GEOMETRY

The University of the State of New York
REGENTS HIGH SCHOOL EXAMINATION

GEOMETRY

Friday, June 21, 2019 — 9:15 a.m. to 12:15 p.m., only

Student Name: ____________________________________________

School Name: ____________________________________________

The possession or use of any communications device is strictly prohibited when taking this examination. If you have or use any communications device, no matter how briefly, your examination will be invalidated and no score will be calculated for you.

Print your name and the name of your school on the lines above.

A separate answer sheet for Part I has been provided to you. Follow the instructions from the proctor for completing the student information on your answer sheet.

This examination has four parts, with a total of 35 questions. You must answer all questions in this examination. Record your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in Parts II, III, and IV directly in this booklet. All work should be written in pen, except for graphs and drawings, which should be done in pencil. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale.

The formulas that you may need to answer some questions in this examination are found at the end of the examination. This sheet is perforated so you may remove it from this booklet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. You may remove this sheet from this booklet. Any work done on this sheet of scrap graph paper will not be scored.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

Notice...
A graphing calculator, a straightedge (ruler), and a compass must be available for you to use while taking this examination.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.
Part I

Answer all 24 questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For each statement or question, choose the word or expression that, of those given, best completes the statement or answers the question. Record your answers on your separate answer sheet."  [48]

1 On the set of axes below, triangle $ABC$ is graphed. Triangles $A'B'C'$ and $A''B''C''$, the images of triangle $ABC$, are graphed after a sequence of rigid motions.

Identify which sequence of rigid motions maps $\triangle ABC$ onto $\triangle A'B'C'$ and then maps $\triangle A'B'C'$ onto $\triangle A''B''C''$.

(1) a rotation followed by another rotation
(2) a translation followed by a reflection
(3) a reflection followed by a translation
(4) a reflection followed by a rotation

Use this space for computations.
2 The table below shows the population and land area, in square miles, of four counties in New York State at the turn of the century.

<table>
<thead>
<tr>
<th>County</th>
<th>2000 Census Population</th>
<th>2000 Land Area (mi²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Broome</td>
<td>200,536</td>
<td>706.82</td>
</tr>
<tr>
<td>Dutchess</td>
<td>280,150</td>
<td>801.59</td>
</tr>
<tr>
<td>Niagara</td>
<td>219,846</td>
<td>522.95</td>
</tr>
<tr>
<td>Saratoga</td>
<td>200,635</td>
<td>811.84</td>
</tr>
</tbody>
</table>

Which county had the greatest population density?
(1) Broome        (3) Niagara
(2) Dutchess      (4) Saratoga

3 If a rectangle is continuously rotated around one of its sides, what is the three-dimensional figure formed?
(1) rectangular prism (3) sphere
(2) cylinder         (4) cone
4 Which transformation carries the parallelogram below onto itself?

(1) a reflection over $y = x$
(2) a reflection over $y = -x$
(3) a rotation of 90° counterclockwise about the origin
(4) a rotation of 180° counterclockwise about the origin

Use this space for computations.

5 After a dilation centered at the origin, the image of $\overline{CD}$ is $\overline{C'D'}$.
If the coordinates of the endpoints of these segments are $C(6, -4)$, $D(2, -8)$, $C'(9, -6)$, and $D'(3, -12)$, the scale factor of the dilation is

(1) $\frac{3}{2}$
(2) $\frac{2}{3}$
(3) 3
(4) $\frac{1}{3}$
6 A tent is in the shape of a right pyramid with a square floor. The square floor has side lengths of 8 feet. If the height of the tent at its center is 6 feet, what is the volume of the tent, in cubic feet?

(1) 48  (3) 192
(2) 128  (4) 384

7 The line $-3x + 4y = 8$ is transformed by a dilation centered at the origin. Which linear equation could represent its image?

(1) $y = \frac{4}{3}x + 8$  
(2) $y = \frac{3}{4}x + 8$
(3) $y = -\frac{3}{4}x - 8$  
(4) $y = -\frac{4}{3}x - 8$

8 In the diagram below, $AC$ and $BD$ intersect at $E$.

Which information is always sufficient to prove $\triangle ABE \cong \triangle CDE$?

(1) $AB \parallel CD$
(2) $AB \cong CD$ and $BE \cong DE$
(3) $E$ is the midpoint of $AC$.
(4) $BD$ and $AC$ bisect each other.

9 The expression $\sin 57^\circ$ is equal to

(1) $\tan 33^\circ$  
(2) $\cos 33^\circ$
(3) $\tan 57^\circ$  
(4) $\cos 57^\circ$
10 What is the volume of a hemisphere that has a diameter of 12.6 cm, to the nearest tenth of a cubic centimeter?

(1) 523.7  (3) 4189.6
(2) 1047.4  (4) 8379.2

11 In the diagram below of \( \triangle ABC \), \( D \) is a point on \( BA \), \( E \) is a point on \( BC \), and \( DE \) is drawn.

If \( BD = 5 \), \( DA = 12 \), and \( BE = 7 \), what is the length of \( BC \) so that \( AC \parallel DE \)?

(1) 23.8  (3) 15.6
(2) 16.8  (4) 8.6

12 A quadrilateral must be a parallelogram if

(1) one pair of sides is parallel and one pair of angles is congruent
(2) one pair of sides is congruent and one pair of angles is congruent
(3) one pair of sides is both parallel and congruent
(4) the diagonals are congruent
13 In the diagram below of circle $O$, chords $JT$ and $ER$ intersect at $M$.

If $EM = 8$ and $RM = 15$, the lengths of $JM$ and $TM$ could be

(1) 12 and 9.5  
(2) 14 and 8.5  
(3) 16 and 7.5  
(4) 18 and 6.5

14 Triangles $JOE$ and $SAM$ are drawn such that $\angle E \equiv \angle M$ and $EJ \equiv MS$. Which mapping would not always lead to $\triangle JOE \equiv \triangle SAM$?

(1) $\angle J$ maps onto $\angle S$  
(2) $\angle O$ maps onto $\angle A$  
(3) $EO$ maps onto $MA$  
(4) $JO$ maps onto $SA$

15 In $\triangle ABC$ shown below, $\angle ACB$ is a right angle, $E$ is a point on $AC$, and $ED$ is drawn perpendicular to hypotenuse $AB$.

If $AB = 9$, $BC = 6$, and $DE = 4$, what is the length of $AE$?

(1) 5  
(2) 6  
(3) 7  
(4) 8
16 Which equation represents a line parallel to the line whose equation is \(-2x + 3y = -4\) and passes through the point (1,3)?

(1) \(y - 3 = -\frac{3}{2}(x - 1)\)  
(3) \(y + 3 = -\frac{3}{2}(x + 1)\)

(2) \(y - 3 = \frac{2}{3}(x - 1)\)  
(4) \(y + 3 = \frac{2}{3}(x + 1)\)

17 In rhombus \(\text{TIGE}\), diagonals \(\overline{TG}\) and \(\overline{IE}\) intersect at \(R\). The perimeter of \(\text{TIGE}\) is 68, and \(TG = 16\).

What is the length of diagonal \(\overline{IE}\)?

(1) 15  
(2) 30  
(3) 34  
(4) 52

18 In circle \(O\) two secants, \(\overline{ABP}\) and \(\overline{CDP}\), are drawn to external point \(P\). If \(m\widehat{AC} = 72^\circ\), and \(m\widehat{BD} = 34^\circ\), what is the measure of \(\angle P\)?

(1) 19°  
(2) 38°  
(3) 53°  
(4) 106°
19 What are the coordinates of point $C$ on the directed segment from $A(-8,4)$ to $B(10,-2)$ that partitions the segment such that $AC:CB$ is 2:1?

(1) (1,1) (3) (2,−2)
(2) (−2,2) (4) (4,0)

20 The equation of a circle is $x^2 + 8x + y^2 − 12y = 144$. What are the coordinates of the center and the length of the radius of the circle?

(1) center $(4,−6)$ and radius 12
(2) center (−4,6) and radius 12
(3) center $(4,−6)$ and radius 14
(4) center (−4,6) and radius 14

21 In parallelogram $PQRS$, $QP$ is extended to point $T$ and $ST$ is drawn.

If $ST \equiv SP$ and $m\angle R = 130^\circ$, what is $m\angle PST$?

(1) $130^\circ$ (3) $65^\circ$
(2) $80^\circ$ (4) $50^\circ$
22 A 12-foot ladder leans against a building and reaches a window 10 feet above ground. What is the measure of the angle, to the nearest degree, that the ladder forms with the ground?

(1) 34  (3) 50
(2) 40  (4) 56

23 In the diagram of equilateral triangle ABC shown below, E and F are the midpoints of AC and BC, respectively.

If \( EF = 2x + 8 \) and \( AB = 7x - 2 \), what is the perimeter of trapezoid ABFE?

(1) 36  (3) 100
(2) 60  (4) 120

24 Which information is not sufficient to prove that a parallelogram is a square?

(1) The diagonals are both congruent and perpendicular.
(2) The diagonals are congruent and one pair of adjacent sides are congruent.
(3) The diagonals are perpendicular and one pair of adjacent sides are congruent.
(4) The diagonals are perpendicular and one pair of adjacent sides are perpendicular.
25 Triangle $A'B'C'$ is the image of triangle $ABC$ after a dilation with a scale factor of $\frac{1}{2}$ and centered at point $A$. Is triangle $ABC$ congruent to triangle $A'B'C'$? Explain your answer.
26 Determine and state the area of triangle $PQR$, whose vertices have coordinates $P(-2, -5)$, $Q(3,5)$, and $R(6,1)$.

[The use of the set of axes below is optional.]
27 A support wire reaches from the top of a pole to a clamp on the ground. The pole is perpendicular to the level ground and the clamp is 10 feet from the base of the pole. The support wire makes a $68^\circ$ angle with the ground. Find the length of the support wire to the nearest foot.
In the diagram below, circle $O$ has a radius of 10.

If $m\overline{AB} = 72^\circ$, find the area of shaded sector $AOB$, in terms of $\pi$. 
29 On the set of axes below, \( \triangle ABC \cong \triangle STU \).

Describe a sequence of rigid motions that maps \( \triangle ABC \) onto \( \triangle STU \).
30 In right triangle $PRT$, $m\angle P = 90^\circ$, altitude $PQ$ is drawn to hypotenuse $RT$, $RT = 17$, and $PR = 15$.

Determine and state, to the nearest tenth, the length of $RQ$. 

![Diagram of right triangle PRT with altitude PQ drawn to hypotenuse RT]
31 Given circle $O$ with radius $OA$, use a compass and straightedge to construct an equilateral triangle inscribed in circle $O$. [Leave all construction marks.]
32 Riley plotted $A(-1,6)$, $B(3,8)$, $C(6,-1)$, and $D(1,0)$ to form a quadrilateral.

Prove that Riley’s quadrilateral $ABCD$ is a trapezoid.

[The use of the set of axes on the next page is optional.]
Riley defines an isosceles trapezoid as a trapezoid with congruent diagonals. Use Riley’s definition to prove that $ABCD$ is not an isosceles trapezoid.
A child-sized swimming pool can be modeled by a cylinder. The pool has a diameter of \( 6 \frac{1}{2} \) feet and a height of 12 inches. The pool is filled with water to \( \frac{2}{3} \) of its height. Determine and state the volume of the water in the pool, to the nearest cubic foot.

One cubic foot equals 7.48 gallons of water. Determine and state, to the nearest gallon, the number of gallons of water in the pool.
Nick wanted to determine the length of one blade of the windmill pictured below. He stood at a point on the ground 440 feet from the windmill's base. Using surveyor's tools, Nick measured the angle between the ground and the highest point reached by the top blade and found it was $38.8^\circ$. He also measured the angle between the ground and the lowest point of the top blade, and found it was $30^\circ$.

Determine and state a blade's length, $x$, to the nearest foot.
Answer the question in this part. A correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for the question to determine your answer. Note that diagrams are not necessarily drawn to scale. For the question in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [-6]

35 Given: Quadrilateral MATH, HM ≅ AT, HT ≅ AM, HE ⊥ MEA, and HA ⊥ AT

Prove: TA • HA = HE • TH

Work space for question 35 is continued on the next page.
Question 35 continued
Scrap Graph Paper — This sheet will *not* be scored.
Scrap Graph Paper — This sheet will *not* be scored.
### High School Math Reference Sheet

1 inch = 2.54 centimeters   1 kilometer = 0.62 mile   1 cup = 8 fluid ounces  
1 meter = 39.37 inches   1 pound = 16 ounces   1 pint = 2 cups  
1 mile = 5280 feet   1 pound = 0.454 kilogram   1 quart = 2 pints  
1 mile = 1760 yards   1 kilogram = 2.2 pounds   1 gallon = 4 quarts  
1 mile = 1.609 kilometers   1 ton = 2000 pounds   1 gallon = 3.785 liters  
   1 liter = 0.264 gallon  
   1 liter = 1000 cubic centimeters

<table>
<thead>
<tr>
<th>Geometry</th>
<th>Formula</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Triangle</td>
<td>$A = \frac{1}{2}bh$</td>
<td></td>
</tr>
<tr>
<td>Parallelogram</td>
<td>$A = bh$</td>
<td></td>
</tr>
<tr>
<td>Circle</td>
<td>$A = \pi r^2$</td>
<td></td>
</tr>
<tr>
<td>Circle</td>
<td>$C = \pi d$ or $C = 2\pi r$</td>
<td></td>
</tr>
<tr>
<td>General Prisms</td>
<td>$V = Bh$</td>
<td></td>
</tr>
<tr>
<td>Cylinder</td>
<td>$V = \pi r^2h$</td>
<td></td>
</tr>
<tr>
<td>Sphere</td>
<td>$V = \frac{4}{3}\pi r^3$</td>
<td></td>
</tr>
<tr>
<td>Cone</td>
<td>$V = \frac{1}{3}\pi r^2h$</td>
<td></td>
</tr>
<tr>
<td>Pyramid</td>
<td>$V = \frac{1}{3}Bh$</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Mathematical Concepts</th>
<th>Formula</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pythagorean Theorem</td>
<td>$a^2 + b^2 = c^2$</td>
<td></td>
</tr>
<tr>
<td>Quadratic Formula</td>
<td>$x = -b \pm \sqrt{b^2 - 4ac} \over 2a$</td>
<td></td>
</tr>
<tr>
<td>Arithmetic Sequence</td>
<td>$a_n = a_1 + (n - 1)d$</td>
<td></td>
</tr>
<tr>
<td>Geometric Sequence</td>
<td>$a_n = a_1r^{n-1}$</td>
<td></td>
</tr>
<tr>
<td>Geometric Series</td>
<td>$S_n = {a_1 - a_1r^n \over 1 - r}$ where $r \neq 1$</td>
<td></td>
</tr>
<tr>
<td>Radians</td>
<td>$1 \text{ radian} = \frac{180}{\pi} \text{ degrees}$</td>
<td></td>
</tr>
<tr>
<td>Degrees</td>
<td>$1 \text{ degree} = \frac{\pi}{180} \text{ radians}$</td>
<td></td>
</tr>
<tr>
<td>Exponential Growth/Decay</td>
<td>$A = A_0e^{kt - t_0} + B_0$</td>
<td></td>
</tr>
</tbody>
</table>

---

Geometry – June ’19 [27]
The chart for determining students’ final examination scores for the June 2019 Regents Examination in Geometry will be posted on the Department's web site at: http://www.p12.nysed.gov/assessment/ on the day of the examination. Conversion charts provided for the previous administrations of the Regents Examination in Geometry must NOT be used to determine students’ final scores for this administration.

### Regents Examination in Geometry – June 2019

**Scoring Key: Part I (Multiple-Choice Questions)**

<table>
<thead>
<tr>
<th>Examination</th>
<th>Date</th>
<th>Question Number</th>
<th>Scoring Key</th>
<th>Question Type</th>
<th>Credit</th>
<th>Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>Geometry</td>
<td>June ’19</td>
<td>1</td>
<td>4</td>
<td>MC</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Geometry</td>
<td>June ’19</td>
<td>2</td>
<td>3</td>
<td>MC</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Geometry</td>
<td>June ’19</td>
<td>3</td>
<td>2</td>
<td>MC</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Geometry</td>
<td>June ’19</td>
<td>4</td>
<td>4</td>
<td>MC</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Geometry</td>
<td>June ’19</td>
<td>5</td>
<td>1</td>
<td>MC</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Geometry</td>
<td>June ’19</td>
<td>6</td>
<td>2</td>
<td>MC</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Geometry</td>
<td>June ’19</td>
<td>7</td>
<td>2</td>
<td>MC</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Geometry</td>
<td>June ’19</td>
<td>8</td>
<td>4</td>
<td>MC</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Geometry</td>
<td>June ’19</td>
<td>9</td>
<td>2</td>
<td>MC</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Geometry</td>
<td>June ’19</td>
<td>10</td>
<td>1</td>
<td>MC</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Geometry</td>
<td>June ’19</td>
<td>11</td>
<td>1</td>
<td>MC</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Geometry</td>
<td>June ’19</td>
<td>12</td>
<td>3</td>
<td>MC</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Geometry</td>
<td>June ’19</td>
<td>13</td>
<td>3</td>
<td>MC</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Geometry</td>
<td>June ’19</td>
<td>14</td>
<td>4</td>
<td>MC</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Geometry</td>
<td>June ’19</td>
<td>15</td>
<td>2</td>
<td>MC</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Geometry</td>
<td>June ’19</td>
<td>16</td>
<td>2</td>
<td>MC</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Geometry</td>
<td>June ’19</td>
<td>17</td>
<td>2</td>
<td>MC</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Geometry</td>
<td>June ’19</td>
<td>18</td>
<td>1</td>
<td>MC</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Geometry</td>
<td>June ’19</td>
<td>19</td>
<td>4</td>
<td>MC</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Geometry</td>
<td>June ’19</td>
<td>20</td>
<td>4</td>
<td>MC</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Geometry</td>
<td>June ’19</td>
<td>21</td>
<td>2</td>
<td>MC</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Geometry</td>
<td>June ’19</td>
<td>22</td>
<td>4</td>
<td>MC</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Geometry</td>
<td>June ’19</td>
<td>23</td>
<td>3</td>
<td>MC</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Geometry</td>
<td>June ’19</td>
<td>24</td>
<td>3</td>
<td>MC</td>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>

**Regents Examination in Geometry – June 2019

Scoring Key: Parts II, III, and IV (Constructed-Response Questions)**

<table>
<thead>
<tr>
<th>Examination</th>
<th>Date</th>
<th>Question Number</th>
<th>Scoring Key</th>
<th>Question Type</th>
<th>Credit</th>
<th>Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>Geometry</td>
<td>June ’19</td>
<td>25</td>
<td>-</td>
<td>CR</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Geometry</td>
<td>June ’19</td>
<td>26</td>
<td>-</td>
<td>CR</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Geometry</td>
<td>June ’19</td>
<td>27</td>
<td>-</td>
<td>CR</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Geometry</td>
<td>June ’19</td>
<td>28</td>
<td>-</td>
<td>CR</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Geometry</td>
<td>June ’19</td>
<td>29</td>
<td>-</td>
<td>CR</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Geometry</td>
<td>June ’19</td>
<td>30</td>
<td>-</td>
<td>CR</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Geometry</td>
<td>June ’19</td>
<td>31</td>
<td>-</td>
<td>CR</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Geometry</td>
<td>June ’19</td>
<td>32</td>
<td>-</td>
<td>CR</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>Geometry</td>
<td>June ’19</td>
<td>33</td>
<td>-</td>
<td>CR</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>Geometry</td>
<td>June ’19</td>
<td>34</td>
<td>-</td>
<td>CR</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>Geometry</td>
<td>June ’19</td>
<td>35</td>
<td>-</td>
<td>CR</td>
<td>6</td>
<td>1</td>
</tr>
</tbody>
</table>

**Key**

- **MC** = Multiple-choice question
- **CR** = Constructed-response question

---

Geometry Scoring Key 1 of 1
RATING GUIDE

Updated information regarding the rating of this examination may be posted on the New York State Education Department's web site during the rating period. Check this web site at: http://www.p12.nysed.gov/assessment/ and select the link “Scoring Information” for any recently posted information regarding this examination. This site should be checked before the rating process for this examination begins and several times throughout the Regents Examination period.

The Department is providing supplemental scoring guidance, the “Model Response Set,” for the Regents Examination in Geometry. This guidance is intended to be part of the scorer training. Schools should use the Model Response Set along with the rubrics in the Scoring Key and Rating Guide to help guide scoring of student work. While not reflective of all scenarios, the Model Response Set illustrates how less common student responses to constructed-response questions may be scored. The Model Response Set will be available on the Department’s web site at: http://www.nysedregents.org/geometryre/.
Mechanics of Rating

The following procedures are to be followed for scoring student answer papers for the Regents Examination in Geometry. More detailed information about scoring is provided in the publication Information Booklet for Scoring the Regents Examination in Geometry.

Do not attempt to correct the student's work by making insertions or changes of any kind. In scoring the constructed-response questions, use check marks to indicate student errors. Unless otherwise specified, mathematically correct variations in the answers will be allowed. Units need not be given when the wording of the questions allows such omissions.

Each student's answer paper is to be scored by a minimum of three mathematics teachers. No one teacher is to score more than approximately one-third of the constructed-response questions on a student's paper. Teachers may not score their own students' answer papers. On the student's separate answer sheet, for each question, record the number of credits earned and the teacher's assigned rater/scorer letter.

Schools are not permitted to rescore any of the constructed-response questions on this exam after each question has been rated once, regardless of the final exam score. Schools are required to ensure that the raw scores have been added correctly and that the resulting scale score has been determined accurately.

Raters should record the student's scores for all questions and the total raw score on the student's separate answer sheet. Then the student's total raw score should be converted to a scale score by using the conversion chart that will be posted on the Department's web site at: http://www.p12.nysed.gov/assessment/ on Friday, June 21, 2019. Because scale scores corresponding to raw scores in the conversion chart may change from one administration to another, it is crucial that, for each administration, the conversion chart provided for that administration be used to determine the student's final score. The student's scale score should be entered in the box provided on the student's separate answer sheet. The scale score is the student's final examination score.
General Rules for Applying Mathematics Rubrics

I. General Principles for Rating
The rubrics for the constructed-response questions on the Regents Examination in Geometry are designed to provide a systematic, consistent method for awarding credit. The rubrics are not to be considered all-inclusive; it is impossible to anticipate all the different methods that students might use to solve a given problem. Each response must be rated carefully using the teacher's professional judgment and knowledge of mathematics; all calculations must be checked. The specific rubrics for each question must be applied consistently to all responses. In cases that are not specifically addressed in the rubrics, raters must follow the general rating guidelines in the publication Information Booklet for Scoring the Regents Examination in Geometry, use their own professional judgment, confer with other mathematics teachers, and/or contact the State Education Department for guidance. During each Regents Examination administration period, rating questions may be referred directly to the Education Department. The contact numbers are sent to all schools before each administration period.

II. Full-Credit Responses
A full-credit response provides a complete and correct answer to all parts of the question. Sufficient work is shown to enable the rater to determine how the student arrived at the correct answer. When the rubric for the full-credit response includes one or more examples of an acceptable method for solving the question (usually introduced by the phrase “such as”), it does not mean that there are no additional acceptable methods of arriving at the correct answer. Unless otherwise specified, mathematically correct alternative solutions should be awarded credit. The only exceptions are those questions that specify the type of solution that must be used; e.g., an algebraic solution or a graphic solution. A correct solution using a method other than the one specified is awarded half the credit of a correct solution using the specified method.

III. Appropriate Work
Full-Credit Responses: The directions in the examination booklet for all the constructed-response questions state: “Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc.” The student has the responsibility of providing the correct answer and showing how that answer was obtained. The student must “construct” the response; the teacher should not have to search through a group of seemingly random calculations scribbled on the student paper to ascertain what method the student may have used.

Responses With Errors: Rubrics that state “Appropriate work is shown, but…” are intended to be used with solutions that show an essentially complete response to the question but contain certain types of errors, whether computational, rounding, graphing, or conceptual. If the response is incomplete; i.e., an equation is written but not solved or an equation is solved but not all of the parts of the question are answered, appropriate work has not been shown. Other rubrics address incomplete responses.

IV. Multiple Errors
Computational Errors, Graphing Errors, and Rounding Errors: Each of these types of errors results in a 1-credit deduction. Any combination of two of these types of errors results in a 2-credit deduction. No more than 2 credits should be deducted for such mechanical errors in a 4-credit question and no more than 3 credits should be deducted in a 6-credit question. The teacher must carefully review the student's work to determine what errors were made and what type of errors they were.

Conceptual Errors: A conceptual error involves a more serious lack of knowledge or procedure. Examples of conceptual errors include using the incorrect formula for the area of a figure, choosing the incorrect trigonometric function, or multiplying the exponents instead of adding them when multiplying terms with exponents.

If a response shows repeated occurrences of the same conceptual error, the student should not be penalized twice. If the same conceptual error is repeated in responses to other questions, credit should be deducted in each response.

For 4- and 6-credit questions, if a response shows one conceptual error and one computational, graphing, or rounding error, the teacher must award credit that takes into account both errors. Refer to the rubric for specific scoring guidelines.
Part II

For each question, use the specific criteria to award a maximum of 2 credits. Unless otherwise specified, mathematically correct alternative solutions should be awarded appropriate credit.

(25) [2] No is indicated, and a correct explanation is written.
[1] No, but the explanation is incomplete or partially correct.
[0] No, but the explanation is missing or incorrect.

or

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

(26) [2] 25, and correct work is shown.
[1] Appropriate work is shown, but one computational or graphing error is made.

or

[1] Appropriate work is shown, but one conceptual error is made.

or

[1] 25, but no work is shown.

[0] Triangle PQR is graphed correctly, but no further correct work is shown.

or

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

(27) [2] 27, and correct work is shown.
[1] Appropriate work is shown, but one computational or rounding error is made.

or

[1] Appropriate work is shown, but one conceptual error is made.

or

[1] A correct relevant trigonometric equation is written, but no further correct work is shown.

or

[1] 27, but no work is shown.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.
(28)  
20π, and correct work is shown.

[2]  
[1]  Appropriate work is shown, but one computational error is made.

or

[1]  Appropriate work is shown, but one conceptual error is made.

or

[1]  Appropriate work is shown, but the answer is not in terms of π.

or

[1]  20π, but no work is shown.

[0]  A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

(29)  
2 A correct sequence of rigid motions is written.

[2]  
[1]  An appropriate sequence of rigid motions is written, but one computational error is made.

or

[1]  An appropriate sequence of rigid motions is written, but one conceptual error is made.

or

[1]  An appropriate sequence of rigid motions is written, but it is incomplete or partially correct.

[0]  A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.
(30) [2] 13.2, and correct work is shown.

[1] Appropriate work is shown, but one computational or rounding error is made.

or

[1] Appropriate work is shown, but one conceptual error is made.

or

[1] A correct equation is written to find $RQ$, but no further correct work is shown.

or

[1] 13.2, but no work is shown.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

(31) [2] A correct construction is drawn showing all appropriate arcs, and the equilateral triangle is drawn.

[1] All appropriate arcs are drawn, but the triangle is not drawn.

or

[1] Appropriate work is shown, but one construction error is made.

or

[1] A correct construction of an equilateral triangle is drawn in circle $O$ using the length of the radius as a side, but it is not inscribed in circle $O$.

[0] A drawing that is not an appropriate construction is shown.

or

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.
Part III

For each question, use the specific criteria to award a maximum of 4 credits. Unless otherwise specified, mathematically correct alternative solutions should be awarded appropriate credit.

(32)  
[4] Correct work is shown to prove $ABCD$ is a trapezoid and not an isosceles trapezoid. Correct concluding statements are made.

[3] Appropriate work is shown, but one computational or graphing error is made. Concluding statements are written.

  or

[3] Appropriate work is shown, but one concluding statement is missing or incorrect.

[2] Appropriate work is shown, but two or more computational or graphing errors are made.

  or

[2] Correct work is shown to prove $ABCD$ is a trapezoid, and a correct concluding statement is written, but no further correct work is shown.

  or

[2] Appropriate work is shown to prove $ABCD$ is not an isosceles trapezoid, and an appropriate concluding statement is written, but no further correct work is shown.

[1] Appropriate work is shown, but two or more computational or graphing errors are made, and one concluding statement is missing or incorrect.

  or

[1] Appropriate work is shown to prove $ABCD$ is a trapezoid, but the concluding statement is missing or incorrect.

  or

[1] Appropriate work is shown to prove $ABCD$ is not an isosceles trapezoid, but a method other than congruent diagonals is used.

  or

[1] Appropriate work is shown to find the slopes of $AD$ and $BC$, but no further correct work is shown.

  or

[1] Appropriate work is shown to find $AC$ and $BD$, but no further correct work is shown.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.
[4] 22 and 165, and correct work is shown.

[3] Appropriate work is shown, but one computational or rounding error is made.

[2] Appropriate work is shown, but two or more computational or rounding errors are made.

or

[2] Correct work is shown to find 22, but no further correct work is shown.

[1] Appropriate work is shown to find the volume of the entire pool in cubic inches or cubic feet, but no further correct work is shown.

or

[1] 22 and 165, but no work is shown.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.
(34)  [4] 100, and correct work is shown.

[3] Appropriate work is shown, but one computational error is made.

   or

[3] Appropriate work is shown to find the height to the bottom of the top blade
and the height to the top of the top blade, but no further correct work is shown.

[2] Appropriate work is shown, but two or more computational or rounding errors
are made.

   or

[2] Appropriate work is shown, but one conceptual error is made.

   or

[2] Appropriate work is shown to find the height to the bottom of the top blade
or the height to the top of the top blade, but no further correct work is shown.

[1] Appropriate work is shown, but one conceptual error and one computational
or rounding error are made.

   or

[1] At least one correct relevant trigonometric equation is written, but no further
correct work is shown.

   or

[1] 100, but no work is shown.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a
correct response that was obtained by an obviously incorrect procedure.
Part IV

For this question, use the specific criteria to award a maximum of 6 credits. Unless otherwise specified, mathematically correct alternative solutions should be awarded appropriate credit.

(35)  

[6] A complete and correct proof that includes a conclusion is written.

[5] A proof is written that demonstrates a thorough understanding of the method of proof and contains no conceptual errors, but one statement and/or reason is missing or incorrect.

[4] A proof is written that demonstrates a good understanding of the method of proof and contains no conceptual errors, but two statements and/or reasons are missing or incorrect.

or

[4] A proof is written that demonstrates a good understanding of the method of proof, but one conceptual error is made.

or

[4] Correct work is shown to prove \( \triangle HEA \sim \triangle TAH \), but no further correct work is shown.

[3] A proof is written that demonstrates a method of proof, but three statements and/or reasons are missing or incorrect.

or

[3] A proof is written that demonstrates a method of proof, but one conceptual error is made and one statement and/or reason is missing or incorrect.

[2] Some correct relevant statements about the proof are made, but four statements and/or reasons are missing or incorrect.

or

[2] Correct work is shown to prove \( \angle HEA \equiv \angle TAH \) or \( \angle EAH \equiv \angle AHT \), but no further correct work is shown.

[1] Only one correct statement and reason are written.

[0] The “given” and/or the “prove” statements are written, but no further correct relevant statements are written.

or

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.
<table>
<thead>
<tr>
<th>Question</th>
<th>Type</th>
<th>Credits</th>
<th>Cluster</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>G-CO.A</td>
</tr>
<tr>
<td>2</td>
<td>Multiple Choice</td>
<td>2</td>
<td>G-MG.A</td>
</tr>
<tr>
<td>3</td>
<td>Multiple Choice</td>
<td>2</td>
<td>G-GMD.B</td>
</tr>
<tr>
<td>4</td>
<td>Multiple Choice</td>
<td>2</td>
<td>G-CO.A</td>
</tr>
<tr>
<td>5</td>
<td>Multiple Choice</td>
<td>2</td>
<td>G-SRT.A</td>
</tr>
<tr>
<td>6</td>
<td>Multiple Choice</td>
<td>2</td>
<td>G-MG.A</td>
</tr>
<tr>
<td>7</td>
<td>Multiple Choice</td>
<td>2</td>
<td>G-SRT.A</td>
</tr>
<tr>
<td>8</td>
<td>Multiple Choice</td>
<td>2</td>
<td>G-CO.C</td>
</tr>
<tr>
<td>9</td>
<td>Multiple Choice</td>
<td>2</td>
<td>G-SRT.C</td>
</tr>
<tr>
<td>10</td>
<td>Multiple Choice</td>
<td>2</td>
<td>G-GMD.A</td>
</tr>
<tr>
<td>11</td>
<td>Multiple Choice</td>
<td>2</td>
<td>G-SRT.B</td>
</tr>
<tr>
<td>12</td>
<td>Multiple Choice</td>
<td>2</td>
<td>G-CO.C</td>
</tr>
<tr>
<td>13</td>
<td>Multiple Choice</td>
<td>2</td>
<td>G-C.A</td>
</tr>
<tr>
<td>14</td>
<td>Multiple Choice</td>
<td>2</td>
<td>G-CO.B</td>
</tr>
<tr>
<td>15</td>
<td>Multiple Choice</td>
<td>2</td>
<td>G-SRT.B</td>
</tr>
<tr>
<td>16</td>
<td>Multiple Choice</td>
<td>2</td>
<td>G-GPE.B</td>
</tr>
<tr>
<td>17</td>
<td>Multiple Choice</td>
<td>2</td>
<td>G-CO.C</td>
</tr>
<tr>
<td>18</td>
<td>Multiple Choice</td>
<td>2</td>
<td>G-C.A</td>
</tr>
<tr>
<td>19</td>
<td>Multiple Choice</td>
<td>2</td>
<td>G-GPE.B</td>
</tr>
<tr>
<td>20</td>
<td>Multiple Choice</td>
<td>2</td>
<td>G-GPE.A</td>
</tr>
<tr>
<td>21</td>
<td>Multiple Choice</td>
<td>2</td>
<td>G-CO.C</td>
</tr>
<tr>
<td>22</td>
<td>Multiple Choice</td>
<td>2</td>
<td>G-SRT.C</td>
</tr>
<tr>
<td>23</td>
<td>Multiple Choice</td>
<td>2</td>
<td>G-CO.C</td>
</tr>
<tr>
<td>24</td>
<td>Multiple Choice</td>
<td>2</td>
<td>G-CO.C</td>
</tr>
<tr>
<td>25</td>
<td>Constructed Response</td>
<td>2</td>
<td>G-CO.B</td>
</tr>
<tr>
<td>26</td>
<td>Constructed Response</td>
<td>2</td>
<td>G-GPE.B</td>
</tr>
<tr>
<td>27</td>
<td>Constructed Response</td>
<td>2</td>
<td>G-SRT.C</td>
</tr>
<tr>
<td>28</td>
<td>Constructed Response</td>
<td>2</td>
<td>G-C.B</td>
</tr>
<tr>
<td>29</td>
<td>Constructed Response</td>
<td>2</td>
<td>G-CO.B</td>
</tr>
<tr>
<td>30</td>
<td>Constructed Response</td>
<td>2</td>
<td>G-SRT.B</td>
</tr>
<tr>
<td>31</td>
<td>Constructed Response</td>
<td>2</td>
<td>G-CO.D</td>
</tr>
<tr>
<td>32</td>
<td>Constructed Response</td>
<td>4</td>
<td>G-GPE.B</td>
</tr>
<tr>
<td>33</td>
<td>Constructed Response</td>
<td>4</td>
<td>G-MG.A</td>
</tr>
<tr>
<td>34</td>
<td>Constructed Response</td>
<td>4</td>
<td>G-SRT.C</td>
</tr>
<tr>
<td>35</td>
<td>Constructed Response</td>
<td>6</td>
<td>G-SRT.B</td>
</tr>
</tbody>
</table>
Online Submission of Teacher Evaluations of the Test to the Department

Suggestions and feedback from teachers provide an important contribution to the test development process. The Department provides an online evaluation form for State assessments. It contains spaces for teachers to respond to several specific questions and to make suggestions. Instructions for completing the evaluation form are as follows:


2. Select the test title.

3. Complete the required demographic fields.

4. Complete each evaluation question and provide comments in the space provided.

5. Click the SUBMIT button at the bottom of the page to submit the completed form.
# Model Response Set

**Table of Contents**

<table>
<thead>
<tr>
<th>Question</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Question 25</td>
<td>2</td>
</tr>
<tr>
<td>Question 26</td>
<td>7</td>
</tr>
<tr>
<td>Question 27</td>
<td>11</td>
</tr>
<tr>
<td>Question 28</td>
<td>16</td>
</tr>
<tr>
<td>Question 29</td>
<td>21</td>
</tr>
<tr>
<td>Question 30</td>
<td>28</td>
</tr>
<tr>
<td>Question 31</td>
<td>33</td>
</tr>
<tr>
<td>Question 32</td>
<td>39</td>
</tr>
<tr>
<td>Question 33</td>
<td>55</td>
</tr>
<tr>
<td>Question 34</td>
<td>63</td>
</tr>
<tr>
<td>Question 35</td>
<td>71</td>
</tr>
</tbody>
</table>
25 Triangle $A'B'C'$ is the image of triangle $ABC$ after a dilation with a scale factor of $\frac{1}{2}$ and centered at point $A$. Is triangle $ABC$ congruent to triangle $A'B'C'$? Explain your answer.

No $\triangle ABC$ isn't congruent to $\triangle A'B'C'$ because a dilation doesn't preserve segment lengths.

Score 2: The student gave a complete and correct response.
25 Triangle $A'B'C'$ is the image of triangle $ABC$ after a dilation with a scale factor of $\frac{1}{2}$ and centered at point $A$. Is triangle $ABC$ congruent to triangle $A'B'C'$? Explain your answer.

No, they're not congruent $\triangle ABC$ had a dilation of $\frac{1}{2}$ so the image will be a lot smaller than $ABC$. It will be the same shape but different size.

Score 2: The student gave a complete and correct response.
25 Triangle $A'B'C'$ is the image of triangle $ABC$ after a dilation with a scale factor of $\frac{1}{2}$ and centered at point $A$. Is triangle $ABC$ congruent to triangle $A'B'C'$? Explain your answer.

No, dilatations always produce similar triangles.

Score 1:  The student wrote an incomplete explanation.
25 Triangle $A'B'C'$ is the image of triangle $ABC$ after a dilation with a scale factor of $\frac{1}{2}$ and centered at point $A$. Is triangle $ABC$ congruent to triangle $A'B'C'$? Explain your answer.

No, a dilation is not a rigid motion.

Score 1: The student wrote an incomplete explanation.
Triangle $A'B'C'$ is the image of triangle $ABC$ after a dilation with a scale factor of $\frac{1}{2}$ and centered at point $A$. Is triangle $ABC$ congruent to triangle $A'B'C'$? Explain your answer.

Yes $\triangle ABC \cong \triangle A'B'C'$ by AAA.
26 Determine and state the area of triangle $PQR$, whose vertices have coordinates $P(-2,-5)$, $Q(3,5)$, and $R(6,1)$.

[The use of the set of axes below is optional.]

\[QR = \sqrt{3^2 + (-4)^2} = \sqrt{25} = 5\]
\[PR = \sqrt{8^2 + 6^2} = \sqrt{100} = 10\]

\[A = \frac{1}{2} \times 5 \times 10 = 25\]

Score 2: The student gave a complete and correct response.
26 Determine and state the area of triangle \( PQR \), whose vertices have coordinates \( P(-2, -5) \), \( Q(3, 5) \), and \( R(6, 1) \).

[The use of the set of axes below is optional.]

\[
\begin{align*}
\text{Area} &= 10 \cdot 6 = 60 \\
B &= \frac{1}{2}(3 \cdot 4) = 6 \\
C &= \frac{1}{2}(6 \cdot 8) = 24 \\
D &= \frac{1}{2}(5 \cdot 10) = 25 \\
A &= 80 - 55
\end{align*}
\]

\( \triangle PQR = 25 \)

Score 2: The student gave a complete and correct response.
Question 26

26 Determine and state the area of triangle $PQR$, whose vertices have coordinates $P(-2,-5)$, $Q(3,5)$, and $R(6,1)$.

[The use of the set of axes below is optional.]

Score 1: The student did not show work to determine the lengths of $QR$ and $PR$. 
26 Determine and state the area of triangle \( PQR \), whose vertices have coordinates \( P(-2, -5) \), \( Q(3, 5) \), and \( R(6, 1) \).

[The use of the set of axes below is optional.]

\[
\begin{align*}
|PQ| &= \sqrt{(-2-3)^2 + (5-5)^2} = 5 \\
|PR| &= \sqrt{(6-(-2))^2 + (5-1)^2} = 8 \\
|QR| &= \sqrt{(3-6)^2 + (5-1)^2} = 5 \\
\end{align*}
\]

\[
\text{Area} = \frac{1}{2} \times 5 \times 8 = 20
\]

**Score 0:** The student did not show enough correct relevant work to receive any credit.
27 A support wire reaches from the top of a pole to a clamp on the ground. The pole is perpendicular to the level ground and the clamp is 10 feet from the base of the pole. The support wire makes a $68^\circ$ angle with the ground. Find the length of the support wire to the nearest foot.

\[
\cos = \frac{A}{H}\\
\cos(68) = \frac{10}{x}\\
\cos(68)x = 10\\
\frac{\cos(68)}{\cos(68)} = \frac{10}{\cos(68)}\\
x = 26.694671163\\
\]

support wire = 27 ft.

Score 2: The student gave a complete and correct response.
A support wire reaches from the top of a pole to a clamp on the ground. The pole is perpendicular to the level ground and the clamp is 10 feet from the base of the pole. The support wire makes a $68^\circ$ angle with the ground. Find the length of the support wire to the nearest foot.

$$\tan 68^\circ = \frac{x}{10}$$

$$x = 24.75$$

$$10^2 + 24.75^2 = c^2$$

$$100 + 601.5625 = c^2$$

$$c = 27$$

Score 2: The student gave a complete and correct response.
27 A support wire reaches from the top of a pole to a clamp on the ground. The pole is perpendicular to the level ground and the clamp is 10 feet from the base of the pole. The support wire makes a 68° angle with the ground. Find the length of the support wire to the nearest foot.

\[
\cos 68° = \frac{10}{x}
\]

\[
\cos 68° \cdot 10 = x
\]

\[
x \approx 4 ft
\]

Score 1: The student wrote a correct trigonometric equation to find the length of the support wire, but no further correct work was shown.
27 A support wire reaches from the top of a pole to a clamp on the ground. The pole is perpendicular to the level ground and the clamp is 10 feet from the base of the pole. The support wire makes a 68° angle with the ground. Find the length of the support wire to the nearest foot.

\[
\frac{\sin(68)}{1} = \frac{10}{x}
\]

\[
x \cdot \sin(68) = 10
\]

\[
x \approx 11 \text{ ft}
\]

The wire to the clamp is approximately 11 ft.

**Score 1:** The student showed appropriate work based on an incorrect location of the angle of elevation.
27 A support wire reaches from the top of a pole to a clamp on the ground. The pole is perpendicular to the level ground and the clamp is 10 feet from the base of the pole. The support wire makes a 68° angle with the ground. Find the length of the support wire to the nearest foot.

\[
\sin 68^\circ = \frac{10}{x}
\]

\[
x = \frac{10}{\sin 68^\circ}
\]

\[
x = 10.8 \text{ feet}
\]

Score 0: The student labeled the diagram incorrectly and did not round the answer to the nearest foot.
28 In the diagram below, circle $O$ has a radius of 10.

If $m\widehat{AB} = 72^\circ$, find the area of shaded sector $AOB$, in terms of $\pi$.

\[
\frac{72}{360} = \frac{x}{100\pi}
\]

\[7200\pi = 360x\]

\[20\pi = x\]

Score 2: The student gave a complete and correct response.
28 In the diagram below, circle $O$ has a radius of 10.

If $m\overarc{AB} = 72^\circ$, find the area of shaded sector $AOB$, in terms of $\pi$.

\[ \frac{72}{360} = \frac{1}{5} \]

\[ \text{Area} \ circle = \pi r^2 \]
\[ = \pi (10)^2 \]
\[ A = 100\pi \]

\[ \frac{1}{5} (100\pi) = \sqrt{20\pi} \]

**Score 2:** The student gave a complete and correct response.
28 In the diagram below, circle O has a radius of 10.

If $m\widehat{AB} = 72^\circ$, find the area of shaded sector $AOB$, in terms of $\pi$.

\[
A_{\text{sector}} = \frac{\theta}{360} \pi r^2
\]

\[
A_{\text{sector}} = \frac{72}{360} \pi \cdot 100
\]

\[
A_{\text{sector}} = 62.8
\]

**Score 1:** The student wrote an appropriate answer, but not in terms of $\pi$. 

---
Question 28

28 In the diagram below, circle $O$ has a radius of 10.

If $m\overarc{AB} = 72^\circ$, find the area of shaded sector $AOB$, in terms of $\pi$.

\[
C = 2\pi r
\]
\[
C = 2\pi (10)
\]
\[
C = 20\pi
\]

**Score 0:** The student gave a correct response that was obtained by an incorrect procedure.
28 In the diagram below, circle $O$ has a radius of 10.

If $m\widehat{AB} = 72^\circ$, find the area of shaded sector $AOB$, in terms of $\pi$.

$$A = \pi r^2$$
$$A = \pi \times 10^2$$
$$A = \pi 100$$

**Score 0:** The student did not show enough relevant work to receive any credit.
29 On the set of axes below, \( \triangle ABC \equiv \triangle STU \).

Describe a sequence of rigid motions that maps \( \triangle ABC \) onto \( \triangle STU \).

\[ \text{A rotation of } 90^\circ \text{ counter clockwise about the origin.} \]

\[ \text{Score 2: The student gave a complete and correct response.} \]
29 On the set of axes below, $\triangle ABC \cong \triangle STU$.

Describe a sequence of rigid motions that maps $\triangle ABC$ onto $\triangle STU$.

To map $\triangle ABC$ onto $\triangle STU$ first it needs to rotate $90^\circ$ counterclockwise from $(-4, 2)$, next translate 6 units down and then 2 units right.

**Score 2:** The student gave a complete and correct response.
On the set of axes below, \(\triangle ABC \cong \triangle STU\).

Describe a sequence of rigid motions that maps \(\triangle ABC\) onto \(\triangle STU\).

Reflect \(\triangle ABC\) over the y-axis, then reflect the image over the x-axis, then rotate around the origin \(90^\circ\) clockwise onto \(\triangle STU\).

Score 2: The student gave a complete and correct response.
On the set of axes below, \( \triangle ABC \cong \triangle STU \).

Describe a sequence of rigid motions that maps \( \triangle ABC \) onto \( \triangle STU \).

A counter-clockwise rotation 90°

**Score 1:** The student did not state the center of rotation.
29 On the set of axes below, $\triangle ABC \cong \triangle STU$.

Describe a sequence of rigid motions that maps $\triangle ABC$ onto $\triangle STU$.

1. Rotate $\triangle STU$ $90^\circ$ clockwise around point $T$
2. Translate $\triangle S'T'U'$ along vector $\overrightarrow{U'C}$ until it maps on to $\triangle ABC$

**Score 1:** The student mapped $\triangle STU$ onto $\triangle ABC$. 
On the set of axes below, $\triangle ABC \cong \triangle STU$.

Describe a sequence of rigid motions that maps $\triangle ABC$ onto $\triangle STU$.

$\textsc{counter clockwise rotation}$

Score 0: The student did not state the number of degrees in the rotation and did not state the center of rotation.
Describe a sequence of rigid motions that maps $\triangle ABC$ onto $\triangle STU$.

$\triangle ABC$ rotated clockwise on point C and did a translation of $(2, -6)$.

**Score 0:** The student did not state the number of degrees in the rotation and wrote an incorrect translation from point C.
In right triangle $PRT$, $m\angle P = 90^\circ$, altitude $\overline{PQ}$ is drawn to hypotenuse $\overline{RT}$, $RT = 17$, and $PR = 15$.

Determine and state, to the nearest tenth, the length of $\overline{RQ}$.

\[
\frac{\overline{PQ}}{\overline{PR}} = \frac{\overline{RT}}{\overline{RT}}
\]

\[
\frac{15}{17} = \frac{x}{17}
\]

\[
17x = 225
\]

\[
x = \frac{225}{17} \approx 13.285 \approx 13.2
\]

**Score 2:** The student gave a complete and correct response.
In right triangle $PRT$, $\angle P = 90^\circ$, altitude $PQ$ is drawn to hypotenuse $RT$, $RT = 17$, and $PR = 15$.

Determine and state, to the nearest tenth, the length of $RQ$.

\[
\frac{8}{17} = \frac{x}{8}
\]

\[
\frac{17x = 64}{17}
\]

\[
x = 3.76471
\]

\[
RQ = 13.2 \text{ units}
\]

Score 2: The student gave a complete and correct response.
30 In right triangle $PRT$, $m\angle P = 90^\circ$, altitude $PQ$ is drawn to hypotenuse $RT$, $RT = 17$, and $PR = 15$.

Determine and state, to the nearest tenth, the length of $RQ$.

$$\cos X = \frac{15}{17}$$

$$X = \cos^{-1} \left( \frac{15}{17} \right)$$

$$X \approx 28.07248694$$

$$\cos(28.07248694) = \frac{y}{15}$$

$$y = 15 \cdot \cos(28.07248694)$$

$$y \approx 13.258412$$

Score 2: The student gave a complete and correct response.
30 In right triangle \( PRT \), \( m\angle P = 90^\circ \), altitude \( \overline{PQ} \) is drawn to hypotenuse \( \overline{RT} \), \( RT = 17 \), and \( PR = 15 \).

\[
\frac{17}{15} = \frac{15}{x}
\]

\[
15x = 255
\]

\[
x = 17
\]

Determine and state, to the nearest tenth, the length of \( RQ \).

Score 1: The student wrote a correct proportion to solve for \( RQ \), but no further correct work was shown.
30 In right triangle \( PRT \), \( \angle P = 90^\circ \), altitude \( \overline{PQ} \) is drawn to hypotenuse \( \overline{RT} \), \( RT = 17 \), and \( PR = 15 \).

Determine and state, to the nearest tenth, the length of \( \overline{RQ} \).

\( \{8, 15, 17\} \) triple \[ \frac{17}{8} = \frac{15}{x} \]

\( x = 7.0588 \)

\[ 9.9412 = QR \]

Score 0: The student wrote an incorrect proportion to solve for \( TQ \), then made a rounding error.
31 Given circle $O$ with radius $OA$, use a compass and straightedge to construct an equilateral triangle inscribed in circle $O$. [Leave all construction marks.]

**Score 2:** The student gave a complete and correct response.
31 Given circle $O$ with radius $\overline{OA}$, use a compass and straightedge to construct an equilateral triangle inscribed in circle $O$. [Leave all construction marks.]

Score 2: The student gave a complete and correct response.
31 Given circle $O$ with radius $OA$, use a compass and straightedge to construct an equilateral triangle inscribed in circle $O$. [Leave all construction marks.]

**Score 2:** The student gave a complete and correct response.
31 Given circle $O$ with radius $OA$, use a compass and straightedge to construct an equilateral triangle inscribed in circle $O$. [Leave all construction marks.]

Score 1: The student constructed an equilateral triangle, but it was not inscribed in circle $O$. 
31 Given circle $O$ with radius $OA$, use a compass and straightedge to construct an equilateral triangle inscribed in circle $O$. [Leave all construction marks.]

Score 1: The student constructed an inscribed isosceles right triangle.
31 Given circle \( O \) with radius \( OA \), use a compass and straightedge to construct an equilateral triangle inscribed in circle \( O \). [Leave all construction marks.]

Score 0: The student gave a drawing that is not a construction.
Riley plotted $A(-1,6)$, $B(3,8)$, $C(6,-1)$, and $D(1,0)$ to form a quadrilateral.

Prove that Riley’s quadrilateral $ABCD$ is a trapezoid.

[The use of the set of axes on the next page is optional.]

\[
\begin{align*}
M_{AD} &= \frac{0-6}{1-(-1)} = \frac{-6}{2} = -3, \\
M_{BC} &= \frac{1-8}{6-3} = \frac{-7}{3} = -\frac{7}{3},
\end{align*}
\]

Riley’s quadrilateral $ABCD$ is a trapezoid because it has a pair of parallel sides.

Score 4: The student gave a complete and correct response.
Riley defines an isosceles trapezoid as a trapezoid with congruent diagonals. Use Riley’s definition to prove that $ABCD$ is not an isosceles trapezoid.

\[
AC = \sqrt{(-1-0)^2 + (6-1)^2} \approx 9.899
\]

\[
BD = \sqrt{(8-0)^2 + (3-1)^2} \approx 8.246
\]

$ABCD$ is not an isosceles trapezoid because its diagonals aren’t congruent.

$9.899 \neq 8.246$
32 Riley plotted $A(-1,6)$, $B(3,8)$, $C(6,-1)$, and $D(1,0)$ to form a quadrilateral.

Prove that Riley’s quadrilateral $ABCD$ is a trapezoid.

[The use of the set of axes on the next page is optional.]

\[
m_{\overline{AB}} = \frac{2}{4}
\]
\[
m_{\overline{BC}} = \frac{9}{3} = -3
\]
\[
m_{\overline{CD}} = -\frac{1}{5}
\]
\[
m_{\overline{AD}} = -\frac{6}{2} = -3
\]

Not same slope; Not \parallel \text{ sides}

Same slope : \parallel \text{ sides}

Since there is only 1 pair of opp sides \parallel, 
Quad $ABCD$ is a trapezoid.

Score 4: The student gave a complete and correct response.
Riley defines an isosceles trapezoid as a trapezoid with congruent diagonals. Use Riley's definition to prove that $ABCD$ is not an isosceles trapezoid.

$$d_{DB} = \sqrt{68}$$
$$d_{AC} = \sqrt{98}$$

Diagonals have different dist.
$\therefore$ They are not $\cong$.

Trapezoid $ABCD$ is Not Isosceles
Question 32

32 Riley plotted $A(-1,6)$, $B(3,8)$, $C(6,-1)$, and $D(1,0)$ to form a quadrilateral. Prove that Riley’s quadrilateral $ABCD$ is a trapezoid. [The use of the set of axes on the next page is optional.]

Plan: Show that $ABCD$ is a trapezoid by proving that its bases are $\parallel$

Slope of $AD = \frac{6-0}{-1-1} = \frac{6}{-2} = -3$

Slope of $BC = \frac{8-1}{3-6} = \frac{7}{-3} = -\frac{7}{3}$

$-3 = -3$

$\therefore \overline{AD} \parallel \overline{BC}$; equal slopes yield $\parallel$ lines and $ABCD$ is a trapezoid. $\parallel$ it is a quadrilateral with at least one pair of opp. sides $\parallel$.

Score 3: The student made a computational error in determining the length of diagonal $\overline{BD}$. 

Geometry – June ’19
Riley defines an isosceles trapezoid as a trapezoid with congruent diagonals. Use Riley’s definition to prove that $ABCD$ is not an isosceles trapezoid.

Plan: Prove that $ABCD$ is not an isosceles trapezoid by showing that the diagonals are not equal.

$BD = \sqrt{(3-1)^2 + (8-0)^2} = \sqrt{4 + 64} = \sqrt{68}$

$AC = \sqrt{(-1-6)^2 + (b-1)^2} = \sqrt{49 + 49} = \sqrt{98}$

$\sqrt{98} \neq \sqrt{68}$

$\therefore$ $ABCD$ is not isosceles b/c its diagonals are not equal.
Riley plotted \( A(-1,6) \), \( B(3,8) \), \( C(6,-1) \), and \( D(1,0) \) to form a quadrilateral.

Prove that Riley’s quadrilateral \( ABCD \) is a trapezoid.

[The use of the set of axes on the next page is optional.]

\[
\begin{align*}
\text{m}_{BC} &= \frac{y_2 - y_1}{x_2 - x_1} \\
\text{m}_{AB} &= \frac{-1 - 8}{6 - 3} = \frac{-9}{3} = -3 \\
\text{m}_{AD} &= \frac{0 - 6}{1 - (-1)} = \frac{-6}{2} = -3 \\
\text{m}_{BC} &= \text{m}_{AD} \\
\therefore \quad \overline{BC} \parallel \overline{AD}
\end{align*}
\]

\( ABCD \) is a trapezoid because it has one pair of parallel sides.

**Score 2:** The student correctly proved quadrilateral \( ABCD \) is a trapezoid, but no further correct work is shown.
Riley defines an isosceles trapezoid as a trapezoid with congruent diagonals. Use Riley’s definition to prove that $ABCD$ is not an isosceles trapezoid.

\[ D = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2} \]
\[ \overline{AD} = \sqrt{(3-6)^2+(8-6)^2} \]
\[ \overline{BC} = \sqrt{4^2+2^2} = \sqrt{16+4} = \sqrt{20} \]
\[ \overline{BD} = \sqrt{} \]
Riley plotted $A(-1,6)$, $B(3,8)$, $C(6,-1)$, and $D(1,0)$ to form a quadrilateral.

Prove that Riley’s quadrilateral $ABCD$ is a trapezoid.

[The use of the set of axes on the next page is optional.]

\[ \overrightarrow{AD} \parallel \overrightarrow{BC} \Rightarrow \text{if lines are } \parallel, \text{ then sides are } \parallel \]

**Score 2:** The student correctly proved the diagonals are not congruent, so quadrilateral $ABCD$ is not an isosceles trapezoid.
Riley defines an isosceles trapezoid as a trapezoid with congruent diagonals. Use Riley’s definition to prove that $ABCD$ is not an isosceles trapezoid.

$$AC = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$
$$= \sqrt{(6+1)^2 + (-1-6)^2}$$
$$= \sqrt{49 + 49}$$
$$= \sqrt{98} \approx 9.8994...$$

$$BD = \sqrt{(3-1)^2 + (8-0)^2}$$
$$= \sqrt{4 + 64}$$
$$= \sqrt{68} \approx 8.2462...$$

Not isosceles due to distances not being equal.
32 Riley plotted $A(-1,6)$, $B(3,8)$, $C(6,-1)$, and $D(1,0)$ to form a quadrilateral.

Prove that Riley’s quadrilateral $ABCD$ is a trapezoid.

[The use of the set of axes on the next page is optional.]

\[ m_{AB} = \frac{-6}{2} = -3 \]
\[ m_{AD} = \frac{-3}{1} \]
\[ m_{BC} = \frac{-9}{3} = -3 \]
\[ m_{AB} = m_{BC} \]

Score 2: Appropriate work is shown, but both concluding statements are missing.
Riley defines an isosceles trapezoid as a trapezoid with congruent diagonals. Use Riley’s definition to prove that $ABCD$ is not an isosceles trapezoid.

\[
\overline{AC} = \sqrt{7^2 + 7^2} = \sqrt{98} \\
\overline{BD} = \sqrt{(3-(-2))^2 + (-1-4)^2} = \sqrt{65}
\]
32 Riley plotted $A(-1,6)$, $B(3,8)$, $C(6,-1)$, and $D(1,0)$ to form a quadrilateral.

Prove that Riley’s quadrilateral $ABCD$ is a trapezoid.

[The use of the set of axes on the next page is optional.]

Score 1: The student found the slopes of bases $AD$ and $BC$, but the concluding statement is incomplete. No further correct work is shown.
Riley defines an isosceles trapezoid as a trapezoid with congruent diagonals. Use Riley’s definition to prove that $ABCD$ is not an isosceles trapezoid.
Riley plotted $A(-1,6)$, $B(3,8)$, $C(6,-1)$, and $D(1,0)$ to form a quadrilateral.

Prove that Riley’s quadrilateral $ABCD$ is a trapezoid.

[The use of the set of axes on the next page is optional.]

Quadrilateral $ABCD$ is a trapezoid

4 sides

One set of parallel lines

Score 0: The student did not show enough correct relevant work to receive any credit.
Riley defines an isosceles trapezoid as a trapezoid with congruent diagonals. Use Riley’s definition to prove that \(ABCD\) is not an isosceles trapezoid.

Both diagonals have different slopes so they are not congruent, therefore it cannot be an isosceles trapezoid.
33 A child-sized swimming pool can be modeled by a cylinder. The pool has a diameter of $6\frac{1}{2}$ feet and a height of 12 inches. The pool is filled with water to $\frac{2}{3}$ of its height. Determine and state the volume of the water in the pool, to the nearest cubic foot.

One cubic foot equals 7.48 gallons of water. Determine and state, to the nearest gallon, the number of gallons of water in the pool.

\[
V = \pi \times 3.28^2 \times \frac{2}{3}
\]

Volume of water = 22.12

\[
\approx 22
\]

height: 12"

\[
\text{Depth: } \frac{2}{3} \text{ of a foot}
\]

\[
7.48 \times 22.12
\]

\[
= 165.45
\]

\[
\approx 165 \text{ gallons}
\]

Score 4: The student gave a complete and correct response.
33 A child-sized swimming pool can be modeled by a cylinder. The pool has a diameter of $6 \frac{1}{2}$ feet and a height of 12 inches. The pool is filled with water to $\frac{2}{3}$ of its height. Determine and state the volume of the water in the pool, to the nearest cubic foot.

\[
V = Bh
\]
\[
V = (\pi \cdot 3\frac{1}{2}) \cdot 1
\]
\[
V = 33.18 \text{ ft}^3 \times \frac{2}{3} = 22 \text{ ft}^3
\]

One cubic foot equals 7.48 gallons of water. Determine and state, to the nearest gallon, the number of gallons of water in the pool.

\[
22.48 \text{ ft}^3 = 165 \text{ gal}
\]

Score 4: The student gave a complete and correct response.
33 A child-sized swimming pool can be modeled by a cylinder. The pool has a diameter of $6\frac{1}{2}$ feet and a height of 12 inches. The pool is filled with water to $\frac{2}{3}$ of its height. Determine and state the volume of the water in the pool, to the nearest cubic foot.

One cubic foot equals 7.48 gallons of water. Determine and state, to the nearest gallon, the number of gallons of water in the pool.

Score 3: The student made an error in converting cubic inches into cubic feet.
33 A child-sized swimming pool can be modeled by a cylinder. The pool has a diameter of 6 1/2 feet and a height of 12 inches. The pool is filled with water to 2/3 of its height. Determine and state the volume of the water in the pool, to the nearest cubic foot.

\[
\text{Volume of cylinder} = \pi r^2 h
\]

\[
\frac{\text{Diameter}}{2} = \text{Radius}
\]

\[
\frac{6.5}{2} = 3.25
\]

Volume = \( \pi \left( 3.5 \right)^2 \times \left( \frac{2}{3} \right) \)

= 26 \, \text{feet}^3

The volume of water is 26 \, \text{feet}^3

One cubic foot equals 7.48 gallons of water. Determine and state, to the nearest gallon, the number of gallons of water in the pool.

\[
\frac{7.48}{1} = \frac{\pi}{26}
\]

\[
\pi \approx 7.48
\]

\[
\pi = 194.48 \, \text{gallons}
\]

The number of gallons of water in this pool is 194 gallons.

Score 3: The student made a transcription error by using a radius of 3.5 instead of 3.25.
33 A child-sized swimming pool can be modeled by a cylinder. The pool has a diameter of $6\frac{1}{2}$ feet and a height of 12 inches. The pool is filled with water to $\frac{2}{3}$ of its height. Determine and state the volume of the water in the pool, to the nearest cubic foot.

\[
V_{\text{cyl}} = \pi r^2 h
\]

\[
V_{\text{cyl}} = (3.25)^2 \pi \cdot \frac{2}{3}
\]

\[
V = 22.1204827
\]

\[
V = 22.1 \text{ ft}^3
\]

One cubic foot equals 7.48 gallons of water. Determine and state, to the nearest gallon, the number of gallons of water in the pool.

\[
\frac{1 \text{ ft}^3}{7.48 \text{ gal}} = 22.1 \text{ ft}^3 \times x
\]

\[
x \text{ ft}^3 = 165.308 \text{ ft}^3 \text{ gal}
\]

\[
x = 165.3 \text{ gallons}
\]

**Score 3:** The student made the same rounding error for both answers.
A child-sized swimming pool can be modeled by a cylinder. The pool has a diameter of $6\frac{1}{2}$ feet and a height of 12 inches. The pool is filled with water to $\frac{2}{3}$ of its height. Determine and state the volume of the water in the pool, to the nearest cubic foot.

One cubic foot equals 7.48 gallons of water. Determine and state, to the nearest gallon, the number of gallons of water in the pool.

**Score 2:** The student made a rounding error in determining the volume and a computational error in determining the number of gallons.
33 A child-sized swimming pool can be modeled by a cylinder. The pool has a diameter of $6 \frac{1}{2}$ feet and a height of 12 inches. The pool is filled with water to $\frac{2}{3}$ of its height. Determine and state the volume of the water in the pool, to the nearest cubic foot.

One cubic foot equals 7.48 gallons of water. Determine and state, to the nearest gallon, the number of gallons of water in the pool.

\[
1 \text{ ft}^3 = 7.48
\]

\[
339 \times 7.48 = 2535.72
\]

\[
\text{2535.72 gallons of water}
\]

**Score 1:** The student found the volume incorrectly and made a rounding error.
33 A child-sized swimming pool can be modeled by a cylinder. The pool has a diameter of $6\frac{1}{2}$ feet and a height of 12 inches. The pool is filled with water to $\frac{2}{3}$ of its height. Determine and state the volume of the water in the pool, to the nearest cubic foot.

\[ V = \left(6\frac{1}{2}\right) (12) \left(\frac{2}{3}\right)\pi \]

\[ = 52\pi \text{ ft}^3 \]

One cubic foot equals 7.48 gallons of water. Determine and state, to the nearest gallon, the number of gallons of water in the pool.

\[ \frac{52\pi}{7.48} = 21.84 \text{ gallons} \]

Score 0: The student gave a completely incorrect response.
34 Nick wanted to determine the length of one blade of the windmill pictured below. He stood at a point on the ground 440 feet from the windmill’s base. Using surveyor’s tools, Nick measured the angle between the ground and the highest point reached by the top blade and found it was 38.8°. He also measured the angle between the ground and the lowest point of the top blade, and found it was 30°.

Determine and state a blade’s length, $x$, to the nearest foot.

$$440 \cdot \tan(30) = \frac{N}{440} \cdot 440 \quad N = 254.031184$$

$$\tan(38.8) = \frac{Y}{440} \cdot Y = 353.7690827$$

$$353.7690827 - 254.031184 = 99.7379043$$

One blade’s length is 100 ft.

Score 4: The student gave a complete and correct response.
Nick wanted to determine the length of one blade of the windmill pictured below. He stood at a point on the ground 440 feet from the windmill's base. Using surveyor’s tools, Nick measured the angle between the ground and the highest point reached by the top blade and found it was $38.8^\circ$. He also measured the angle between the ground and the lowest point of the top blade, and found it was $30^\circ$.

Determine and state a blade’s length, $x$, to the nearest foot.

\[
\tan 30^\circ = \frac{z}{440} \\
440 \tan 30^\circ = z \\
z = 254
\]

\[
\tan 38.8^\circ = \frac{y}{440} \\
y = 353.8
\]

Score 4: The student gave a complete and correct response.
Nick wanted to determine the length of one blade of the windmill pictured below. He stood at a point on the ground 440 feet from the windmill’s base. Using surveyor’s tools, Nick measured the angle between the ground and the highest point reached by the top blade and found it was 38.8°. He also measured the angle between the ground and the lowest point of the top blade, and found it was 30°.

Determine and state a blade’s length, \( x \), to the nearest foot.

\[
\begin{align*}
\tan(30°) &= \frac{Y}{440} \\
Y &= 254 \\
(\cos(30°) &= \frac{440}{x} \\
x &= \frac{440 \cdot \cos(30°)}{\cos(30°)} \\
&= 343.7 \\
&= 344
\end{align*}
\]

\[\frac{2844}{-254} = 90.44\]

**Score 3:** The student made a transcription error by using 38° instead of 38.8°.
34 Nick wanted to determine the length of one blade of the windmill pictured below. He stood at a point on the ground 440 feet from the windmill’s base. Using surveyor’s tools, Nick measured the angle between the ground and the highest point reached by the top blade and found it was 38.8°. He also measured the angle between the ground and the lowest point of the top blade, and found it was 30°.

Determine and state a blade’s length, $x$, to the nearest foot.

**Score 3:** The student made an error using radian measure.
Nick wanted to determine the length of one blade of the windmill pictured below. He stood at a point on the ground 440 feet from the windmill’s base. Using surveyor’s tools, Nick measured the angle between the ground and the highest point reached by the top blade and found it was $38.8^\circ$. He also measured the angle between the ground and the lowest point of the top blade, and found it was $30^\circ$.

Determine and state a blade’s length, $x$, to the nearest foot.

\[
\tan \angle L = \frac{O}{A} \\
\tan (38.8^\circ) = \frac{y}{440} \\
y = 353.769 \\
y = 354
\]

**Score 2:** The student correctly found the height to the top of the top blade.
Question 34

34 Nick wanted to determine the length of one blade of the windmill pictured below. He stood at a point on the ground 440 feet from the windmill's base. Using surveyor's tools, Nick measured the angle between the ground and the highest point reached by the top blade and found it was 38.8°. He also measured the angle between the ground and the lowest point of the top blade, and found it was 30°.

Determine and state a blade's length, \( x \), to the nearest foot.

\[
\frac{440}{\sqrt{3}} = \frac{y}{\sqrt{3}}
\]

\[
\frac{440 \sqrt{3}}{\sqrt{3} \sqrt{3}} = y \frac{\sqrt{3}}{3}
\]

\[
\frac{440 \sqrt{3}}{3} = y
\]

\[
y = 254.031 \quad y = 254
\]

Score 2: The student correctly found the height to the bottom of the top blade.
34 Nick wanted to determine the length of one blade of the windmill pictured below. He stood at a point on the ground 440 feet from the windmill’s base. Using surveyor’s tools, Nick measured the angle between the ground and the highest point reached by the top blade and found it was 38.8°. He also measured the angle between the ground and the lowest point of the top blade, and found it was 30°.

Determine and state a blade’s length, $x$, to the nearest foot.

\[
\tan 30^\circ = \frac{y}{440}
\]

\[
y = -2818.3457 \text{ ft}
\]

**Score 1:** The student wrote a correct trigonometric equation.
Nick wanted to determine the length of one blade of the windmill pictured below. He stood at a point on the ground 440 feet from the windmill’s base. Using surveyor’s tools, Nick measured the angle between the ground and the highest point reached by the top blade and found it was 38.8°. He also measured the angle between the ground and the lowest point of the top blade, and found it was 30°.

Determine and state a blade’s length, \( x \), to the nearest foot.

\[
\tan(38.8°) = \frac{x}{440} = \frac{ \tan(38.8°) }{440}
\]

\[
x = \frac{ \tan(38.8°) }{440} = 68.115
\]

The blade is 70 ft long.

**Score 0:** The student gave a completely incorrect response.
35 Given: Quadrilateral \( MATH, \overline{HM} \parallel \overline{AT}, \overline{HT} \parallel \overline{AM}, \overline{HE} \perpendicular \overline{MEA}, \) and \( \overline{HA} \perpendicular \overline{AT} \)

Prove: \( TA \cdot HA = HE \cdot TH \)

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Quad ( MATH, \overline{HE} \perpendicular \overline{MEA}, \overline{HM} \parallel \overline{AT}, \overline{HT} \parallel \overline{AM}, \overline{HA} \perpendicular \overline{AT} )</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ( \angle EPA, \angle TAH ) are rt ( \angle )s</td>
<td>2. ( \perp ) lines form right ( \angle )s.</td>
</tr>
<tr>
<td>3. ( \angle HEP \equiv \angle TAH )</td>
<td>3. All right ( \angle )s are ( \equiv ).</td>
</tr>
<tr>
<td>4. ( MATH ) is a ( \square )</td>
<td>4. If a quadrilateral has 2 pairs of ( \equiv ) opp sides, the quad. is a ( \square ).</td>
</tr>
<tr>
<td>5. ( \overline{MA} \parallel \overline{TH} )</td>
<td>5. Opposite sides of a ( \square ) are ( \parallel ).</td>
</tr>
<tr>
<td>6. ( \angle THA \leq \angle EPA )</td>
<td>6. Alt int ( \angle )s of ( \parallel ) lines and a transversal are ( \leq ).</td>
</tr>
<tr>
<td>7. ( \triangle HEP \sim \triangle TAH )</td>
<td>7. AA</td>
</tr>
<tr>
<td>8. ( \frac{HE}{TH} = \frac{HA}{TA} )</td>
<td>8. Corresponding sides of similar ( \triangle )s are ( \propto ) proportion.</td>
</tr>
<tr>
<td>9. ( TA \cdot HA = HE \cdot TH )</td>
<td>9. In a ( \propto ) proportion, the product of the means equals the product of the extremes.</td>
</tr>
</tbody>
</table>

Score 6: The student gave a complete and correct response.
35 Given: Quadrilateral $MATH$, $HM \parallel AT$, $HT \parallel AM$, $HE \perp MEA$, and $HA \perp AT$

Prove: $TA \cdot HA = HE \cdot TH$

1. Given
2. Reflexive
3. $SSS \equiv SSS$
4. CPCTC
5. Perpendicular lines form right $\angle$s
6. All right $\angle$s are $\cong$
7. $AA \equiv AA$
8. Corresponding sides of similar triangles are in proportion
9. The product of the means equals the product of the extremes.

Score 6: The student gave a complete and correct response.
35 Given: Quadrilateral $MATH$, $HM \cong AT$, $HT \cong AM$, $HE \perp MEA$, and $HA \perp AT$

Prove: $TA \cdot HA = HE \cdot TH$

Since $HE \perp MEA$ and $HA \perp AT$, $\angle HEA$ and $\angle TAH$ are right angles. $\angle HEA \cong \angle TAH$ because all right angles are $\cong$. The opposite sides of quad. $MATH$ are $\cong$; therefore $MATH$ is a $\square$ so by definition $HT \parallel MEA$. These parallel lines are cut by a transversal, $HA$, which forms $\cong$ alternate interior angles $\angle THA \cong \angle HAE$. \( \triangle THA \sim \triangle HAE \) by $AA \cong AA$. In similar $\triangle$'s the corresponding sides are in proportion so $\frac{TA}{HE} = \frac{TH}{HA}$. In a proportion the cross products are equal therefore $TA \cdot HA = HE \cdot TH$.

Score 6: The student gave a complete and correct response.
Question 35

35 Given: Quadrilateral $MATH$, $HM \cong AT$, $HT \cong AM$, $HE \perp MEA$, and $HA \perp AT$

Prove: $TA \cdot HA = HE \cdot TH$

Score 5: The student had an incomplete reason in step 8.
35 Given: Quadrilateral $MATH$, $HM \equiv AT$, $HT \equiv AM$, $HE \perp MEA$, and $HA \perp AT$

Prove: $TA \cdot HA = HE \cdot TH$

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $HM \equiv AT$, $HT \equiv AM$, $HE \perp MEA$, $HA \perp AT$</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. $\angle 1 \equiv \angle 2$</td>
<td>2. All right angles are congruent.</td>
</tr>
<tr>
<td>3. $HMAT$ is a parallelogram</td>
<td>3. A parallelogram is a quad with both pairs opposite sides $\parallel$</td>
</tr>
<tr>
<td>4. $\angle 3 \equiv \angle 4$</td>
<td>4. Alternate interior angles formed by parallel lines $\parallel$</td>
</tr>
<tr>
<td>5. $\triangle HEA \sim \triangle TAH$</td>
<td>5. AA $\sim$</td>
</tr>
<tr>
<td>6. $\frac{TA}{HE} = \frac{TH}{HA}$</td>
<td>6. Corresponding sides of similar triangles are in proportion</td>
</tr>
<tr>
<td>7. $TA \cdot HA = HE \cdot TH$</td>
<td>7. In a proportion, the product of the means is equal to the product of the extremes</td>
</tr>
</tbody>
</table>

Score 4: The student had two missing statements and reasons: stating angles 1 and 2 are right angles and stating $HT \parallel MA$. 

Geometry – June ’19
35 Given: Quadrilateral $MATH$, $HM \equiv AT$, $HT \equiv AM$, $HE \perp MEA$, and $HA \perp AT$

Prove: $TA \cdot HA = HE \cdot TH$

We were given the opposite sides of quadrilateral $MATH$ are congruent, therefore $MATH$ is a parallelogram. By definition, the opposite sides $HT$ and $AM$ are parallel as well with $HA$ as a transversal. The alternate interior angles $\angle THA \equiv \angle HAE$ are congruent ($a \equiv a$). Given $\overline{HE} \perp MEA$ and $\overline{HA} \perp AT$, right angles are formed, $\angle HEA \equiv \angle TAH$ ($a \equiv a$). By the reflexive property, $\overline{HA} \equiv \overline{HA}$ ($s \equiv s$). So by $AAS \equiv AAS$, $\triangle HEA \equiv \triangle TAH$, therefore $\triangle HEA \sim \triangle TAH$.

Since the corresponding sides of similar triangles are proportional, the proportion $\frac{TA}{TH} = \frac{HE}{HA}$ can be derived, therefore $(TA)(HA) = (HE)(TH)$.

**Score 3:** The student incorrectly proved $\triangle HEA$ and $\triangle TAH$ congruent ($AAS \equiv ASA$) and had a missing reason for $(TA)(HA) = (HE)(TH)$. 
35 Given: Quadrilateral $MATH$, $HM \equiv AT$, $HT \equiv AM$, $HE \perp MEA$, and $HA \perp AT$

Prove: $TA \cdot HA = HE \cdot TH$

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) $HM \equiv AT$, $HT \equiv AM$, $HE \perp MEA$ and $HA \perp AT$</td>
<td>1) Given</td>
</tr>
<tr>
<td>2) $\angle 1$ and $\angle 2$ are $r + 45'$</td>
<td>2) $\perp$ lines form $r + 45'$s</td>
</tr>
<tr>
<td>3) $\angle 1 \cong \angle 2$</td>
<td>3) all $r + 45'$s are $\cong$</td>
</tr>
<tr>
<td>4) $HA \equiv HA$</td>
<td>4) reflexive property</td>
</tr>
</tbody>
</table>

Score 2: The student correctly proved $\triangle HEA \cong \triangle TAH$, but step 4 is not relevant in proving $\triangle HEA \sim \triangle TAH$. 
35 Given: Quadrilateral $MATH$, $HM \equiv AT$, $HT \equiv AM$, $HE \perp MEA$, and $HA \perp AT$

Prove: $TA \cdot HA = HE \cdot TH$

Score 2: The student wrote some correct relevant statements and reasons (steps 2 and 4).
35 Given: Quadrilateral $MATH$, $HM \cong AT$, $HT \cong AM$, $HE \perp MEA$, and $HA \perp AT$

Prove: $TA \cdot HA = HE \cdot TH$

\begin{align*}
1. & \quad \frac{HM}{HE} = \frac{AT}{MEA}, \quad HA \perp AT \\
2. & \quad \angle HEA \cong \angle HAT \text{ are right} \\
3. & \quad \angle M \cong \angle T \\
4. & \quad \triangle HEA \cong \triangle HAT \\
5. & \quad \frac{TA}{HE} = \frac{TH}{HA} \\
6. & \quad TA \cdot HA = HE \cdot TH
\end{align*}

1. Given
2. Perpendicular lines form right \( \perp \)
3. Opposite angles \( \cong \)
4. AAS
5. CPCTC
6. Cross multiply

**Score 1:** The student had only one correct relevant statement and reason in step 2.
35 Given: Quadrilateral $MATH$, $HM \cong AT$, $HT \cong AM$, $HE \perp MEA$, and $HA \perp AT$

Prove: $TA \cdot HA = HE \cdot TH$

Score 0: The student did not show enough correct relevant work to receive any credit.
Given: Quadrilateral $MATH$, $HM \equiv AT$, $HT \equiv AM$, $HE \perp MEA$, and $HA \perp AT$

Prove: $TA \cdot HA = HE \cdot TH$

1.) $HM \equiv AT$
2.) $HT \equiv AM$
3.) $HE \perp MEA$
4.) $HA \perp AT$
5.) $\angle 1, \angle 2, \text{ and } \angle 3$
   are right $\angle$s
6.) $\angle 1 \equiv \angle 2$
7.) $T$ is a reflexive $\angle$
8.) $\triangle THA \sim \triangle HEA$
9.) $TA \cdot HA = HE \cdot TH$

Score 0: The student did not show enough correct relevant work to receive any credit.
Schools are not permitted to rescore any of the open-ended questions on this exam after each question has been rated once, regardless of the final exam score. Schools are required to ensure that the raw scores have been added correctly and that the resulting scale score has been determined accurately.

Because scale scores corresponding to raw scores in the conversion chart change from one administration to another, it is crucial that for each administration the conversion chart provided for that administration be used to determine the student’s final score. The chart above is usable only for this administration of the Regents Examination in Geometry.

**Regents Examination in Geometry – June 2019**

**Chart for Converting Total Test Raw Scores to Final Exam Scores (Scale Scores)**

(Use for the June 2019 exam only.)

<table>
<thead>
<tr>
<th>Raw Score</th>
<th>Scale Score</th>
<th>Performance Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>80</td>
<td>100</td>
<td>5</td>
</tr>
<tr>
<td>79</td>
<td>99</td>
<td>5</td>
</tr>
<tr>
<td>78</td>
<td>98</td>
<td>5</td>
</tr>
<tr>
<td>77</td>
<td>97</td>
<td>5</td>
</tr>
<tr>
<td>76</td>
<td>96</td>
<td>5</td>
</tr>
<tr>
<td>75</td>
<td>95</td>
<td>5</td>
</tr>
<tr>
<td>74</td>
<td>94</td>
<td>5</td>
</tr>
<tr>
<td>73</td>
<td>93</td>
<td>5</td>
</tr>
<tr>
<td>72</td>
<td>92</td>
<td>5</td>
</tr>
<tr>
<td>71</td>
<td>91</td>
<td>5</td>
</tr>
<tr>
<td>70</td>
<td>90</td>
<td>5</td>
</tr>
<tr>
<td>69</td>
<td>90</td>
<td>5</td>
</tr>
<tr>
<td>68</td>
<td>89</td>
<td>5</td>
</tr>
<tr>
<td>67</td>
<td>88</td>
<td>5</td>
</tr>
<tr>
<td>66</td>
<td>87</td>
<td>5</td>
</tr>
<tr>
<td>65</td>
<td>87</td>
<td>5</td>
</tr>
<tr>
<td>64</td>
<td>86</td>
<td>5</td>
</tr>
<tr>
<td>63</td>
<td>86</td>
<td>5</td>
</tr>
<tr>
<td>62</td>
<td>85</td>
<td>5</td>
</tr>
<tr>
<td>61</td>
<td>84</td>
<td>4</td>
</tr>
<tr>
<td>60</td>
<td>84</td>
<td>4</td>
</tr>
<tr>
<td>59</td>
<td>83</td>
<td>4</td>
</tr>
<tr>
<td>58</td>
<td>82</td>
<td>4</td>
</tr>
<tr>
<td>57</td>
<td>82</td>
<td>4</td>
</tr>
<tr>
<td>56</td>
<td>81</td>
<td>4</td>
</tr>
<tr>
<td>55</td>
<td>81</td>
<td>4</td>
</tr>
<tr>
<td>54</td>
<td>80</td>
<td>4</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Raw Score</th>
<th>Scale Score</th>
<th>Performance Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>53</td>
<td>80</td>
<td>4</td>
</tr>
<tr>
<td>52</td>
<td>79</td>
<td>3</td>
</tr>
<tr>
<td>51</td>
<td>79</td>
<td>3</td>
</tr>
<tr>
<td>50</td>
<td>78</td>
<td>3</td>
</tr>
<tr>
<td>49</td>
<td>78</td>
<td>3</td>
</tr>
<tr>
<td>48</td>
<td>77</td>
<td>3</td>
</tr>
<tr>
<td>47</td>
<td>77</td>
<td>3</td>
</tr>
<tr>
<td>46</td>
<td>76</td>
<td>3</td>
</tr>
<tr>
<td>45</td>
<td>76</td>
<td>3</td>
</tr>
<tr>
<td>44</td>
<td>75</td>
<td>3</td>
</tr>
<tr>
<td>43</td>
<td>75</td>
<td>3</td>
</tr>
<tr>
<td>42</td>
<td>74</td>
<td>3</td>
</tr>
<tr>
<td>41</td>
<td>74</td>
<td>3</td>
</tr>
<tr>
<td>40</td>
<td>73</td>
<td>3</td>
</tr>
<tr>
<td>39</td>
<td>73</td>
<td>3</td>
</tr>
<tr>
<td>38</td>
<td>72</td>
<td>3</td>
</tr>
<tr>
<td>37</td>
<td>71</td>
<td>3</td>
</tr>
<tr>
<td>36</td>
<td>71</td>
<td>3</td>
</tr>
<tr>
<td>35</td>
<td>70</td>
<td>3</td>
</tr>
<tr>
<td>34</td>
<td>69</td>
<td>3</td>
</tr>
<tr>
<td>33</td>
<td>68</td>
<td>3</td>
</tr>
<tr>
<td>32</td>
<td>67</td>
<td>3</td>
</tr>
<tr>
<td>31</td>
<td>66</td>
<td>3</td>
</tr>
<tr>
<td>30</td>
<td>65</td>
<td>3</td>
</tr>
<tr>
<td>29</td>
<td>64</td>
<td>2</td>
</tr>
<tr>
<td>28</td>
<td>63</td>
<td>2</td>
</tr>
<tr>
<td>27</td>
<td>62</td>
<td>2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Raw Score</th>
<th>Scale Score</th>
<th>Performance Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>26</td>
<td>61</td>
<td>2</td>
</tr>
<tr>
<td>25</td>
<td>60</td>
<td>2</td>
</tr>
<tr>
<td>24</td>
<td>58</td>
<td>2</td>
</tr>
<tr>
<td>23</td>
<td>57</td>
<td>2</td>
</tr>
<tr>
<td>22</td>
<td>55</td>
<td>2</td>
</tr>
<tr>
<td>21</td>
<td>54</td>
<td>1</td>
</tr>
<tr>
<td>20</td>
<td>52</td>
<td>1</td>
</tr>
<tr>
<td>19</td>
<td>51</td>
<td>1</td>
</tr>
<tr>
<td>18</td>
<td>49</td>
<td>1</td>
</tr>
<tr>
<td>17</td>
<td>47</td>
<td>1</td>
</tr>
<tr>
<td>16</td>
<td>45</td>
<td>1</td>
</tr>
<tr>
<td>15</td>
<td>43</td>
<td>1</td>
</tr>
<tr>
<td>14</td>
<td>41</td>
<td>1</td>
</tr>
<tr>
<td>13</td>
<td>39</td>
<td>1</td>
</tr>
<tr>
<td>12</td>
<td>36</td>
<td>1</td>
</tr>
<tr>
<td>11</td>
<td>34</td>
<td>1</td>
</tr>
<tr>
<td>10</td>
<td>32</td>
<td>1</td>
</tr>
<tr>
<td>9</td>
<td>29</td>
<td>1</td>
</tr>
<tr>
<td>8</td>
<td>26</td>
<td>1</td>
</tr>
<tr>
<td>7</td>
<td>23</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>20</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>17</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>14</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>11</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
<td>1</td>
</tr>
</tbody>
</table>
| 0         |  0          | 1                 

To determine the student’s final examination score (scale score), find the student’s total test raw score in the column labeled “Raw Score” and then locate the scale score that corresponds to that raw score. The scale score is the student’s final examination score. Enter this score in the space labeled “Scale Score” on the student’s answer sheet.