C(4, 1) \).

Graph and label \( \triangle A'B'C' \), the image of \( \triangle ABC \) after it is reflected over the \( y \)-axis.

Graph and label \( \triangle A''B''C'' \), the image of \( \triangle A'B'C' \) after it is reflected over the \( x \)-axis.

State a single transformation that will map \( \triangle ABC \) onto \( \triangle A''B''C'' \).
36 On the set of axes below, solve the following system of equations graphically and state the coordinates of all points in the solution.

\[
\begin{align*}
y &= x^2 + 4x + 2 \\
y - 2x &= 5
\end{align*}
\]
17 Given: Triangle \( RST \) has coordinates \( R(-1,7), S(3,-1), \) and \( T(9,2) \)

Prove: \( \triangle RST \) is a right triangle

[The use of the set of axes below is optional.]
Part IV

Answer the question in this part. A correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. A correct numerical answer with no work shown will receive only 1 credit. The answer should be written in pen, except for graphs and drawings, which should be done in pencil. [6]

38 In right triangle $FGH$ shown below, $m \angle GHF = 90$, altitude $HJ$ is drawn to $FG$, $FJ = 16$, and $HG = 15$.

Determine and state the length of $JG$. [Only an algebraic solution can receive full credit.]

Determine and state the length of $HJ$. [Only an algebraic solution can receive full credit.]
FOR TEACHERS ONLY

The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

GEOMETRY

Tuesday, January 26, 2016 — 1:15 to 4:15 p.m., only

SCORING KEY AND RATING GUIDE

Mechanics of Rating

The following procedures are to be followed for scoring student answer papers for the Regents Examination in Geometry. More detailed information about scoring is provided in the publication Information Booklet for Scoring the Regents Examinations in Mathematics.

Do not attempt to correct the student’s work by making insertions or changes of any kind. In scoring the open-ended questions, use check marks to indicate student errors. Unless otherwise specified, mathematically correct variations in the answers will be allowed. Units need not be given when the wording of the questions allows such omissions.

Each student’s answer paper is to be scored by a minimum of three mathematics teachers. No one teacher is to score more than approximately one-third of the open-ended questions on a student’s paper. Teachers may not score their own students’ answer papers. On the student’s separate answer sheet, for each question, record the number of credits earned and the teacher’s assigned rater/scorer letter.

Schools are not permitted to rescore any of the open-ended questions on this exam after each question has been rated once, regardless of the final exam score. Schools are required to ensure that the raw scores have been added correctly and that the resulting scale score has been determined accurately.

Raters should record the student’s scores for all questions and the total raw score on the student’s separate answer sheet. Then the student’s total raw score should be converted to a scale score by using the conversion chart that will be posted on the Department’s web site at: http://www.p12.nysed.gov/assessment/ on Tuesday, January 26, 2016. Because scale scores corresponding to raw scores in the conversion chart may change from one administration to another, it is crucial that, for each administration, the conversion chart provided for that administration be used to determine the student’s final score. The student’s scale score should be entered in the box provided on the student’s separate answer sheet. The scale score is the student’s final examination score.
If the student’s responses for the multiple-choice questions are being hand scored prior to being scanned, the scorer must be careful not to make any marks on the answer sheet except to record the scores in the designated score boxes. Marks elsewhere on the answer sheet will interfere with the accuracy of the scanning.

Part I

Allow a total of 56 credits, 2 credits for each of the following. Allow credit if the student has written the correct answer instead of the numeral 1, 2, 3, or 4.

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Updated information regarding the rating of this examination may be posted on the New York State Education Department’s web site during the rating period. Check this web site at: http://www.p12.nysed.gov/assessment/ and select the link “Scoring Information” for any recently posted information regarding this examination. This site should be checked before the rating process for this examination begins and several times throughout the Regents Examination period.

Beginning in June 2013, the Department is providing supplemental scoring guidance, the “Sample Response Set,” for the Regents Examination in Geometry. This guidance is not required as part of the scorer training. It is at the school’s discretion to incorporate it into the scorer training or to use it as supplemental information during scoring. While not reflective of all scenarios, the sample student responses selected for the Sample Response Set illustrate how less common student responses to open-ended questions may be scored. The Sample Response Set will be available on the Department’s web site at: http://www.nysedregents.org/Geometry/.
General Rules for Applying Mathematics Rubrics

I. General Principles for Rating
The rubrics for the constructed-response questions on the Regents Examination in Geometry are designed to provide a systematic, consistent method for awarding credit. The rubrics are not to be considered all-inclusive; it is impossible to anticipate all the different methods that students might use to solve a given problem. Each response must be rated carefully using the teacher's professional judgment and knowledge of mathematics; all calculations must be checked. The specific rubrics for each question must be applied consistently to all responses. In cases that are not specifically addressed in the rubrics, raters must follow the general rating guidelines in the publication Information Booklet for Scoring the Regents Examinations in Mathematics, use their own professional judgment, confer with other mathematics teachers, and/or contact the State Education Department for guidance. During each Regents Examination administration period, rating questions may be referred directly to the Education Department. The contact numbers are sent to all schools before each administration period.

II. Full-Credit Responses
A full-credit response provides a complete and correct answer to all parts of the question. Sufficient work is shown to enable the rater to determine how the student arrived at the correct answer.

When the rubric for the full-credit response includes one or more examples of an acceptable method for solving the question (usually introduced by the phrase "such as"), it does not mean that there are no additional acceptable methods of arriving at the correct answer. Unless otherwise specified, mathematically correct alternative solutions should be awarded credit. The only exceptions are those questions that specify the type of solution that must be used; e.g., an algebraic solution or a graphic solution. A correct solution using a method other than the one specified is awarded half the credit of a correct solution using the specified method.

III. Appropriate Work
Full-Credit Responses: The directions in the examination booklet for all the constructed-response questions state: "Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc." The student has the responsibility of providing the correct answer and showing how that answer was obtained. The student must "construct" the response; the teacher should not have to search through a group of seemingly random calculations scribbled on the student paper to ascertain what method the student may have used.

Responses With Errors: Rubrics that state "Appropriate work is shown, but..." are intended to be used with solutions that show an essentially complete response to the question but contain certain types of errors, whether computational, rounding, graphing, or conceptual. If the response is incomplete; i.e., an equation is written but not solved or an equation is solved but not all of the parts of the question are answered, appropriate work has not been shown. Other rubrics address incomplete responses.

IV. Multiple Errors
Computational Errors, Graphing Errors, and Rounding Errors: Each of these types of errors results in a 1-credit deduction. Any combination of two of these types of errors results in a 2-credit deduction. No more than 2 credits should be deducted for such mechanical errors in any response. The teacher must carefully review the student's work to determine what errors were made and what type of errors they were.

Conceptual Errors: A conceptual error involves a more serious lack of knowledge or procedure. Examples of conceptual errors include using the incorrect formula for the area of a figure, choosing the incorrect trigonometric function, or multiplying the exponents instead of adding them when multiplying terms with exponents.

If a response shows repeated occurrences of the same conceptual error, the student should not be penalized twice. If the same conceptual error is repeated in responses to other questions, credit should be deducted in each response.

For 4- and 6-credit questions, if a response shows one conceptual error and one computational, graphing, or rounding error, the teacher must award credit that takes into account both errors. Refer to the rubric for specific scoring guidelines.
Part II

For each question, use the specific criteria to award a maximum of 2 credits. Unless otherwise specified, mathematically correct alternative solutions should be awarded appropriate credit.

(29) [2] 16, and correct work is shown.

[1] Appropriate work is shown, but one computational error is made.

or

[1] Appropriate work is shown, but one conceptual error is made.

or

[1] A correct proportion is written, but no further correct work is shown.

or

[1] 16, but no work is shown.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

(30) [2] $\triangle A'B'C'$ is graphed and labeled correctly.

[1] Appropriate work is shown, but one graphing error is made.

or

[1] Appropriate work is shown, but one conceptual error is made, such as a dilation of 2.

or

[1] The image of $\triangle ABC$ is graphed correctly, but it is not labeled or is labeled incorrectly.

or

[1] $A'(3,-2), B'(0,1), \text{ and } C'(3,1)$ are stated, but no further correct work is shown.

[0] (3, −2), (0,1), and (3,1) are stated, but are not labeled or are labeled incorrectly.

or

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.
(31) [2] 21.5, and correct work is shown.

[1] Appropriate work is shown, but one computational error is made.

or

[1] Appropriate work is shown, but one conceptual error is made.

or

[1] 21.5, but no work is shown.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

(32) [2] 17 or \( \sqrt{289} \), and correct work is shown.

[1] Appropriate work is shown, but one computational error is made.

or

[1] Appropriate work is shown, but one conceptual error is made.

or

[1] 17, but no work is shown.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.
(33) [2] 160, and correct work is shown.

[1] Appropriate work is shown, but one computational error is made.

or

[1] Appropriate work is shown, but one conceptual error is made.

or

[1] 160, but no work is shown.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

(34) [2] A correct construction is drawn showing all appropriate arcs, and the angle bisector is drawn.

[1] A correct construction for an angle bisector is drawn for an angle other than $\angle D$.

or

[1] All construction arcs are drawn, but the angle bisector is not drawn.

[0] A drawing that is not an appropriate construction is shown.

or

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.
Part III

For each question, use the specific criteria to award a maximum of 4 credits. Unless otherwise specified, mathematically correct alternative solutions should be awarded appropriate credit.

(35)  

[4] $\triangle A'B'C'$ and $\triangle A''B''C''$ are graphed and labeled correctly, and a rotation of $180^\circ$ or a reflection through the origin is stated.

[3] Appropriate work is shown, but one computational, graphing, or labeling error is made.

or

[3] $\triangle A'B'C'$ and $\triangle A''B''C''$ are graphed and labeled correctly, but a single transformation is not stated or is stated incorrectly.

[2] Appropriate work is shown, but two or more computational, graphing, or labeling errors are made.

or

[2] Appropriate work is shown, but one conceptual error is made.

or

[2] $\triangle A'B'C'$ and $\triangle A''B''C''$ are graphed correctly, but are not labeled or are labeled incorrectly. No further correct work is shown.

[1] Appropriate work is shown, but one conceptual error and one computational, graphing, or labeling error are made.

or

[1] $\triangle A'B'C'$ is graphed and labeled correctly, but no further correct work is shown.

or

[1] Appropriate work is shown to find $A'(-2,-5)$, $B'(-5,-7)$, and $C'(-4,-1)$, but no graphs are shown.

or

[1] A rotation of $180^\circ$ or a reflection through the origin is stated, but no work is shown.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.
Both equations are graphed correctly, and \((-3, -1)\) and \((1, 7)\) are stated.

Appropriate work is shown, but one computational or graphing error is made.

or

Both equations are graphed correctly, but only \((-3, -1)\) or \((1, 7)\) is stated.

Appropriate work is shown, but two or more computational or graphing errors are made.

or

Appropriate work is shown, but one conceptual error is made.

or

Both equations are graphed correctly, but the points of intersection are not stated or are stated incorrectly.

or

Appropriate work is shown to find \((-3, -1)\) and \((1, 7)\), but a method other than graphing is used.

Appropriate work is shown, but one conceptual error and one computational or graphing error are made.

or

One equation is graphed correctly, but no further correct work is shown.

or

\((-3, -1)\) and \((1, 7)\) are stated, but no work is shown.

A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.
[4] A complete and correct proof that includes a concluding statement is written.

[3] A proof is written that includes a concluding statement, but one computational or graphing error is made.

    or

[3] A proof is written that includes a concluding statement, but one reason is missing.

    or

[3] Appropriate work is shown to prove $\angle RST$ is a right angle, but a concluding statement is missing or is incorrect.

    or

[3] Appropriate work is shown to demonstrate that the Pythagorean Theorem is satisfied, but a concluding statement is missing or is incorrect.

[2] A proof is written that includes a concluding statement, but two or more computational or graphing errors are made.

    or

[2] Appropriate work is shown, but one conceptual error is made.

    or

[2] Appropriate work is shown to prove $\overline{RS} \perp \overline{ST}$, but no further correct work is shown.

    or

[2] A correct substitution is made into the Pythagorean Theorem, but no further correct work is shown.

[1] A proof is written, but one conceptual error and one computational or graphing error are made.

    or

[1] Appropriate work is shown to find the lengths of all three sides, but no further correct work is shown.

    or

[1] Appropriate work is shown to find the slopes of $\overline{RS}$ and $\overline{ST}$, but no further correct work is shown.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.
Part IV

For this question, use the specific criteria to award a maximum of 6 credits. Unless otherwise specified, mathematically correct alternative solutions should be awarded appropriate credit.

(38)  [6] 9 and 12, and correct algebraic work is shown.

[5] Appropriate work is shown, but one computational or factoring error is made.

or

[5] Correct work is shown to find 9, and a correct equation is written to find HI, but no further correct work is shown.

[4] Appropriate work is shown, but two computational or factoring errors are made.

or

[4] Appropriate work is shown, but one conceptual error is made in finding JG.

or

[4] Correct work is shown to find 9, but no further correct work is shown.

[3] Appropriate work is shown, but three or more computational or factoring errors are made.

or

[3] Appropriate work is shown, but one conceptual error and one computational or factoring error are made.

or

[3] 9 and 12, but a method other than algebraic is used.

[2] Appropriate work is shown, but two conceptual errors are made.

or

[2] Appropriate work is shown, but one conceptual error and two or more computational or factoring errors are made.

or

[2] A correct quadratic equation is written to find JG, but no further correct work is shown.

or
[2] Correct proportions for $JG$ and $HJ$ are written, but no further correct work is shown.

or

[2] An appropriate solution for the length of $HJ$ is found based upon an incorrectly obtained value for the length of $JG$.

[1] Appropriate work is shown, but two conceptual errors and one computational or factoring error are made.

or

[1] A correct proportion for either $JG$ or $HJ$ is written, but no further correct work is shown.

or

[1] 9 and 12, but no work is shown.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.
Map to Core Curriculum

<table>
<thead>
<tr>
<th>Content Band</th>
<th>Item Numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Geometric Relationships</td>
<td>6, 21, 23, 33</td>
</tr>
<tr>
<td>Constructions</td>
<td>28, 34</td>
</tr>
<tr>
<td>Locus</td>
<td>4, 9</td>
</tr>
<tr>
<td>Informal and Formal Proofs</td>
<td>2, 3, 5, 7, 8, 11, 12, 15, 16, 18, 19, 24, 25, 26, 27, 29, 31, 38</td>
</tr>
<tr>
<td>Transformational Geometry</td>
<td>17, 22, 30, 35</td>
</tr>
<tr>
<td>Coordinate Geometry</td>
<td>1, 10, 13, 14, 20, 32, 36, 37</td>
</tr>
</tbody>
</table>

Regents Examination in Geometry

January 2016

Chart for Converting Total Test Raw Scores to Final Examination Scores (Scale Scores)

The Chart for Determining the Final Examination Score for the January 2016 Regents Examination in Geometry will be posted on the Department’s web site at: http://www.p12.nysed.gov/assessment/ on Tuesday, January 26, 2016. Conversion charts provided for previous administrations of the Regents Examination in Geometry must NOT be used to determine students’ final scores for this administration.

Online Submission of Teacher Evaluations of the Test to the Department

Suggestions and feedback from teachers provide an important contribution to the test development process. The Department provides an online evaluation form for State assessments. It contains spaces for teachers to respond to several specific questions and to make suggestions. Instructions for completing the evaluation form are as follows:


2. Select the test title.

3. Complete the required demographic fields.

4. Complete each evaluation question and provide comments in the space provided.

5. Click the SUBMIT button at the bottom of the page to submit the completed form.
The University of the State of New York
REGENTS HIGH SCHOOL EXAMINATION

GEOMETRY

Tuesday, January 26, 2016 — 1:15 p.m.

SAMPLE RESPONSE SET

Table of Contents

Question 29 ....................... 2
Question 30 ....................... 7
Question 31 ....................... 12
Question 32 ....................... 17
Question 33 ....................... 20
Question 34 ....................... 24
Question 35 ....................... 29
Question 36 ....................... 36
Question 37 ....................... 44
Question 38 ....................... 52
The sides of a triangle measure 7, 4, and 9. If the longest side of a similar triangle measures 36, determine and state the length of the shortest side of this triangle.

\[
\frac{9}{36} = \frac{4}{x}
\]

\[9x = (36)(4)\]

\[9x = 144\]

\[x = 16\]

**Score 2:** The student has a complete and correct response.
The sides of a triangle measure 7, 4, and 9. If the longest side of a similar triangle measures 36, determine and state the length of the shortest side of this triangle.

\[
\frac{9}{4} = \frac{36}{x}
\]

\[x = 16\]

**Score 2:** The student has a complete and correct response.
29 The sides of a triangle measure 7, 4, and 9. If the longest side of a similar triangle measures 36, determine and state the length of the shortest side of this triangle.

\[ \begin{align*}
4 \times 4 &= 16 \\
7 \times 4 &= 28 \\
9 \times 4 &= 36
\end{align*} \]

Shortest side 36

Score 1: The student wrote shortest side next to \(4 \times 4 = 16\) but circled “shortest side = 36” as the answer.
29 The sides of a triangle measure 7, 4, and 9. If the longest side of a similar triangle measures 36, determine and state the length of the shortest side of this triangle.

\[
\frac{9}{4} = \frac{39}{x} \quad \text{and} \quad 9x = 156
\]

\[
x = 17.3
\]

**Score 1:** The student made a transcription error, but found an appropriate length for the shortest side.
29 The sides of a triangle measure 7, 4, and 9. If the longest side of a similar triangle measures 36, determine and state the length of the shortest side of this triangle.

\[ a^2 + b^2 = c^2 \]
\[ 7^2 + 4^2 = 9^2 \]
\[ 49 + 16 = 81 \]
\[ 65 \]
\[ \frac{16}{16} = 1 \]
\[ x = 2 \text{ is the shortest side} \]

Score 0: The student's work was completely incorrect.
30 Triangle $ABC$ has coordinates $A(6,-4)$, $B(0,2)$, and $C(6,2)$. On the set of axes below, graph and label $\triangle A'B'C'$, the image of $\triangle ABC$ after a dilation of $\frac{1}{2}$.

**Score 2:** The student has a complete and correct response.
30 Triangle $ABC$ has coordinates $A(6,-4)$, $B(0,2)$, and $C(6,2)$. On the set of axes below, graph and label $\triangle A'B'C'$, the image of $\triangle ABC$ after a dilation of $\frac{1}{2}$.

**Score 2:** The student has a complete and correct response.
30 Triangle $ABC$ has coordinates $A(6, -4)$, $B(0, 2)$, and $C(6, 2)$. On the set of axes below, graph and label $\triangle A'B'C'$, the image of $\triangle ABC$ after a dilation of $\frac{1}{2}$.

$A' (3, -2)$

$B' (0, 1)$

$C' (3, 1)$

Score 1: The student stated the coordinates of each corresponding point of the image of $\triangle ABC$. 

Geometry – Jan. ’16
30 Triangle $ABC$ has coordinates $A(6,-4)$, $B(0,2)$, and $C(6,2)$. On the set of axes below, graph and label $\triangle A'B'C'$, the image of $\triangle ABC$ after a dilation of $\frac{1}{2}$.

\[
\begin{align*}
A(6,-4) \rightarrow A'(3, -2) \\
B(0,2) \rightarrow B'(0, 1) \\
C(6,2) \rightarrow C'(3, 1)
\end{align*}
\]

Score 1: The student made an error graphing $B'$. 
Triangle $ABC$ has coordinates $A(6,-4)$, $B(0,2)$, and $C(6,2)$. On the set of axes below, graph and label $\triangle A'B'C'$, the image of $\triangle ABC$ after a dilation of $\frac{1}{2}$.

**Score 0:** The student’s work was completely incorrect.
31 In parallelogram $RSTU$, $m\angle R = 5x - 2$ and $m\angle S = 3x + 10$. Determine and state the value of $x$.

\[\begin{array}{ccc}
R & 105.5 & S \\
5x - 2 & 3x + 10 \\
3x + 10 & 5x - 2 \\
\end{array}\]

\[
360 = 5x - 2 + 5x - 2 + 3x + 10 + 3x + 10 \\
360 = 10x - 4 + 6x + 20 \\
360 = 16x + 16 \\
\quad -16 \\
344 = 16x \\
\quad -16 \\
21.5 = x
\]

**Score 2:** The student has a complete and correct response.
31 In parallelogram $RSTU$, $m\angle R = 5x - 2$ and $m\angle S = 3x + 10$.
Determine and state the value of $x$.

\[
\begin{align*}
5x - 2 + 3x + 10 &= 180 \\
8x + 8 &= 180 \\
8x &= 172 \\
x &= 21.5
\end{align*}
\]

Score 2: The student has a complete and correct response.
31. In parallelogram $RSTU$, $m\angle R = 5x - 2$ and $m\angle S = 3x + 10$. Determine and state the value of $x$.

\[
8x + 8 = 180 \\
-8 -8 \\
\frac{8x}{8} = \frac{172}{8} = 22.5
\]

\[
16x + 16 = 360
\]

\[
x = 22.5
\]

Score 1: The student made an error in division.
31 In parallelogram $RSTU$, $m\angle R = 5x - 2$ and $m\angle S = 3x + 10$.
Determine and state the value of $x$.

Score 1: The student made an error by setting the consecutive angles of the parallelogram equal.
31 In parallelogram $RSTU$, $m\angle R = 5x - 2$ and $m\angle S = 3x + 10$.
Determine and state the value of $x$.

\[
\begin{align*}
5x - 2 + 3x + 10 &= 360 \\
8x + 8 &= 360 \\
8x &= 352 \\
x &= 44
\end{align*}
\]

**Score 0:** The student wrote an incorrect equation and made an error in solving it.
32 Determine and state the length of a line segment whose endpoints are (6, 4) and (−9, −4).

\[ \sqrt{(6+9)^2 + (4+4)^2} = \sqrt{(15)^2 + (8)^2} \]
\[ = \sqrt{225 + 64} \]
\[ = \sqrt{289} = 17 \]

**Score 2:** The student has a complete and correct response.
32 Determine and state the length of a line segment whose endpoints are \((6,4)\) and \((-9,-4)\).

\[
\sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}
\]

\[
\sqrt{(0+9)^2 + (4+4)^2}
\]

\[
\sqrt{(-3)^2 + (0)^2}
\]

\[
\sqrt{9 + 0}
\]

\[
= \sqrt{9}
\]

Score 1: The student used an incorrect formula for the length of a line segment, but found an appropriate solution.
32 Determine and state the length of a line segment whose endpoints are (6,4) and (−9,−4).

\[ d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \]
\[ d = \sqrt{(-9 - 6)^2 + (-4 - 6)^2} \]
\[ d = \sqrt{(-15)^2 + (-10)^2} \]
\[ d = \sqrt{225 + 100} \]
\[ d = \sqrt{325} \]
\[ d = 18 \]

**Score 0:** The student made an incorrect substitution into the formula and did not show the entire display of the calculator.
33 The base of a right pentagonal prism has an area of 20 square inches. If the prism has an altitude of 8 inches, determine and state the volume of the prism, in cubic inches.

\[ V = 20 \times 8 \]

\[ V = 160 \]

Score 2: The student has a complete and correct response.
33 The base of a right pentagonal prism has an area of 20 square inches. If the prism has an altitude of 8 inches, determine and state the volume of the prism, in cubic inches.

\[ V = 160 \text{ in}^3 \]

**Score 1:** The student showed no work.
33 The base of a right pentagonal prism has an area of 20 square inches. If the prism has an altitude of 8 inches, determine and state the volume of the prism, in cubic inches.

\[
(20)(8) = 160 \text{ sq. in.}
\]

**Score 1:** The student wrote incorrect units in the answer.
33 The base of a right pentagonal prism has an area of 20 square inches. If the prism has an altitude of 8 inches, determine and state the volume of the prism, in cubic inches.

\[
\frac{1}{2}(8)(20) \quad \text{Volume} = 100 \text{ in}^3
\]

**Score 0:** The student used an incorrect formula and made an error in multiplication.
34 Using a compass and a straightedge, construct the bisector of \( \angle CDE \).

[Leave all construction marks.]

**Score 2:** The student has a complete and correct response.
34 Using a compass and a straightedge, construct the bisector of \( \angle CDE \).

[Leave all construction marks.]

**Score 1:** The student drew a correct construction on an angle other than \( \angle CDE \).
34 Using a compass and a straightedge, construct the bisector of $\angle CDE$.

[Leave all construction marks.]

**Score 1:** The student did not complete the construction.
34 Using a compass and a straightedge, construct the bisector of \( \angle CDE \).

[Leave all construction marks.]

**Score 0:** The student did not demonstrate a proper method of construction.
34 Using a compass and a straightedge, construct the bisector of \( \angle CDE \).

[Leave all construction marks.]

**Score 0:** The student drew a construction that was irrelevant to the problem.
35 The coordinates of $\triangle ABC$, shown on the graph below, are $A(2,5)$, $B(5,7)$, and $C(4,1)$.

Graph and label $\triangle A'B'C'$, the image of $\triangle ABC$ after it is reflected over the $y$-axis.

Graph and label $\triangle A''B''C''$, the image of $\triangle A'B'C'$ after it is reflected over the $x$-axis.

State a single transformation that will map $\triangle ABC$ onto $\triangle A''B''C''$.

Score 4: The student has a complete and correct response.
The coordinates of $\triangle ABC$, shown on the graph below, are $A(2,5)$, $B(5,7)$, and $C(4,1)$. Graph and label $\triangle A'B'C'$, the image of $\triangle ABC$ after it is reflected over the $y$-axis. Graph and label $\triangle A''B''C''$, the image of $\triangle A'B'C'$ after it is reflected over the $x$-axis. State a single transformation that will map $\triangle ABC$ onto $\triangle A''B''C''$.

Score 4: The student has a complete and correct response.
35 The coordinates of $\triangle ABC$, shown on the graph below, are $A(2,5)$, $B(5,7)$, and $C(4,1)$.
Graph and label $\triangle A'B'C'$, the image of $\triangle ABC$ after it is reflected over the $y$-axis.
Graph and label $\triangle A''B''C''$, the image of $\triangle A'B'C'$ after it is reflected over the $x$-axis.
State a single transformation that will map $\triangle ABC$ onto $\triangle A''B''C''$.

Score 3: The student did not state the single transformation completely.
35 The coordinates of $\triangle ABC$, shown on the graph below, are $A(2,5)$, $B(5,7)$, and $C(4,1)$.

Graph and label $\triangle A'B'C'$, the image of $\triangle ABC$ after it is reflected over the $y$-axis.

Graph and label $\triangle A''B''C''$, the image of $\triangle A'B'C'$ after it is reflected over the $x$-axis.

State a single transformation that will map $\triangle ABC$ onto $\triangle A''B''C''$.

---

**Score 2**: The student graphed the two images correctly, but did not label them or state a single transformation.
35 The coordinates of $\triangle ABC$, shown on the graph below, are $A(2,5)$, $B(5,7)$, and $C(4,1)$. Graph and label $\triangle A'B'C'$, the image of $\triangle ABC$ after it is reflected over the $y$-axis. Graph and label $\triangle A''B''C''$, the image of $\triangle A'B'C'$ after it is reflected over the $x$-axis. State a single transformation that will map $\triangle ABC$ onto $\triangle A''B''C''$. $\begin{align*} A(2,5) &\rightarrow (5,-2) \rightarrow (5,2) \\ B(5,7) &\rightarrow (7,-5) \rightarrow (7,5) \\ C(4,1) &\rightarrow (1,-4) \rightarrow (1,4) \end{align*}$

**Score 2:** The student graphed the first transformation incorrectly, but graphed and labeled the second correctly and stated an appropriate single transformation.
35 The coordinates of \( \triangle ABC \), shown on the graph below, are \( A(2,5) \), \( B(5,7) \), and \( C(4,1) \).

Graph and label \( \triangle A'B'C' \), the image of \( \triangle ABC \) after it is reflected over the \( y \)-axis.

Graph and label \( \triangle A''B''C'' \), the image of \( \triangle A'B'C' \) after it is reflected over the \( x \)-axis.

State a single transformation that will map \( \triangle ABC \) onto \( \triangle A''B''C'' \).

**Score 1:** The student graphed and labeled \( \triangle A'B'C' \) correctly. When graphing \( \triangle A''B''C'' \) the student reflected \( \triangle ABC \) over the \( x \)-axis, and stated an incorrect single transformation.
35 The coordinates of $\triangle ABC$, shown on the graph below, are $A(2,5)$, $B(5,7)$, and $C(4,1)$.

Graph and label $\triangle A'B'C'$, the image of $\triangle ABC$ after it is reflected over the $y$-axis.

Graph and label $\triangle A''B''C''$, the image of $\triangle A'B'C'$ after it is reflected over the $x$-axis.

State a single transformation that will map $\triangle ABC$ onto $\triangle A''B''C''$.

Score 0: The student did not show any work that was relevant to the problem.
36 On the set of axes below, solve the following system of equations graphically and state the coordinates of all points in the solution.

\[ y = x^2 + 4x + 2 \]

\[ y = 2x + 5 \]

\[ y - 2x = 5 \]

\[ y = 2x + 5 \]

Score 4: The student has a complete and correct response.
36 On the set of axes below, solve the following system of equations graphically and state the coordinates of all points in the solution.

\[
\begin{align*}
y &= x^2 + 4x + 2 \\
y - 2x &= 5 \\
y &= 2x + 5
\end{align*}
\]

Score 3: The student graphed both equations accurately, but only showed and stated one solution.
36 On the set of axes below, solve the following system of equations graphically and state the coordinates of all points in the solution.

\[ y = x^2 + 4x + 2 \]
\[ y - 2x = 5 \]
\[ y = 2x + 5 \]

\[ \frac{y}{2} = 2 \]
\[ y_i = 5 \]

Score 2: The student confused the slope and y-intercept, but stated appropriate solutions.
36 On the set of axes below, solve the following system of equations graphically and state the coordinates of all points in the solution.

\[ y = x^2 + 4x + 2 \]
\[ y - 2x = 5 \]
\[ y > 2x + 5 \]

\[ 2y + 5 = x^2 + 4x + 2 \]
\[ 0 = x^2 + 2x - 3 \]
\[ 0 = (x-1)(x+3) \]
\[ x-1=0 \]
\[ x+3=0 \]
\[ x=1 \]
\[ x=-3 \]
\[ y = 7 \]
\[ y = -7 \]

**Score 2:** The student found both solutions by a method other than graphing.
36 On the set of axes below, solve the following system of equations graphically and state the coordinates of all points in the solution.

\[
y = x^2 + 4x + 2 \\
y - 2x = 5
\]

**Score 2:** The student graphed both equations correctly, but did not state the coordinates of the solution.
36 On the set of axes below, solve the following system of equations graphically and state the coordinates of all points in the solution.

\[ y = x^2 + 4x + 2 \]
\[ y - 2x = 5 \]

\[ y = 2x + 5 \]

**Score 1:** The student only graphed the quadratic equation correctly.
36 On the set of axes below, solve the following system of equations graphically and state the coordinates of all points in the solution.

\[ y = x^2 + 4x + 2 \quad \text{and} \quad y = -2x + 5 \]

Score 1: The student only graphed the linear equation correctly.
36 On the set of axes below, solve the following system of equations graphically and state the coordinates of all points in the solution.

\[ y = x^2 + 4x + 2 \]

\[ y = 2x + 5 \]

\[ y = x^2 + 4x + 2 \]

**Score 0:** The student did not show any work that was relevant to the problem.
Given: Triangle $RST$ has coordinates $R(-1,7)$, $S(3,-1)$, and $T(9,2)$

Prove: $\triangle RST$ is a right triangle

[The use of the set of axes below is optional.]

$$\text{Slope } \overline{RS} = -\frac{2}{3}$$
$$\text{Slope } \overline{ST} = \frac{2}{3} \cdot \frac{1}{2}$$

Lines $RS$ and $ST$ are perpendicular because they have negative reciprocal slopes of each other. Perpendicular lines composed form right angle.

Thus, angle $RST$ is a right angle, a triangle needs 1 right angles to be a right triangle and it has one.
37 Given: Triangle $RST$ has coordinates $R(-1,7)$, $S(3,-1)$, and $T(9,2)$

Prove: $\triangle RST$ is a right triangle

[The use of the set of axes below is optional.]

1) Get side $RS$ by doing Pythagorean theorem $8^2 + 4^2 + x^2 = 10 \sqrt{10}$
2) Get side $RT$ by doing Pythagorean theorem $10^2 + 16^2 + x^2 = 1175$
3) Get side $ST$ by doing Pythagorean theorem $6^2 + 9^2 + x^2 =\sqrt{105}$
4) $(\sqrt{10} + \sqrt{10})^2 = 125$
5) $\sqrt{80} + 125
6) $\triangle RST$ is a right $\Delta$ because it's sides worked in! Pythagorean theorem

Score 4: The student has a complete and correct response.
37 Given: Triangle RST has coordinates R(-1,7), S(3,-1), and T(9,2).

Prove: ΔRST is a right triangle.

[The use of the set of axes below is optional.]

\[
\text{Slope of \overline{ST}} = \frac{\Delta y}{\Delta x} = \frac{2-(-1)}{9-(-3)} = \frac{3}{6} = \frac{1}{2} = \text{slope of \overline{ST}}
\]

\[
\text{Slope of \overline{RS}} = \frac{2-(-1)}{7-3} = \frac{3}{4} = -\frac{1}{2}
\]

\[\text{m}_{\overline{RS}} \text{ is the negative reciprocal of } m_{\overline{ST}} \text{ so } \overline{RS} \text{ is perpendicular to } \overline{ST} \text{ forming a 90° or right angle.}\]

Score 3: The student wrote an incomplete concluding statement.
37 Given: Triangle $RST$ has coordinates $R(-1,7)$, $S(3,-1)$, and $T(9,2)$

Prove: $\triangle RST$ is a right triangle

[The use of the set of axes below is optional.]

\[ D = \sqrt{\Delta x^2 + \Delta y^2} \]

\[ \overline{RT} = \sqrt{(-1-9)^2 + (7-2)^2} \]

\[ \overline{RS} = \sqrt{(-1-3)^2 + (-1-1)^2} \]

\[ \overline{ST} = \sqrt{(3-9)^2 + (-1-2)^2} \]

\[ D = 25 \]

\[ \overline{RT} = \sqrt{125} \]

\[ \overline{RS} = \sqrt{80} \]

\[ \overline{ST} = \sqrt{95} \]

\[ a^2 + b^2 = c^2 \]

\[ \sqrt{80} + \sqrt{95} = \sqrt{125} \]

Score 3: The student did not write a concluding statement.
37 Given: Triangle $RST$ has coordinates $R(-1,7)$, $S(3,-1)$, and $T(9,2)$

Prove: $\triangle RST$ is a right triangle

[The use of the set of axes below is optional.]

$$\text{Slope} \quad \frac{RS}{RS} = \frac{-1 - 7}{3 + 1} = \frac{-8}{4} = -2$$

$$\text{Slope} \quad \frac{ST}{ST} = \frac{2 + 1}{9 - 3} = \frac{3}{6} = \frac{1}{2}$$

Line $RS$ is perpendicular to line $ST$ because the slopes are neg reciprocals.

Score 2: The student proved $\overline{RS} \perp \overline{ST}$. 
37 Given: Triangle RST has coordinates R(−1,7), S(3,−1), and T(9,2)

Prove: ΔRST is a right triangle

[The use of the set of axes below is optional.]

Score 2: The student did not show that the Pythagorean Theorem was satisfied, and did not write a concluding statement.
37 Given: Triangle \( RST \) has coordinates \( R(-1,7) \), \( S(3,-1) \), and \( T(9,2) \).

Prove: \( \triangle RST \) is a right triangle.

[The use of the set of axes below is optional.]

\[
\begin{align*}
\text{d} &= \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2} \\
d_{RS} &= \sqrt{(3-(-1))^2 + (-1-7)^2} \\
&= \sqrt{16 + 64} \\
&= \sqrt{80} \\
&= \sqrt{16 \cdot 5} \\
&= 4\sqrt{5}
\end{align*}
\]

\[
\begin{align*}
\text{d}_{RT} &= \sqrt{(9-(-1))^2 + (2-7)^2} \\
&= \sqrt{100 + 25} \\
&= 5\sqrt{5}
\end{align*}
\]

\[
\begin{align*}
\text{d}_{ST} &= \sqrt{(9-3)^2 + (2-(-1))^2} \\
&= \sqrt{36 + 9} \\
&= \sqrt{45}
\end{align*}
\]

Score 1: The student only found the lengths of all three sides.
37 Given: Triangle $RST$ has coordinates $R(-1,7)$, $S(3,-1)$, and $T(9,2)$

Prove: $\triangle RST$ is a right triangle

[The use of the set of axes below is optional.]

\[
\begin{align*}
&\sqrt{(1-x)^2 + (y-7)^2} = x+y \\
&\sqrt{(-4)^2 + (6)^2} = 16 + 64 = 80 \\
&\sqrt{(3-x)^2 + (-1-y)^2} = \sqrt{(-6)^2 + (-1-2)^2} = 36 + 9 = 45 \\
&\sqrt{(9-x)^2 + (2-y)^2} = \sqrt{(10)^2 + (2-2)^2} = 100 + 25 = 125
\end{align*}
\]

Score 0: The student attempted to find the lengths of all three sides.
38 In right triangle $FGH$ shown below, $m\angle GHF = 90$, altitude $\overline{HJ}$ is drawn to $\overline{FG}$, $FJ = 16$, and $HG = 15$.

Determine and state the length of $\overline{JG}$. [Only an algebraic solution can receive full credit.]

\[
\begin{align*}
\frac{16 \cdot x}{16} & = \frac{15}{x} \\
16x + x^2 &= 225 \\
x^2 + 16x &= 225 \\
x^2 + 16x - 225 &= 0 \\
(x+25)(x-9) &= 0 \\
x &= -25, 9
\end{align*}
\]

JG = 9

Determine and state the length of $\overline{HJ}$. [Only an algebraic solution can receive full credit.]

\[
\begin{align*}
C^2 &= a^2 + b^2 \\
15^2 &= 9^2 + x^2 \\
36 &= 9 + x^2 \\
-9 &= x^2 \\
\sqrt{144 - x^2} &= 12
\end{align*}
\]

Score 6: The student has a complete and correct response.
38 In right triangle $FGH$ shown below, $m \angle GHF = 90$, altitude $\overline{HJ}$ is drawn to $\overline{FG}$, $FJ = 16$, and $HG = 15$.

Determine and state the length of $\overline{GJ}$. [Only an algebraic solution can receive full credit.]

\[ \frac{x}{15} \times \frac{15}{16 + x} \quad 16x + x^2 = 225 \]
\[ x^2 + 16x - 225 = 0 \]
\[ (x + 25)(x - 9) = 0 \]
\[ x = -25 \quad \text{or} \quad x = 9 \]
\[ GJ = 9 \]
\[ \frac{9}{15} = \frac{15}{16 + 9} \]
\[ \frac{16 \times 9}{25} (9) = 225 \]

Determine and state the length of $\overline{HJ}$. [Only an algebraic solution can receive full credit.]

\[ \frac{16}{x} \times \frac{x}{9} \quad x^2 = 144 \]
\[ \sqrt{144} = 12 \quad x = 12 \]
\[ HJ = 12 \]

**Score 6**: The student has a complete and correct response.
In right triangle $FGH$ shown below, $m\angle GHF = 90$, altitude $\overline{HJ}$ is drawn to $\overline{FG}$, $FJ = 16$, and $HG = 15$.

Determine and state the length of $\overline{GJ}$. [Only an algebraic solution can receive full credit.]

\[
\begin{align*}
x + 16 &= \frac{15}{x} \\
x^2 + 16x &= 225 \\
x^2 + 16x - 225 &= 0 \\
(x - 9)(x + 25) &= 0 \\
x &= 9 \quad x &= -25
\end{align*}
\]

Determine and state the length of $\overline{HJ}$. [Only an algebraic solution can receive full credit.]

\[
\begin{align*}
16 &= \sqrt{9} \\
16 &= 3y \\
y &= \sqrt{16} \\
y &= 15.4
\end{align*}
\]

**Score 5:** The student made a multiplication error.
38 In right triangle $FGH$ shown below, $m\angle GHF = 90$, altitude $\overline{HJ}$ is drawn to $\overline{FG}$, $FJ = 16$, and $HG = 15$.

Determine and state the length of $\overline{JG}$. [Only an algebraic solution can receive full credit.]

\[
\begin{align*}
x &= \frac{15}{16} \quad \text{[Or]} \quad x = \frac{15}{16} \\
15^2 &= (x + 16)(x - 9) \\
x^2 + 16x - 225 &= 0 \\
(x + 25)(x - 9) &= 0 \\
x &= -25 \quad \text{[Or]} \quad x = 9 \\
JG &= 9
\end{align*}
\]

Determine and state the length of $\overline{HJ}$. [Only an algebraic solution can receive full credit.]

\[
\begin{align*}
y &= \frac{9}{15} \\
\sqrt{y^2} &= \sqrt{135} \\
y &= \sqrt{135} \\
y &= 3\sqrt{15} \\
HJ &= 3\sqrt{15}
\end{align*}
\]

**Score 4:** The student found the length of $\overline{JG}$, but no further correct work was shown.
In right triangle $FGH$ shown below, $\angle GHF = 90$, altitude $\overline{HJ}$ is drawn to $\overline{FG}$, $FJ = 16$, and $HG = 15$.

Determine and state the length of $\overline{JG}$. [Only an algebraic solution can receive full credit.]

\[
\frac{x + 16}{15} = \frac{15}{x} \quad \Rightarrow \quad (x - 25)(x + 9) = 0
\]

\[
x = 25 \quad \text{and} \quad x = -9.
\]

Determine and state the length of $\overline{HJ}$. [Only an algebraic solution can receive full credit.]

\[
25^2 = 15^2 + y^2
\]

\[
625 = 225 + y^2
\]

\[
400 = y^2
\]

\[
y = 20
\]

**Score 3**: The student made one factoring error when finding $JG$, and substituted into the Pythagorean Theorem incorrectly.
In right triangle $FGH$ shown below, $\angle GHF = 90^\circ$, altitude $\overline{HJ}$ is drawn to $\overline{FG}$, $FJ = 16$, and $HG = 15$.

Determine and state the length of $\overline{JG}$. [Only an algebraic solution can receive full credit.]

\[
\frac{16 + x}{15} = \frac{15}{x}
\]

\[225 = 16x + x^2\]

\[-x^2 - 16x + 225 = 0\]

Determine and state the length of $\overline{HJ}$. [Only an algebraic solution can receive full credit.]

\[
\frac{x}{y} = \frac{y}{16}
\]

**Score 3:** The student wrote a correct quadratic equation for $JG$ and a correct proportion for $HJ$. 
38 In right triangle $FGH$ shown below, $m\angle GHF = 90$, altitude $\overline{HJ}$ is drawn to $\overline{FG}$, $FJ = 16$, and $HG = 15$.

Determine and state the length of $\overline{JG}$. [Only an algebraic solution can receive full credit.]

$$\frac{x + 16}{15} = \frac{15}{x}$$

Determine and state the length of $\overline{HJ}$. [Only an algebraic solution can receive full credit.]

$$\frac{x}{y} = \frac{y}{16}$$

$$y^2 = 16x$$

**Score 2:** The student wrote correct proportions for both $JG$ and $HJ$. 

Geometry – Jan. ’16
38 In right triangle $FGH$ shown below, $\angle GHF = 90$, altitude $\overline{HJ}$ is drawn to $\overline{FG}$, $FJ = 16$, and $HG = 15$.

Determine and state the length of $\overline{JG}$. [Only an algebraic solution can receive full credit.]

\[
\frac{x}{15} = \frac{15}{x+16}
\]

Determine and state the length of $\overline{HJ}$. [Only an algebraic solution can receive full credit.]

**Score 1:** The student wrote a correct proportion to find $\overline{JG}$. 
38 In right triangle $FGH$ shown below, $m\angle GHF = 90$, altitude $\overline{HJ}$ is drawn to $\overline{FG}$, $FJ = 16$, and $HG = 15$.

Determine and state the length of $\overline{JG}$. [Only an algebraic solution can receive full credit.]

\[3 \times 5 = 15\]

\[3 \times 3 = 9\]

Determine and state the length of $\overline{HJ}$. [Only an algebraic solution can receive full credit.]

\[4 \times 3 = 12\]

Score 1: The student did not show sufficient work for 9 and 12.
38 In right triangle $FGH$ shown below, $\measuredangle GHF = 90$, altitude $\overline{HJ}$ is drawn to $\overline{FG}$, $FJ = 16$, and $HG = 15$.

Determine and state the length of $\overline{JG}$. [Only an algebraic solution can receive full credit.]

\[
\frac{16}{x} = \frac{x}{15} \implies \sqrt{x^2} = 1240 \implies x = 4\sqrt{15}
\]

Determine and state the length of $\overline{HJ}$. [Only an algebraic solution can receive full credit.]

Score 0: The student’s work was completely incorrect.
The State Education Department / The University of the State of New York


Chart for Converting Total Test Raw Scores to Final Examination Scores (Scale Scores)

<table>
<thead>
<tr>
<th>Raw Score</th>
<th>Scale Score</th>
<th>Raw Score</th>
<th>Scale Score</th>
<th>Raw Score</th>
<th>Scale Score</th>
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To determine the student’s final examination score, find the student’s total test raw score in the column labeled “Raw Score” and then locate the scale score that corresponds to that raw score. The scale score is the student’s final examination score. Enter this score in the space labeled “Scale Score” on the student’s answer sheet.

**Schools are not permitted to rescore any of the open-ended questions on this exam after each question has been rated once, regardless of the final exam score. Schools are required to ensure that the raw scores have been added correctly and that the resulting scale score has been determined accurately.**

Because scale scores corresponding to raw scores in the conversion chart change from one administration to another, it is crucial that for each administration the conversion chart provided for that administration be used to determine the student’s final score. The chart above is usable only for this administration of the Regents Examination in Geometry (2005).