

The University of the State of New York
REGENTS HIGH SCHOOL EXAMINATION

GEOMETRY (Common Core)

Friday, June 17, 2016 — 1:15 to 4:15 p.m., only

Student Name:

Mr. Sibal

School Name:

JMAP

The possession or use of any communications device is strictly prohibited when taking this examination. If you have or use any communications device, no matter how briefly, your examination will be invalidated and no score will be calculated for you.

Print your name and the name of your school on the lines above.

A separate answer sheet for Part I has been provided to you. Follow the instructions from the proctor for completing the student information on your answer sheet.

This examination has four parts, with a total of 36 questions. You must answer all questions in this examination. Write your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in Parts II, III, and IV directly in this booklet. All work should be written in pen, except for graphs and drawings, which should be done in pencil. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale.

The formulas that you may need to answer some questions in this examination are found at the end of the examination. This sheet is perforated so you may remove it from this booklet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. You may remove this sheet from this booklet. Any work done on this sheet of scrap graph paper will *not* be scored.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

Notice...

A graphing calculator, a straightedge (ruler), and a compass must be available for you to use while taking this examination.

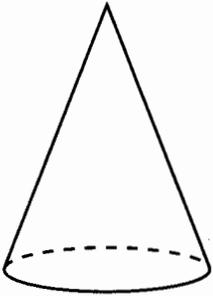
DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.

Part I

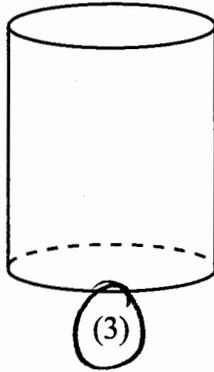
Answer all 24 questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For each statement or question, choose the word or expression that, of those given, best completes the statement or answers the question. Record your answers on your separate answer sheet. [48]

Use this space for computations.

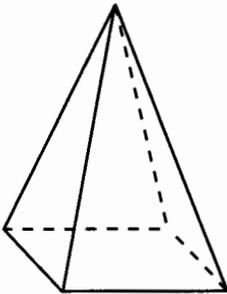
1 A student has a rectangular postcard that he folds in half lengthwise. Next, he rotates it continuously about the folded edge. Which three-dimensional object below is generated by this rotation?



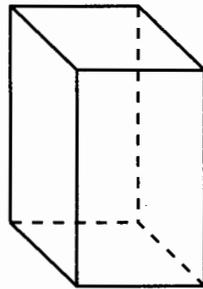
(1)



(3)



(2)



(4)

2 A three-inch line segment is dilated by a scale factor of 6 and centered at its midpoint. What is the length of its image?

(1) 9 inches

(3) 15 inches

(2) 2 inches

(4) 18 inches

$$3 \times 6 = 18$$

Use this space for
computations.

3 Kevin's work for deriving the equation of a circle is shown below.

$$x^2 + 4x = -(y^2 - 20)$$

STEP 1 $x^2 + 4x = -y^2 + 20$

STEP 2 $x^2 + 4x + 4 = -y^2 + 20 - 4$

STEP 3 $(x + 2)^2 = -y^2 + 20 - 4$

STEP 4 $(x + 2)^2 + y^2 = 16$

add 4 to
both sides

In which step did he make an error in his work?

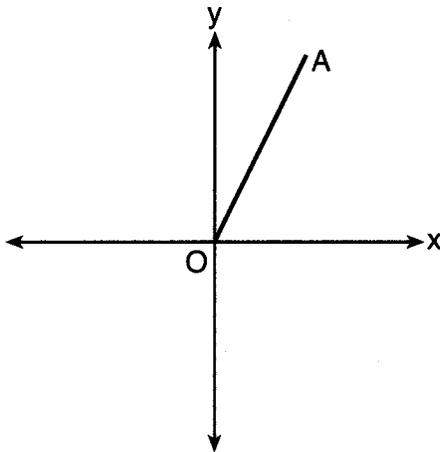
(1) Step 1

(3) Step 3

(2) Step 2

(4) Step 4

4 Which transformation of \overline{OA} would result in an image parallel to \overline{OA} ?



(1) a translation of two units down

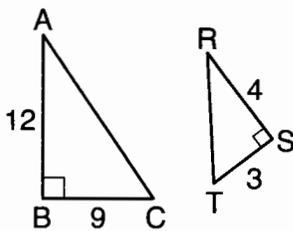
(2) a reflection over the x -axis

(3) a reflection over the y -axis

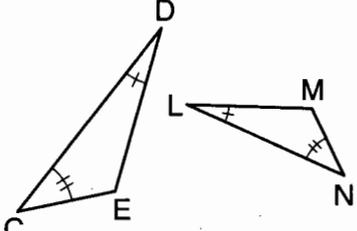
(4) a clockwise rotation of 90° about the origin

Use this space for computations.

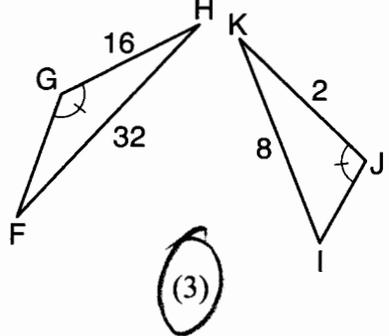
5 Using the information given below, which set of triangles can *not* be proven similar?



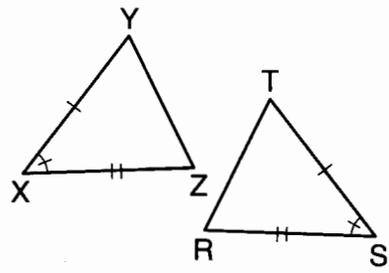
 $\frac{12}{9} = \frac{4}{3}$ (1)



 AA (2)



 $\frac{32}{16} \neq \frac{8}{2}$ (3)



 SAS (4)

6 A company is creating an object from a wooden cube with an edge length of 8.5 cm. A right circular cone with a diameter of 8 cm and an altitude of 8 cm will be cut out of the cube. Which expression represents the volume of the remaining wood?

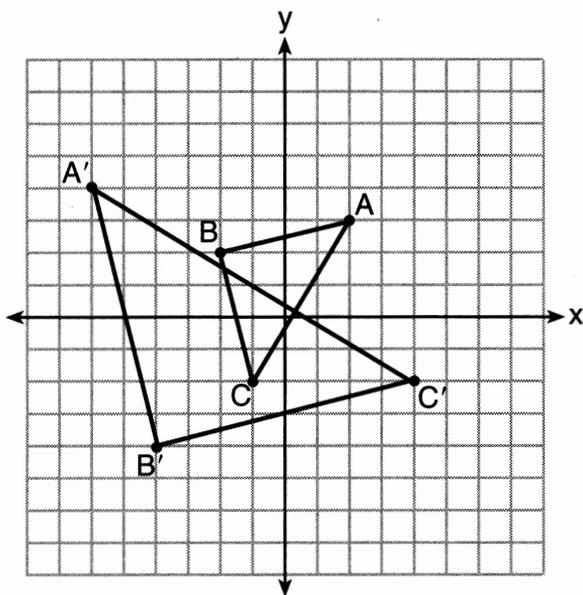
- (1) $(8.5)^3 - \pi(8)^2(8)$ (3) $(8.5)^3 - \frac{1}{3}\pi(8)^2(8)$
 (2) $(8.5)^3 - \pi(4)^2(8)$ (4) $(8.5)^3 - \frac{1}{3}\pi(4)^2(8)$

7 Two right triangles must be congruent if

- (1) an acute angle in each triangle is congruent *only proves similarity AA*
 (2) the lengths of the hypotenuses are equal *need congruent legs for HL*
 (3) the corresponding legs are congruent *SAS*
 (4) the areas are equal *only proves product of altitude & base are equal*

Use this space for computations.

8 Which sequence of transformations will map $\triangle ABC$ onto $\triangle A'B'C'$?



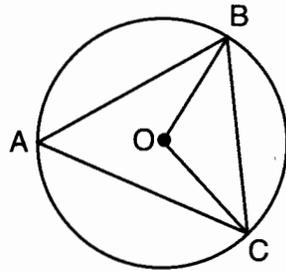
- (1) reflection and translation
- (2) rotation and reflection
- (3) translation and dilation
- (4) dilation and rotation

9 In parallelogram $ABCD$, diagonals \overline{AC} and \overline{BD} intersect at E . Which statement does *not* prove parallelogram $ABCD$ is a rhombus?

- (1) $\overline{AC} \cong \overline{DB}$ opposite sides
- (2) $\overline{AB} \cong \overline{BC}$ adjacent sides
- (3) $\overline{AC} \perp \overline{DB}$ perpendicular diagonals
- (4) \overline{AC} bisects $\angle DCB$. diagonal bisects angle

Use this space for computations.

- 10 In the diagram below of circle O , \overline{OB} and \overline{OC} are radii, and chords \overline{AB} , \overline{BC} , and \overline{AC} are drawn.

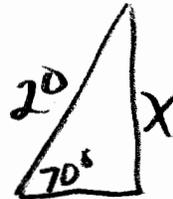


Which statement must always be true?

- (1) $\angle BAC \cong \angle BOC$
 (2) $m\angle BAC = \frac{1}{2}m\angle BOC$
 (3) $\triangle BAC$ and $\triangle BOC$ are isosceles.
 (4) The area of $\triangle BAC$ is twice the area of $\triangle BOC$.

- 11 A 20-foot support post leans against a wall, making a 70° angle with the ground. To the nearest tenth of a foot, how far up the wall will the support post reach?

- (1) 6.8
 (2) 6.9
 (3) 18.7
 (4) 18.8



$$\sin 70 = \frac{x}{20}$$

$$x = 20 \sin 70$$

$$x \approx 18.8$$

- 12 Line segment \overline{NY} has endpoints $N(-11,5)$ and $Y(5,-7)$. What is the equation of the perpendicular bisector of \overline{NY} ?

(1) $y + 1 = \frac{4}{3}(x + 3)$ (3) $y - 6 = \frac{4}{3}(x - 8)$

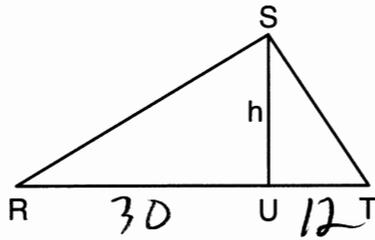
(2) $y + 1 = -\frac{3}{4}(x + 3)$ (4) $y - 6 = -\frac{3}{4}(x - 8)$

Midpoint $\left(\frac{-11+5}{2}, \frac{5+(-7)}{2} \right)$
 $(-3, -1)$

Slope = $\frac{5-(-7)}{-11-5}$
 $= \frac{12}{-16}$
 $= -\frac{3}{4}$ $M_{\perp} = \frac{4}{3}$

Use this space for computations.

13 In $\triangle RST$ shown below, altitude \overline{SU} is drawn to \overline{RT} at U .



$$h^2 = 30 \cdot 12$$

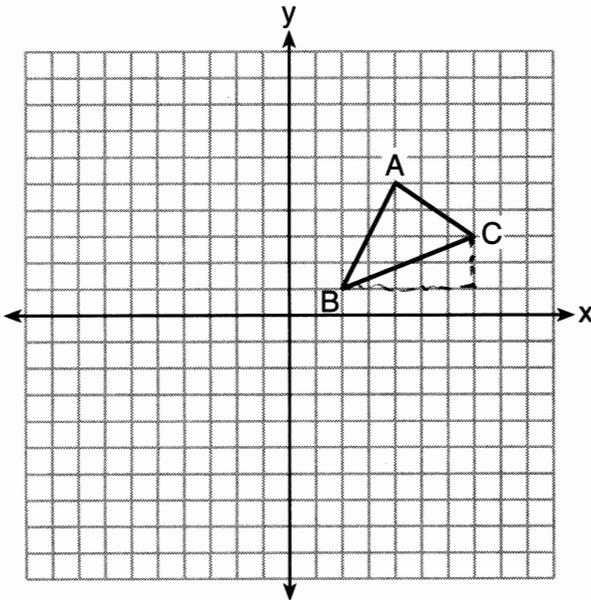
$$h^2 = 360$$

$$h = \sqrt{360} = 6\sqrt{10}$$

If $SU = h$, $UT = 12$, and $RT = 42$, which value of h will make $\triangle RST$ a right triangle with $\angle RST$ as a right angle?

- (1) $6\sqrt{3}$ (3) $6\sqrt{14}$
 (2) $6\sqrt{10}$ (4) $6\sqrt{35}$

14 In the diagram below, $\triangle ABC$ has vertices $A(4,5)$, $B(2,1)$, and $C(7,3)$.



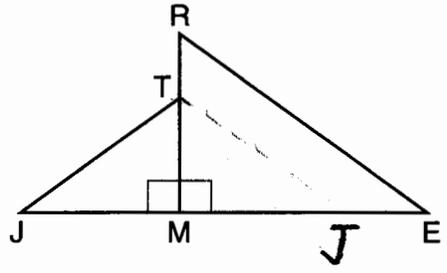
Slope of \overline{BC} is $\frac{2}{5}$
 Altitude is \perp , so
 slope is $-\frac{5}{2}$

What is the slope of the altitude drawn from A to \overline{BC} ?

- (1) $\frac{2}{5}$ (3) $-\frac{1}{2}$
 (2) $\frac{3}{2}$ (4) $-\frac{5}{2}$

Use this space for computations.

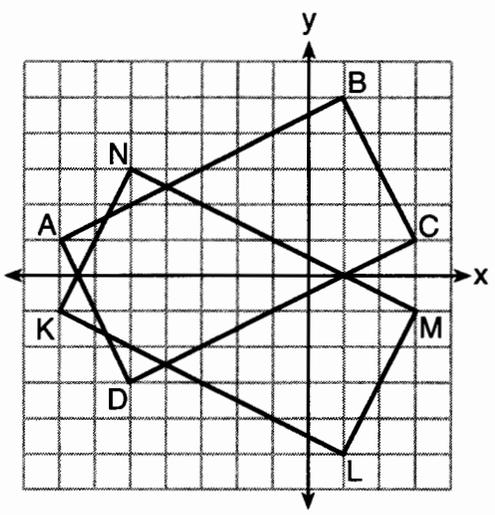
15 In the diagram below, $\triangle ERM \sim \triangle JTM$.



Which statement is always true?

- (1) $\cos J = \frac{RM}{RE}$ $\frac{JM}{JT}$ (3) $\tan T = \frac{RM}{EM}$ $\frac{JM}{TM}$
 (2) $\cos R = \frac{JM}{JT}$ $\frac{RM}{JT}$ (4) $\tan E = \frac{TM}{JM}$ $\frac{RM}{EM}$

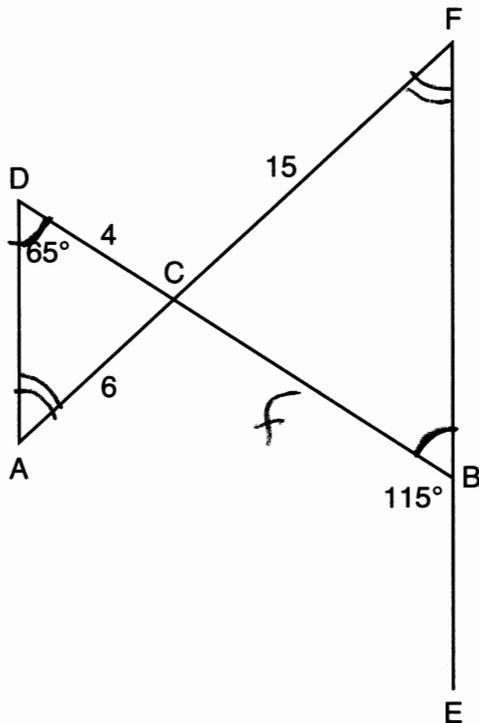
16 On the set of axes below, rectangle $ABCD$ can be proven congruent to rectangle $KLMN$ using which transformation?



- (1) rotation
 (2) translation
 (3) reflection over the x -axis
 (4) reflection over the y -axis

17 In the diagram below, \overline{DB} and \overline{AF} intersect at point C , and \overline{AD} and \overline{FE} are drawn.

Use this space for computations.



$$\frac{f}{4} = \frac{15}{6}$$

$$6f = 60$$

$$f = 10$$

If $AC = 6$, $DC = 4$, $FC = 15$, $m\angle D = 65^\circ$, and $m\angle CBE = 115^\circ$, what is the length of \overline{CB} ?

- (1) 10 (3) 17
 (2) 12 (4) 22.5

18 Seawater contains approximately 1.2 ounces of salt per liter on average. How many gallons of seawater, to the nearest tenth of a gallon, would contain 1 pound of salt?

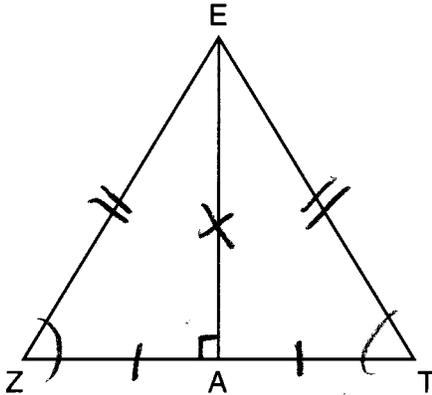
- (1) 3.3 (3) 4.7
 (2) 3.5 (4) 13.3

$$\frac{1.2 \text{ oz}}{1.2 \text{ oz}} \left(\frac{16 \text{ oz}}{1 \text{ lb}} \right) = \frac{13.3 \text{ l}}{\text{lb}}$$

$$\frac{13.3 \text{ l}}{1 \text{ lb}} \frac{1 \text{ g}}{3.785 \text{ l}} \approx \frac{3.5 \text{ g}}{1 \text{ lb}}$$

Use this space for
computations.

- 19 Line segment \overline{EA} is the perpendicular bisector of \overline{ZT} , and \overline{ZE} and \overline{TE} are drawn.



Which conclusion can *not* be proven?

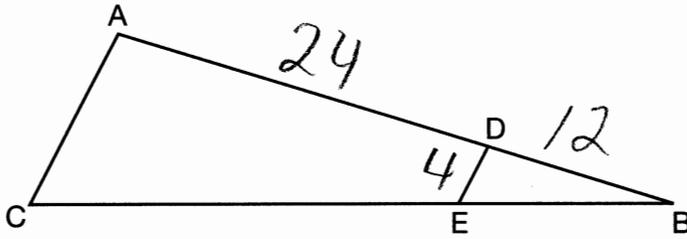
- (1) \overline{EA} bisects angle ZET .
(2) Triangle EZT is equilateral.
(3) \overline{EA} is a median of triangle EZT .
(4) Angle Z is congruent to angle T .
- 20 A hemispherical water tank has an inside diameter of 10 feet. If water has a density of 62.4 pounds per cubic foot, what is the weight of the water in a full tank, to the *nearest pound*?

- (1) 16,336
(2) 32,673
(3) 130,690
(4) 261,381

$$\frac{1}{2} \left(\frac{4}{3} \right) \pi \cdot 5^3 \cdot 62.4 \approx 16,336$$

Use this space for computations.

21 In the diagram of $\triangle ABC$, points D and E are on \overline{AB} and \overline{CB} , respectively, such that $\overline{AC} \parallel \overline{DE}$.



$$\frac{12}{4} = \frac{36}{x}$$

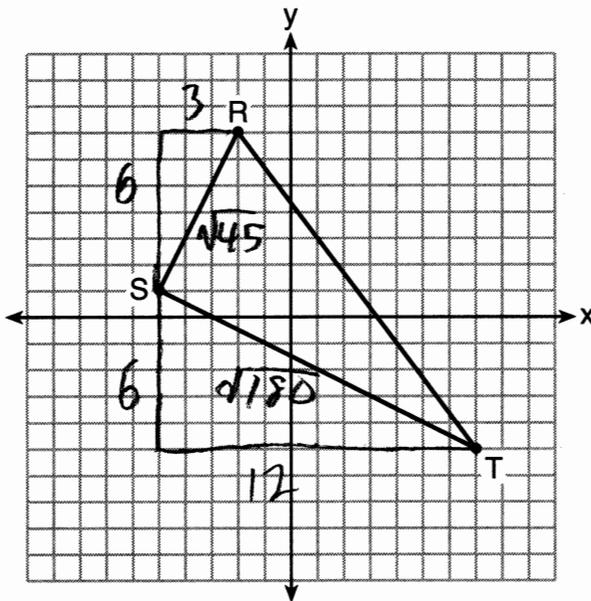
$$12x = 144$$

$$x = 12$$

If $AD = 24$, $DB = 12$, and $DE = 4$, what is the length of \overline{AC} ?

- (1) 8
- (2) 12
- (3) 16
- (4) 72

22 Triangle RST is graphed on the set of axes below.



$$\sqrt{45} = 3\sqrt{5}$$

$$\sqrt{180} = 6\sqrt{5}$$

$$A = \frac{1}{2}(3\sqrt{5})(6\sqrt{5})$$

$$= \frac{1}{2} \cdot 18 \cdot 5$$

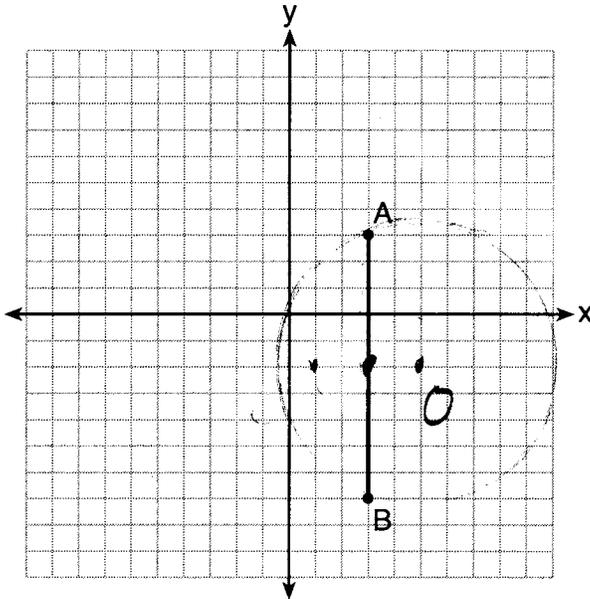
$$= 45$$

How many square units are in the area of $\triangle RST$?

- (1) $9\sqrt{3} + 15$
- (2) $9\sqrt{5} + 15$
- (3) 45
- (4) 90

Use this space for computations.

- 23 The graph below shows \overline{AB} , which is a chord of circle O . The coordinates of the endpoints of \overline{AB} are $A(3,3)$ and $B(3,-7)$. The distance from the midpoint of \overline{AB} to the center of circle O is 2 units.



center is either $(5, -2)$ or $(1, -2)$

What could be a correct equation for circle O ?

- (1) $(x - 1)^2 + (y + 2)^2 = 29$
 (2) ~~$(x + 5)^2 + (y - 2)^2 = 29$~~
 (3) ~~$(x - 1)^2 + (y - 2)^2 = 25$~~
 (4) ~~$(x - 5)^2 + (y + 2)^2 = 25$~~

$$r = \sqrt{2^2 + 5^2}$$

$$r = \sqrt{29}$$

$$r^2 = 29$$

- 24 What is the area of a sector of a circle with a radius of 8 inches and formed by a central angle that measures 60° ?

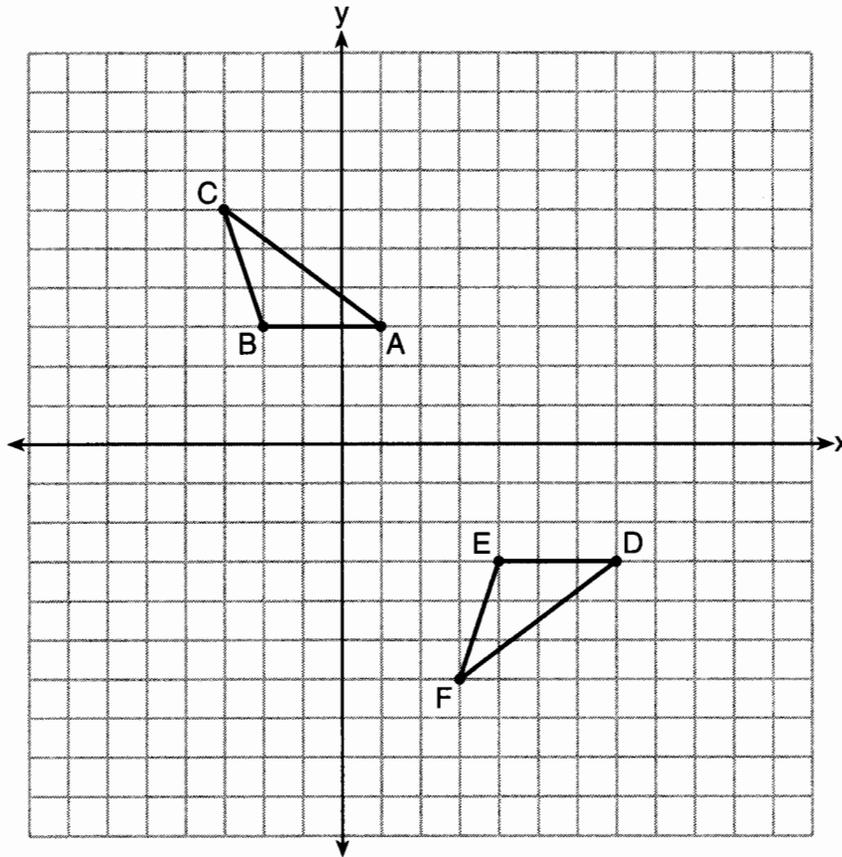
- (1) $\frac{8\pi}{3}$
 (2) $\frac{16\pi}{3}$
 (3) $\frac{32\pi}{3}$
 (4) $\frac{64\pi}{3}$

$$\pi 8^2 \cdot \frac{60}{360} = \frac{64\pi}{6} = \frac{32\pi}{3}$$

Part II

Answer all 7 questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [14]

25 Describe a sequence of transformations that will map $\triangle ABC$ onto $\triangle DEF$ as shown below.



$T_{6,0} \circ R_{x\text{-axis}}$

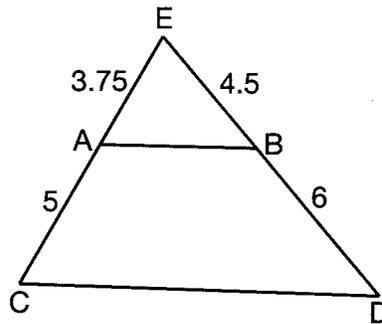
26 Point P is on segment AB such that $AP:PB$ is 4:5. If A has coordinates $(4,2)$, and B has coordinates $(22,2)$, determine and state the coordinates of P .

$$\frac{4}{9}(22-4) = 8 \quad \frac{4}{9}(2-2) = 0$$

$$(4+8, 2+0)$$

$$(12, 2)$$

27 In $\triangle CED$ as shown below, points A and B are located on sides \overline{CE} and \overline{ED} , respectively. Line segment \overline{AB} is drawn such that $AE = 3.75$, $AC = 5$, $EB = 4.5$, and $BD = 6$.



Explain why \overline{AB} is parallel to \overline{CD} .

$$\frac{3.75}{5} = \frac{4.5}{6}$$

$$39.375 = 39.375$$

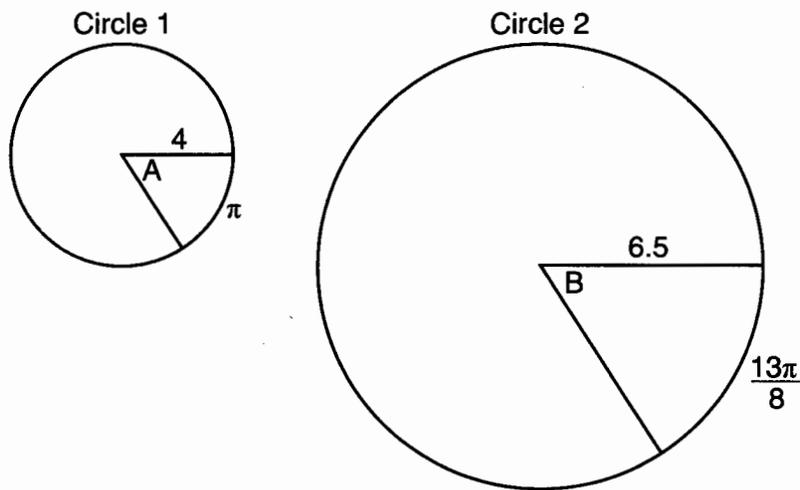
$\overline{AB} \parallel \overline{CD}$ because \overline{AB}
divides the sides
proportionately

- 28 Find the value of R that will make the equation $\sin 73^\circ = \cos R$ true when $0^\circ < R < 90^\circ$.
Explain your answer.

$$90 - 73 = 17$$

Equal cofunctions
are complementary

29 In the diagram below, Circle 1 has radius 4, while Circle 2 has radius 6.5. Angle A intercepts an arc of length π , and angle B intercepts an arc of length $\frac{13\pi}{8}$.

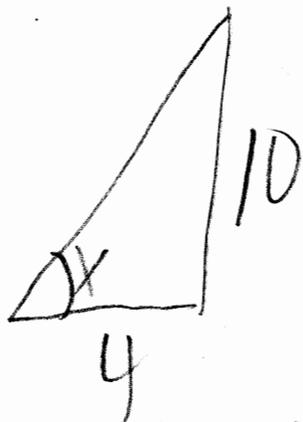


Dominic thinks that angles A and B have the same radian measure. State whether Dominic is correct or not. Explain why.

$$\begin{aligned}
 s &= \theta r & s &= \theta r \\
 \pi &= A \cdot 4 & \frac{13\pi}{8} &= B \cdot 6.5 \\
 \frac{\pi}{4} &= A & \frac{\pi}{4} &= B
 \end{aligned}$$

Yes, both angles are equal

30 A ladder leans against a building. The top of the ladder touches the building 10 feet above the ground. The foot of the ladder is 4 feet from the building. Find, to the *nearest degree*, the angle that the ladder makes with the level ground.

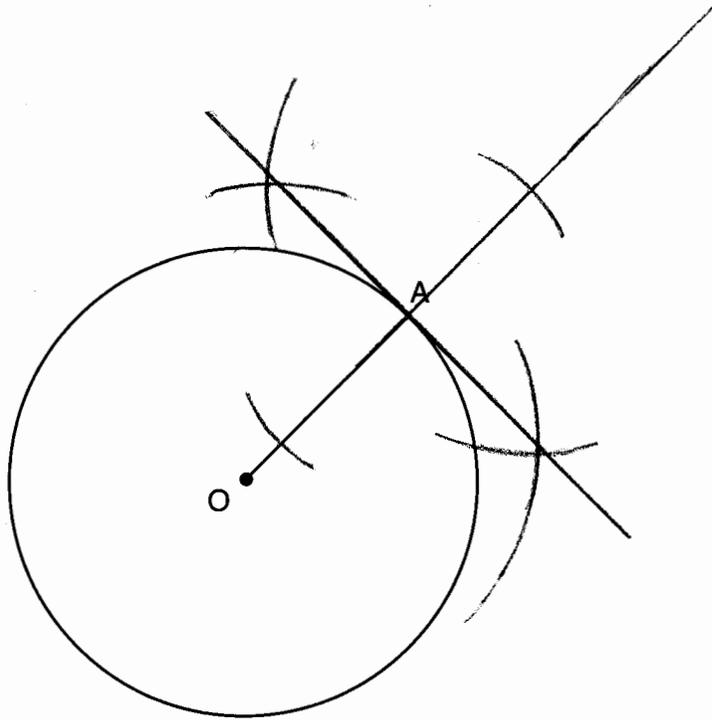


$$\tan x = \frac{10}{4}$$

$$x = \tan^{-1} \frac{10}{4}$$

$$x \approx 68$$

31 In the diagram below, radius \overline{OA} is drawn in circle O . Using a compass and a straightedge, construct a line tangent to circle O at point A . [Leave all construction marks.]



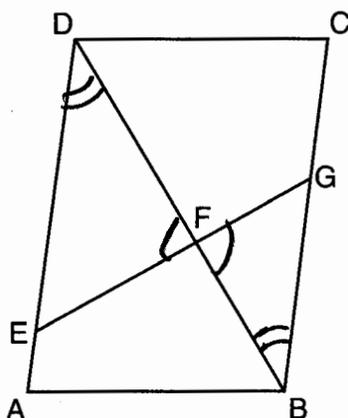
Part III

Answer all 3 questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [12]

- 32 A barrel of fuel oil is a right circular cylinder where the inside measurements of the barrel are a diameter of 22.5 inches and a height of 33.5 inches. There are 231 cubic inches in a liquid gallon. Determine and state, to the *nearest tenth*, the gallons of fuel that are in a barrel of fuel oil.

$$\frac{\pi \cdot 11.25^2 \cdot 33.5}{231} \approx 57.7$$

33 Given: Parallelogram $ABCD$, \overline{EFG} , and diagonal \overline{DFB}



Prove: $\triangle DEF \sim \triangle BGF$

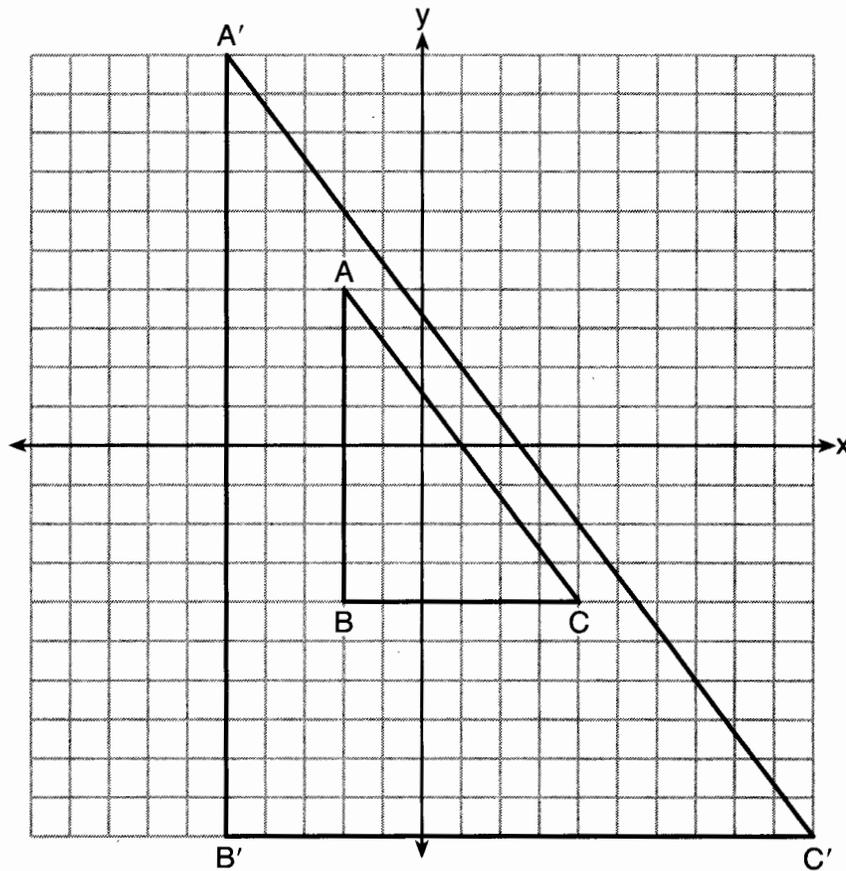
STATEMENT

REASON

- ① Parallelogram $ABCD$, \overline{EFG} & diagonal \overline{DFB}
- ② $\angle DFE \cong \angle BFG$
- ③ $\overline{AD} \parallel \overline{CB}$
- ④ $\angle EDF \cong \angle GBF$
- ⑤ $\triangle DEF \sim \triangle BGF$

- ① Given
- ② Vertical angles
- ③ Opposite sides of a parallelogram are \parallel
- ④ Alternate interior angles
- ⑤ AA

34 In the diagram below, $\triangle A'B'C'$ is the image of $\triangle ABC$ after a transformation.



Describe the transformation that was performed.

A dilation of $\frac{5}{2}$ about the origin

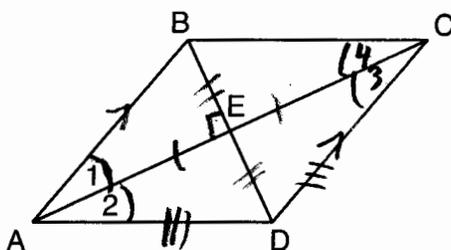
Explain why $\triangle A'B'C' \sim \triangle ABC$.

Dilations preserve angle measure, so the triangles are similar by AA.

Part IV

Answer the 2 questions in this part. Each correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [12]

35 Given: Quadrilateral $ABCD$ with diagonals \overline{AC} and \overline{BD} that bisect each other, and $\angle 1 \cong \angle 2$



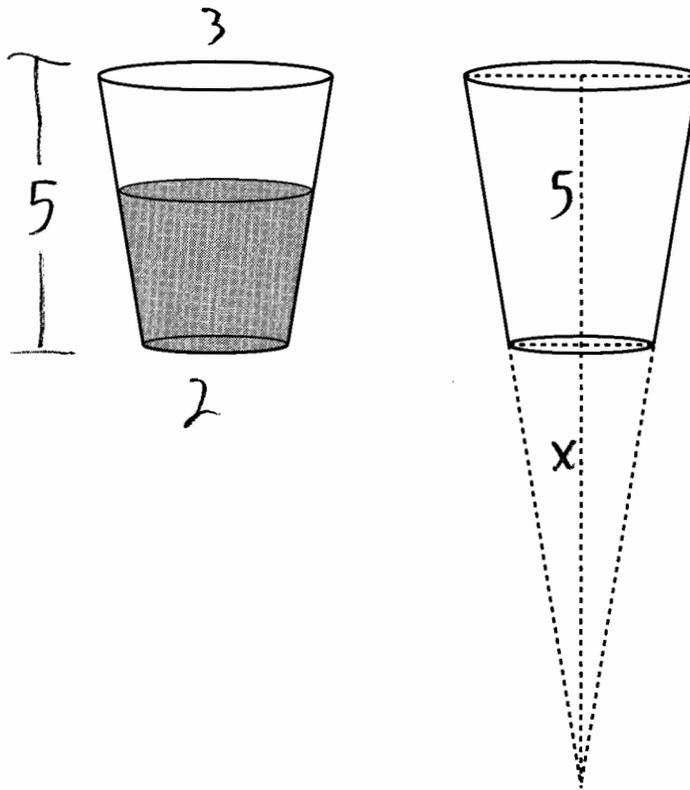
Prove: $\triangle ACD$ is an isosceles triangle and $\triangle AEB$ is a right triangle

STATEMENT

REASON

- | | |
|--|--|
| ① Quad $ABCD$ with diagonals \overline{AC} & \overline{BD} that bisect each other
$\angle 1 \cong \angle 2$ | ① Given |
| ② Quad $ABCD$ is a parallelogram | ② The diagonals of a parallelogram bisect each other |
| ③ $\overline{AB} \parallel \overline{CD}$ | ③ Opposite sides of a parallelogram are \parallel |
| ④ $\angle 1 \cong \angle 3$; $\angle 2 \cong \angle 4$ | ④ Alternate interior angles formed by parallel lines & a transversal are \cong |
| ⑤ $\angle 2 \cong \angle 3$; $\angle 3 \cong \angle 4$ | ⑤ substitution |
| ⑥ $\triangle ACD$ is isosceles | ⑥ The base angles of an isosceles triangle are congruent |
| ⑦ $\overline{AD} \cong \overline{DC}$ | ⑦ The sides of an isosceles \triangle are \cong |
| ⑧ Quad $ABCD$ is a rhombus | ⑧ A rhombus has consecutive congruent sides |
| ⑨ $\overline{AE} \perp \overline{BE}$ | ⑨ The diagonals of a rhombus are \perp |
| ⑩ $\angle BBA$ is a right \angle | ⑩ \perp lines form right \angle 's |
| ⑪ $\triangle AEB$ is a right \triangle | ⑪ A right \triangle has a right \angle . |

36 A water glass can be modeled by a truncated right cone (a cone which is cut parallel to its base) as shown below.



The diameter of the top of the glass is 3 inches, the diameter at the bottom of the glass is 2 inches, and the height of the glass is 5 inches.

The base with a diameter of 2 inches must be parallel to the base with a diameter of 3 inches in order to find the height of the cone. Explain why.

Similar Δ s are required to set up & solve a proportion

Question 36 is continued on the next page.

Question 36 continued

Determine and state, in inches, the height of the larger cone.

$$\frac{x+5}{1.5} = \frac{x}{1}$$
$$x+5 = 1.5x$$
$$5 = .5x$$
$$10 = x$$
$$10+5 = 15$$

Determine and state, to the nearest tenth of a cubic inch, the volume of the water glass.

$$V_{\text{large cone}} = \frac{1}{3}\pi(1.5)^2(15) \approx 35.343$$
$$V_{\text{small cone}} = \frac{1}{3}\pi(1)^2(10) \approx \underline{10.472}$$
$$V_{\text{glass}} \approx 24.9$$