

GEOMETRY (Common Core)

Tuesday, June 2, 2015 — 1:15 to 4:15 p.m., only

Student Name: _____

School Name: _____

The possession or use of any communications device is strictly prohibited when taking this examination. If you have or use any communications device, no matter how briefly, your examination will be invalidated and no score will be calculated for you.

Print your name and the name of your school on the lines above.

A separate answer sheet for Part I has been provided to you. Follow the instructions from the proctor for completing the student information on your answer sheet.

This examination has four parts, with a total of 36 questions. You must answer all questions in this examination. Record your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in Parts II, III, and IV directly in this booklet. All work should be written in pen, except for graphs and drawings, which should be done in pencil. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale.

The formulas that you may need to answer some questions in this examination are found at the end of the examination. This sheet is perforated so you may remove it from this booklet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. You may remove this sheet from this booklet. Any work done on this sheet of scrap graph paper will *not* be scored.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

Notice...

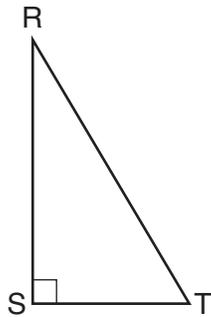
A graphing calculator, a straightedge (ruler), and a compass must be available for you to use while taking this examination.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.

Part I

Answer all 24 questions in this part. Each correct answer will receive 2 credits. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For each statement or question, choose the word or expression that, of those given, best completes the statement or answers the question. [48]

- 1 Which object is formed when right triangle RST shown below is rotated around leg RS ?

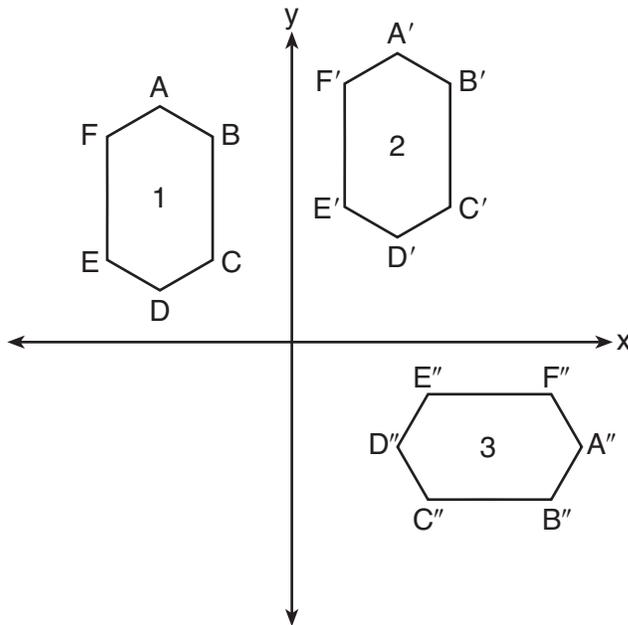


Use this space for
computations.

- (1) a pyramid with a square base (3) a right triangle
(2) an isosceles triangle (4) a cone
- 2 The vertices of $\triangle JKL$ have coordinates $J(5,1)$, $K(-2,-3)$, and $L(-4,1)$. Under which transformation is the image $\triangle J'K'L'$ *not* congruent to $\triangle JKL$?
- (1) a translation of two units to the right and two units down
(2) a counterclockwise rotation of 180 degrees around the origin
(3) a reflection over the x -axis
(4) a dilation with a scale factor of 2 and centered at the origin
- 3 The center of circle Q has coordinates $(3,-2)$. If circle Q passes through $R(7,1)$, what is the length of its diameter?
- (1) 50 (3) 10
(2) 25 (4) 5

Use this space for computations.

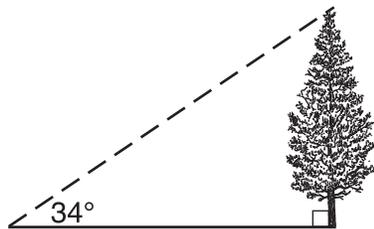
4 In the diagram below, congruent figures 1, 2, and 3 are drawn.



Which sequence of transformations maps figure 1 onto figure 2 and then figure 2 onto figure 3?

- (1) a reflection followed by a translation
- (2) a rotation followed by a translation
- (3) a translation followed by a reflection
- (4) a translation followed by a rotation

5 As shown in the diagram below, the angle of elevation from a point on the ground to the top of the tree is 34° .

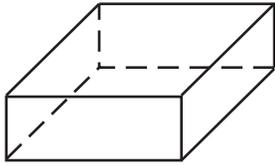


If the point is 20 feet from the base of the tree, what is the height of the tree, to the *nearest tenth of a foot*?

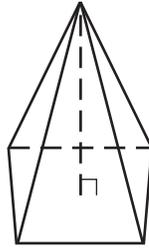
- (1) 29.7
- (2) 16.6
- (3) 13.5
- (4) 11.2

**Use this space for
computations.**

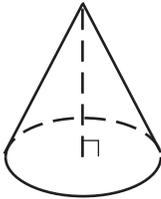
6 Which figure can have the same cross section as a sphere?



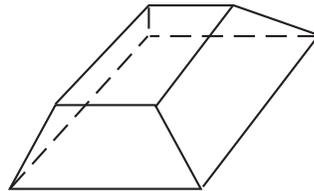
(1)



(3)



(2)



(4)

7 A shipping container is in the shape of a right rectangular prism with a length of 12 feet, a width of 8.5 feet, and a height of 4 feet. The container is completely filled with contents that weigh, on average, 0.25 pound per cubic foot. What is the weight, in pounds, of the contents in the container?

(1) 1,632

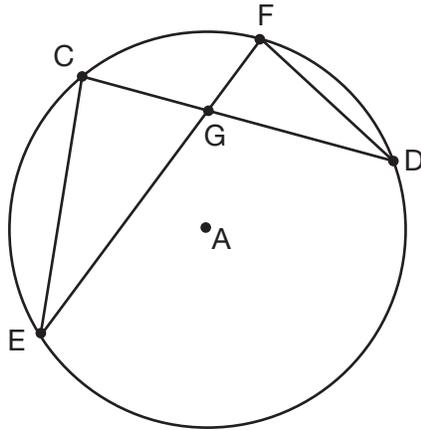
(3) 102

(2) 408

(4) 92

Use this space for
computations.

- 8 In the diagram of circle A shown below, chords \overline{CD} and \overline{EF} intersect at G , and chords \overline{CE} and \overline{FD} are drawn.



Which statement is *not* always true?

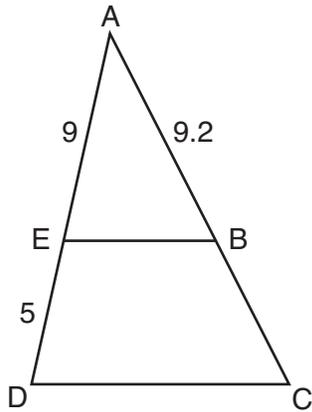
- (1) $\overline{CG} \cong \overline{FG}$ (3) $\frac{CE}{EG} = \frac{FD}{DG}$
(2) $\angle CEG \cong \angle FDG$ (4) $\triangle CEG \sim \triangle FDG$
- 9 Which equation represents a line that is perpendicular to the line represented by $2x - y = 7$?
- (1) $y = -\frac{1}{2}x + 6$ (3) $y = -2x + 6$
(2) $y = \frac{1}{2}x + 6$ (4) $y = 2x + 6$

Use this space for computations.

10 Which regular polygon has a minimum rotation of 45° to carry the polygon onto itself?

- (1) octagon
- (2) decagon
- (3) hexagon
- (4) pentagon

11 In the diagram of $\triangle ADC$ below, $\overline{EB} \parallel \overline{DC}$, $AE = 9$, $ED = 5$, and $AB = 9.2$.

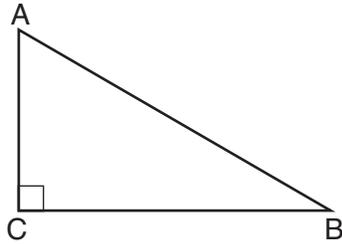


What is the length of \overline{AC} , to the *nearest tenth*?

- (1) 5.1
- (2) 5.2
- (3) 14.3
- (4) 14.4

Use this space for
computations.

12 In scalene triangle ABC shown in the diagram below, $m\angle C = 90^\circ$.



Which equation is always true?

- (1) $\sin A = \sin B$ (3) $\cos A = \sin C$
(2) $\cos A = \cos B$ (4) $\sin A = \cos B$

13 Quadrilateral $ABCD$ has diagonals \overline{AC} and \overline{BD} . Which information is *not* sufficient to prove $ABCD$ is a parallelogram?

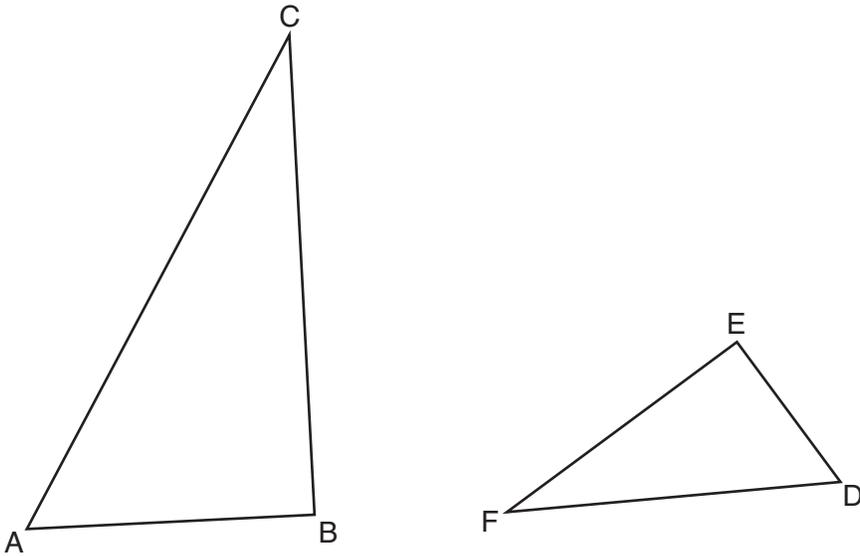
- (1) \overline{AC} and \overline{BD} bisect each other.
(2) $\overline{AB} \cong \overline{CD}$ and $\overline{BC} \cong \overline{AD}$
(3) $\overline{AB} \cong \overline{CD}$ and $\overline{AB} \parallel \overline{CD}$
(4) $\overline{AB} \cong \overline{CD}$ and $\overline{BC} \parallel \overline{AD}$

14 The equation of a circle is $x^2 + y^2 + 6y = 7$. What are the coordinates of the center and the length of the radius of the circle?

- (1) center $(0,3)$ and radius 4
(2) center $(0,-3)$ and radius 4
(3) center $(0,3)$ and radius 16
(4) center $(0,-3)$ and radius 16

Use this space for
computations.

15 Triangles ABC and DEF are drawn below.



If $AB = 9$, $BC = 15$, $DE = 6$, $EF = 10$, and $\angle B \cong \angle E$, which statement is true?

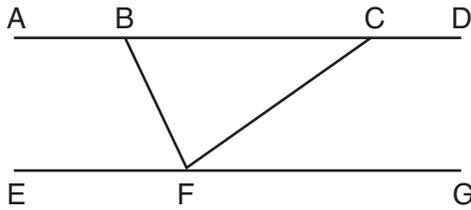
- (1) $\angle CAB \cong \angle DEF$ (3) $\triangle ABC \sim \triangle DEF$
(2) $\frac{AB}{CB} = \frac{FE}{DE}$ (4) $\frac{AB}{DE} = \frac{FE}{CB}$

16 If $\triangle ABC$ is dilated by a scale factor of 3, which statement is true of the image $\triangle A'B'C'$?

- (1) $3A'B' = AB$ (3) $m\angle A' = 3(m\angle A)$
(2) $B'C' = 3BC$ (4) $3(m\angle C') = m\angle C$

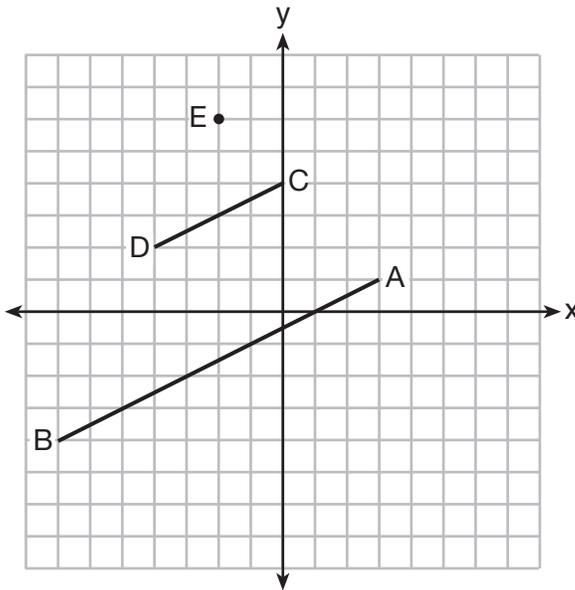
Use this space for computations.

- 17 Steve drew line segments $ABCD$, EFG , BF , and CF as shown in the diagram below. Scalene $\triangle BFC$ is formed.



Which statement will allow Steve to prove $\overline{ABCD} \parallel \overline{EFG}$?

- (1) $\angle CFG \cong \angle FCB$ (3) $\angle EFB \cong \angle CFB$
 (2) $\angle ABF \cong \angle BFC$ (4) $\angle CBF \cong \angle GFC$
- 18 In the diagram below, \overline{CD} is the image of \overline{AB} after a dilation of scale factor k with center E .



Which ratio is equal to the scale factor k of the dilation?

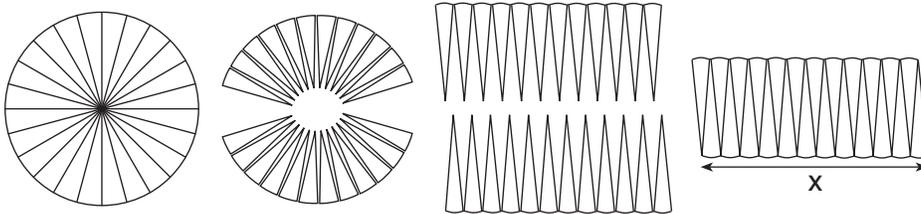
- (1) $\frac{EC}{EA}$ (3) $\frac{EA}{BA}$
 (2) $\frac{BA}{EA}$ (4) $\frac{EA}{EC}$

Use this space for computations.

22 The line $3y = -2x + 8$ is transformed by a dilation centered at the origin. Which linear equation could be its image?

- (1) $2x + 3y = 5$ (3) $3x + 2y = 5$
(2) $2x - 3y = 5$ (4) $3x - 2y = 5$

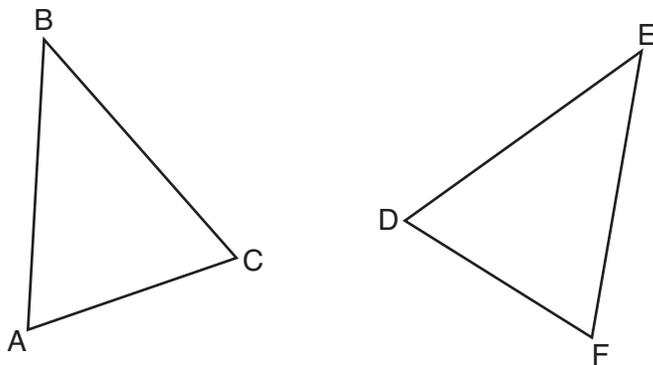
23 A circle with a radius of 5 was divided into 24 congruent sectors. The sectors were then rearranged, as shown in the diagram below.



To the *nearest integer*, the value of x is

- (1) 31 (3) 12
(2) 16 (4) 10

24 Which statement is sufficient evidence that $\triangle DEF$ is congruent to $\triangle ABC$?

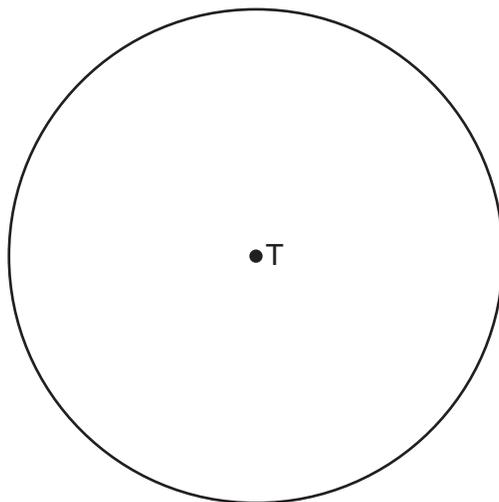


- (1) $AB = DE$ and $BC = EF$
(2) $\angle D \cong \angle A$, $\angle B \cong \angle E$, $\angle C \cong \angle F$
(3) There is a sequence of rigid motions that maps \overline{AB} onto \overline{DE} , \overline{BC} onto \overline{EF} , and \overline{AC} onto \overline{DF} .
(4) There is a sequence of rigid motions that maps point A onto point D , \overline{AB} onto \overline{DE} , and $\angle B$ onto $\angle E$.

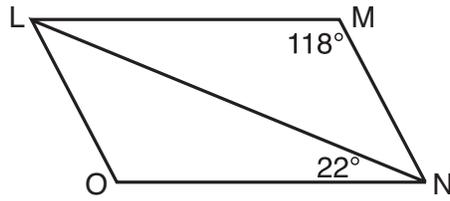
Part II

Answer all 7 questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [14]

- 25 Use a compass and straightedge to construct an inscribed square in circle T shown below.
[Leave all construction marks.]

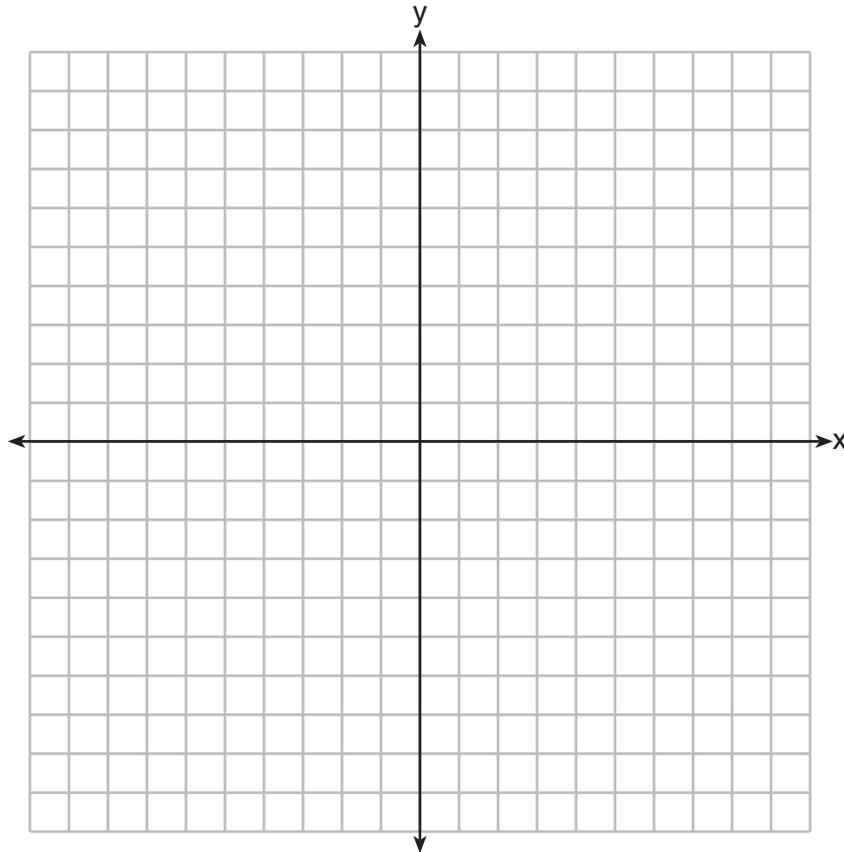


26 The diagram below shows parallelogram $LMNO$ with diagonal \overline{LN} , $m\angle M = 118^\circ$, and $m\angle LNO = 22^\circ$.

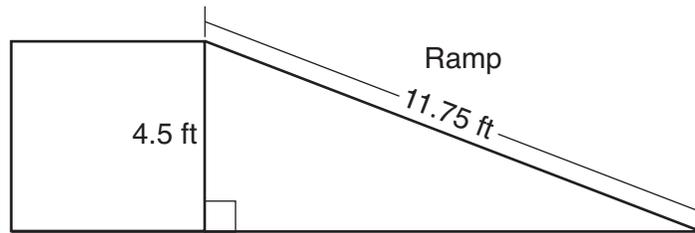


Explain why $m\angle NLO$ is 40 degrees.

- 27** The coordinates of the endpoints of \overline{AB} are $A(-6, -5)$ and $B(4, 0)$. Point P is on \overline{AB} . Determine and state the coordinates of point P , such that $AP:PB$ is $2:3$.
[The use of the set of axes below is optional.]

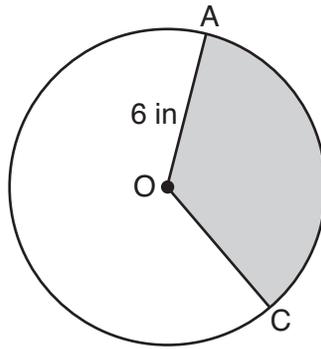


28 The diagram below shows a ramp connecting the ground to a loading platform 4.5 feet above the ground. The ramp measures 11.75 feet from the ground to the top of the loading platform.



Determine and state, to the *nearest degree*, the angle of elevation formed by the ramp and the ground.

29 In the diagram below of circle O , the area of the shaded sector AOC is $12\pi \text{ in}^2$ and the length of \overline{OA} is 6 inches. Determine and state $m\angle AOC$.



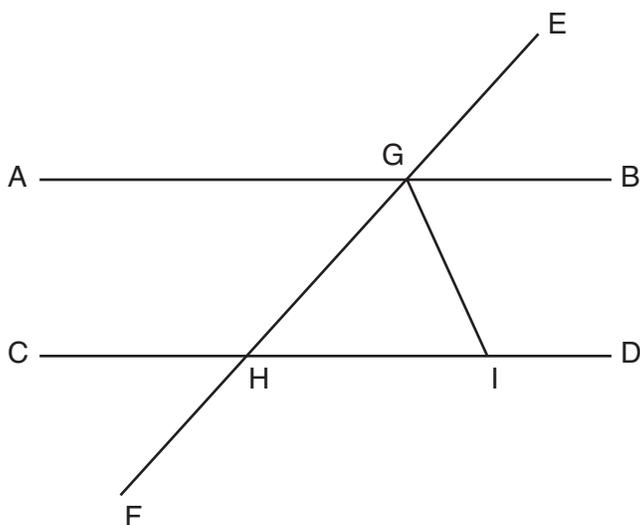
30 After a reflection over a line, $\triangle A'B'C'$ is the image of $\triangle ABC$. Explain why triangle ABC is congruent to triangle $A'B'C'$.

31 A flagpole casts a shadow 16.60 meters long. Tim stands at a distance of 12.45 meters from the base of the flagpole, such that the end of Tim's shadow meets the end of the flagpole's shadow. If Tim is 1.65 meters tall, determine and state the height of the flagpole to the *nearest tenth of a meter*.

Part III

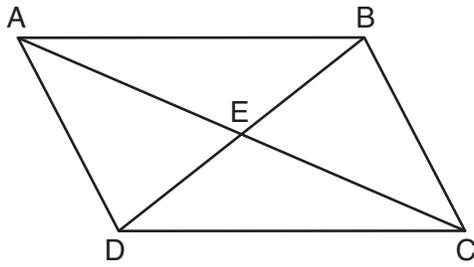
Answer all 3 questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [12]

32 In the diagram below, \overline{EF} intersects \overline{AB} and \overline{CD} at G and H , respectively, and \overline{GI} is drawn such that $\overline{GH} \cong \overline{IH}$.



If $m\angle EGB = 50^\circ$ and $m\angle DIG = 115^\circ$, explain why $\overline{AB} \parallel \overline{CD}$.

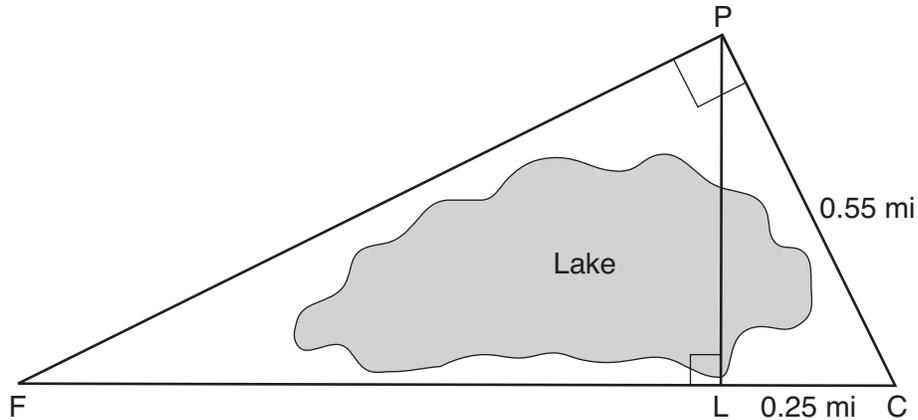
33 Given: Quadrilateral $ABCD$ is a parallelogram with diagonals \overline{AC} and \overline{BD} intersecting at E



Prove: $\triangle AED \cong \triangle CEB$

Describe a single rigid motion that maps $\triangle AED$ onto $\triangle CEB$.

34 In the diagram below, the line of sight from the park ranger station, P , to the lifeguard chair, L , on the beach of a lake is perpendicular to the path joining the campground, C , and the first aid station, F . The campground is 0.25 mile from the lifeguard chair. The straight paths from both the campground and first aid station to the park ranger station are perpendicular.



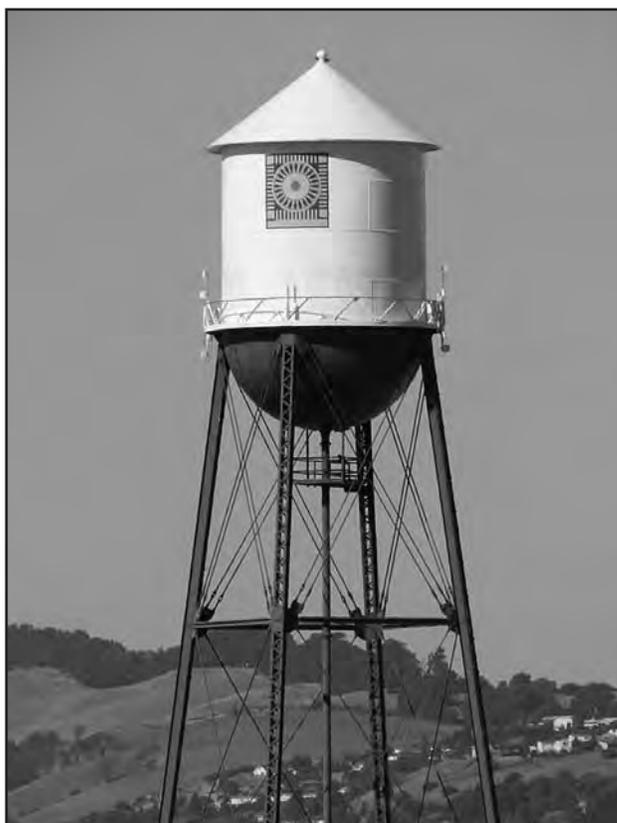
If the path from the park ranger station to the campground is 0.55 mile, determine and state, to the *nearest hundredth of a mile*, the distance between the park ranger station and the lifeguard chair.

Gerald believes the distance from the first aid station to the campground is at least 1.5 miles. Is Gerald correct? Justify your answer.

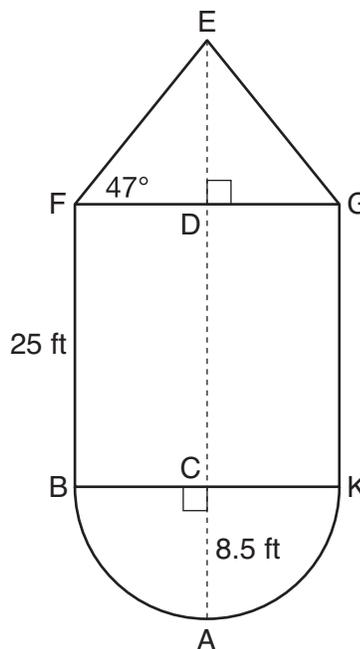
Part IV

Answer the 2 questions in this part. Each correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [12]

- 35 The water tower in the picture below is modeled by the two-dimensional figure beside it. The water tower is composed of a hemisphere, a cylinder, and a cone. Let C be the center of the hemisphere and let D be the center of the base of the cone.



Source: <http://en.wikipedia.org>



Question 35 is continued on the next page.

Question 35 continued

If $AC = 8.5$ feet, $BF = 25$ feet, and $m\angle EFD = 47^\circ$, determine and state, to the *nearest cubic foot*, the volume of the water tower.

The water tower was constructed to hold a maximum of 400,000 pounds of water. If water weighs 62.4 pounds per cubic foot, can the water tower be filled to 85% of its volume and *not* exceed the weight limit? Justify your answer.

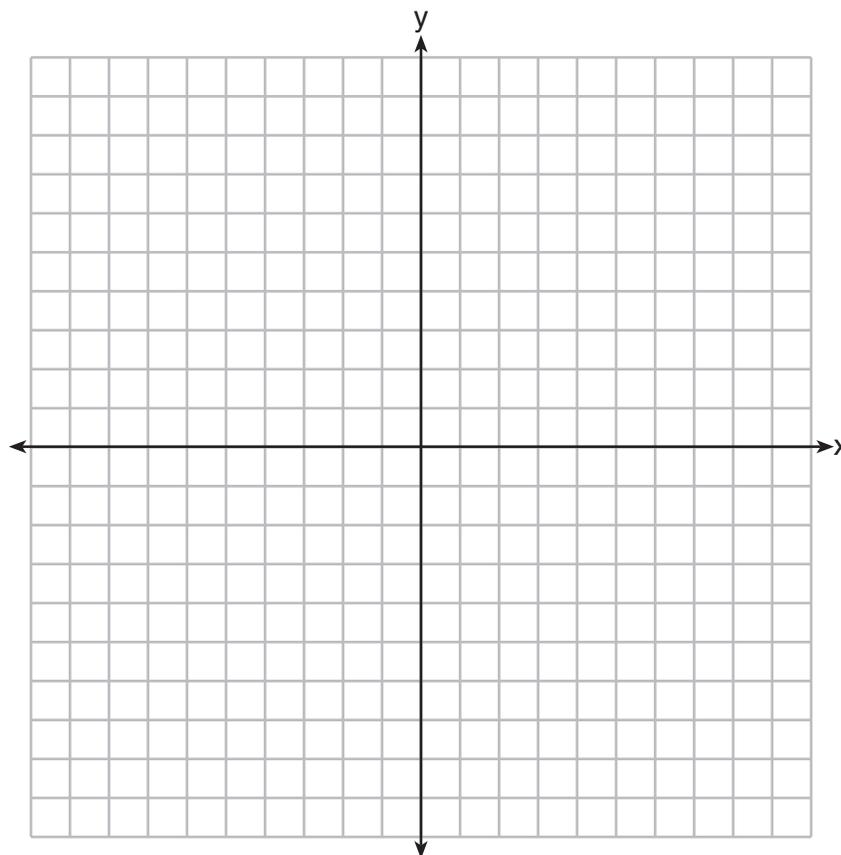
- 36** In the coordinate plane, the vertices of $\triangle RST$ are $R(6, -1)$, $S(1, -4)$, and $T(-5, 6)$.
Prove that $\triangle RST$ is a right triangle.
[The use of the set of axes on the next page is optional.]

State the coordinates of point P such that quadrilateral $RSTP$ is a rectangle.

Question 36 is continued on the next page.

Question 36 continued

Prove that your quadrilateral $RSTP$ is a rectangle.
[The use of the set of axes below is optional.]



High School Math Reference Sheet

1 inch = 2.54 centimeters	1 kilometer = 0.62 mile	1 cup = 8 fluid ounces
1 meter = 39.37 inches	1 pound = 16 ounces	1 pint = 2 cups
1 mile = 5280 feet	1 pound = 0.454 kilogram	1 quart = 2 pints
1 mile = 1760 yards	1 kilogram = 2.2 pounds	1 gallon = 4 quarts
1 mile = 1.609 kilometers	1 ton = 2000 pounds	1 gallon = 3.785 liters
		1 liter = 0.264 gallon
		1 liter = 1000 cubic centimeters

Triangle	$A = \frac{1}{2}bh$
Parallelogram	$A = bh$
Circle	$A = \pi r^2$
Circle	$C = \pi d$ or $C = 2\pi r$
General Prisms	$V = Bh$
Cylinder	$V = \pi r^2 h$
Sphere	$V = \frac{4}{3}\pi r^3$
Cone	$V = \frac{1}{3}\pi r^2 h$
Pyramid	$V = \frac{1}{3}Bh$

Pythagorean Theorem	$a^2 + b^2 = c^2$
Quadratic Formula	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
Arithmetic Sequence	$a_n = a_1 + (n - 1)d$
Geometric Sequence	$a_n = a_1 r^{n - 1}$
Geometric Series	$S_n = \frac{a_1 - a_1 r^n}{1 - r}$ where $r \neq 1$
Radians	1 radian = $\frac{180}{\pi}$ degrees
Degrees	1 degree = $\frac{\pi}{180}$ radians
Exponential Growth/Decay	$A = A_0 e^{k(t - t_0)} + B_0$

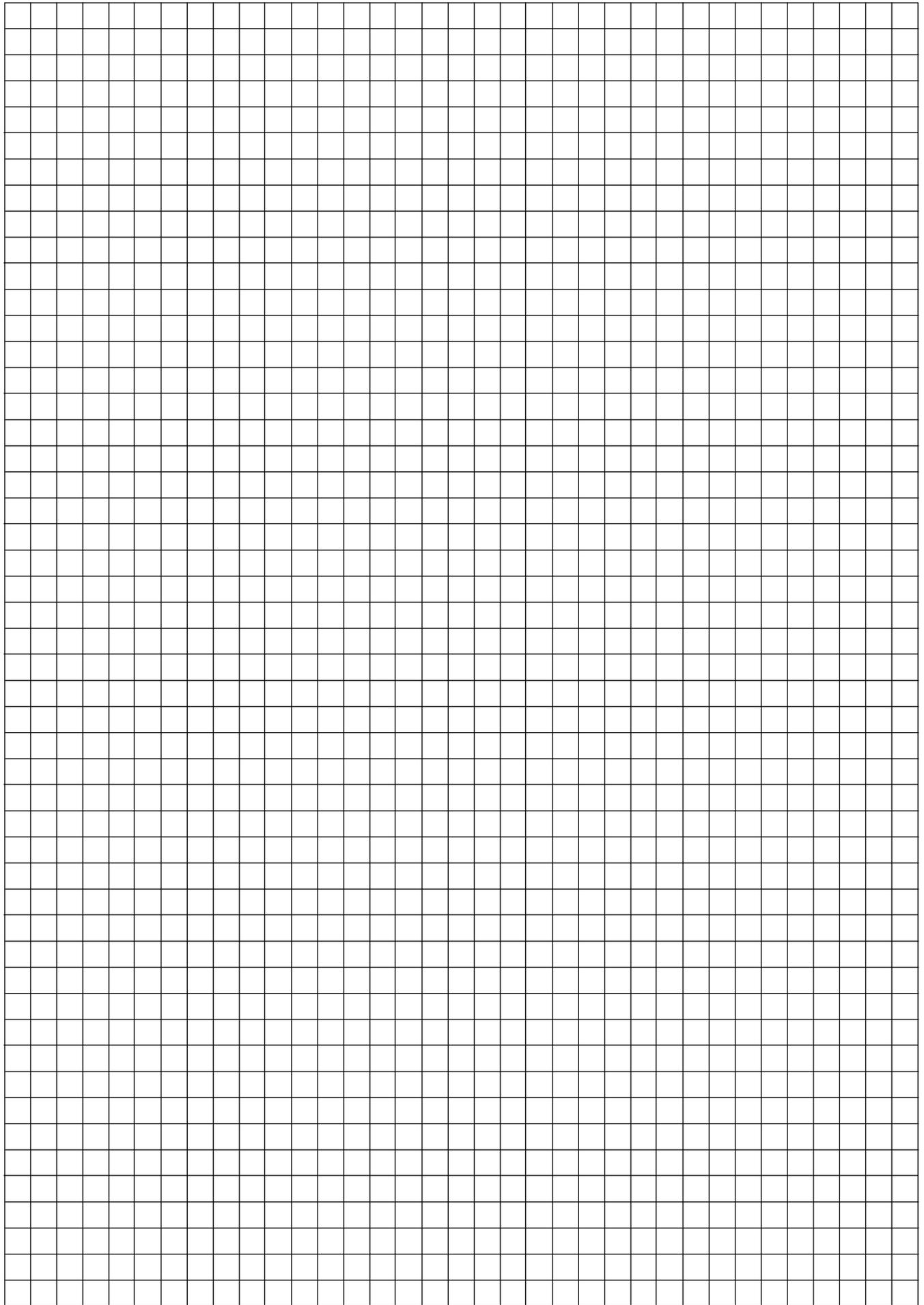
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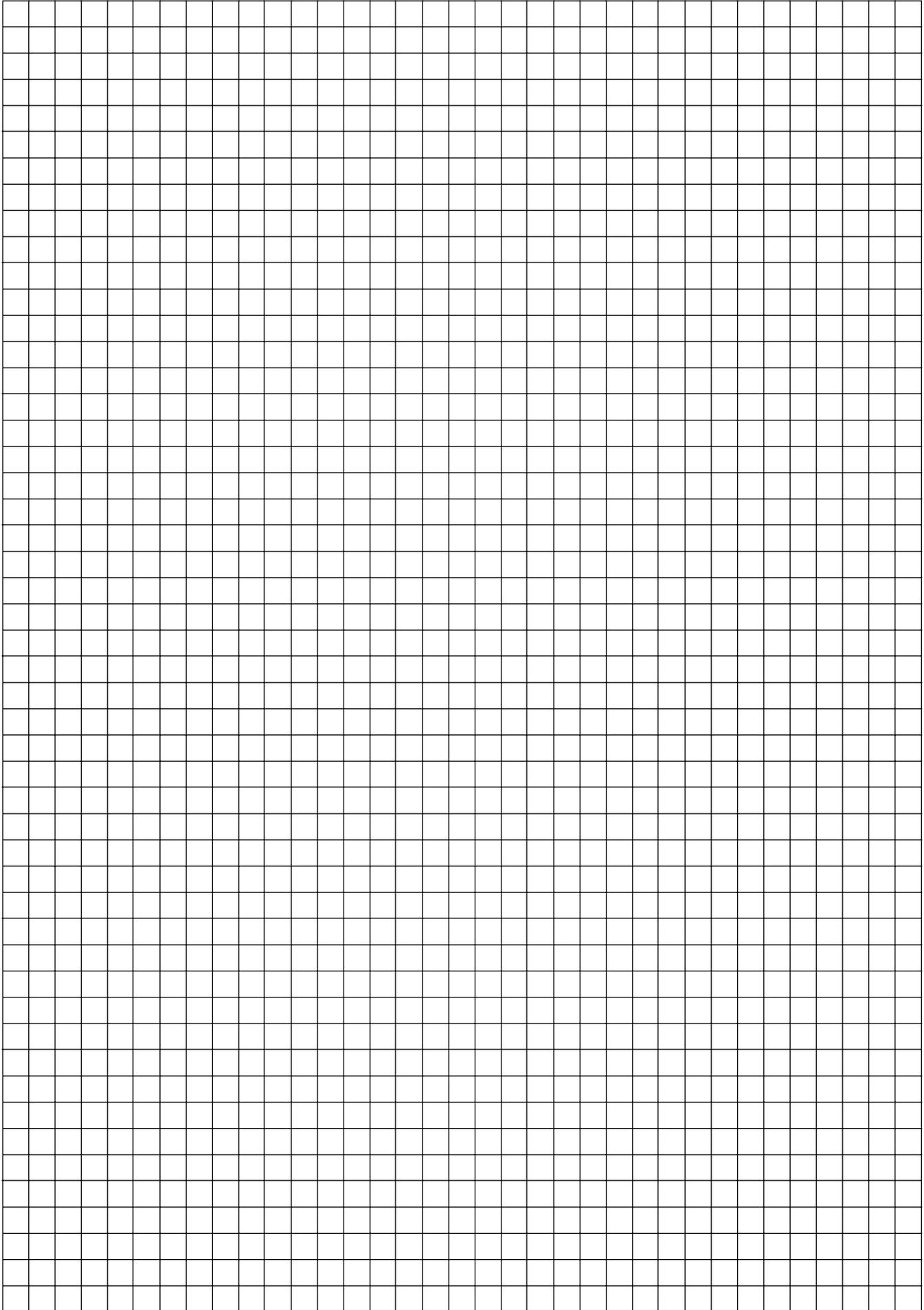
Scrap Graph Paper — This sheet will *not* be scored.



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Scrap Graph Paper — This sheet will *not* be scored.



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GEOMETRY (COMMON CORE)

Printed on Recycled Paper

GEOMETRY (COMMON CORE)

FOR TEACHERS ONLY

The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

GEOMETRY (COMMON CORE)

Tuesday, June 2, 2015 — 1:15 to 4:15 p.m., only

SCORING KEY AND RATING GUIDE

Mechanics of Rating

The following procedures are to be followed for scoring student answer papers for the Regents Examination in Geometry (Common Core). More detailed information about scoring is provided in the publication *Information Booklet for Scoring the Regents Examination in Geometry (Common Core)*.

Do *not* attempt to correct the student's work by making insertions or changes of any kind. In scoring the open-ended questions, use check marks to indicate student errors. Unless otherwise specified, mathematically correct variations in the answers will be allowed. Units need not be given when the wording of the questions allows such omissions.

Each student's answer paper is to be scored by a minimum of three mathematics teachers. No one teacher is to score more than approximately one-third of the open-ended questions on a student's paper. Teachers may not score their own students' answer papers. On the student's separate answer sheet, for each question, record the number of credits earned and the teacher's assigned rater/scorer letter.

Schools are not permitted to rescore any of the open-ended questions on this exam after each question has been rated once, regardless of the final exam score. Schools are required to ensure that the raw scores have been added correctly and that the resulting scale score has been determined accurately.

Raters should record the student's scores for all questions and the total raw score on the student's separate answer sheet. Then the student's total raw score should be converted to a scale score by using the conversion chart that will be posted on the Department's web site at: <http://www.p12.nysed.gov/assessment/> no later than Thursday, June 25, 2015. Because scale scores corresponding to raw scores in the conversion chart may change from one administration to another, it is crucial that, for each administration, the conversion chart provided for that administration be used to determine the student's final score. The student's scale score should be entered in the box provided on the student's separate answer sheet. The scale score is the student's final examination score.

If the student’s responses for the multiple-choice questions are being hand scored prior to being scanned, the scorer must be careful not to make any marks on the answer sheet except to record the scores in the designated score boxes. Marks elsewhere on the answer sheet will interfere with the accuracy of the scanning.

Part I

Allow a total of 48 credits, 2 credits for each of the following. Allow credit if the student has written the correct answer instead of the numeral 1, 2, 3, or 4.

(1) 4	(9) 1	(17) 1
(2) 4	(10) 1	(18) 1
(3) 3	(11) 3	(19) 2
(4) 4	(12) 4	(20) 1
(5) 3	(13) 4	(21) 4
(6) 2	(14) 2	(22) 1
(7) 3	(15) 3	(23) 2
(8) 1	(16) 2	(24) 3

Updated information regarding the rating of this examination may be posted on the New York State Education Department’s web site during the rating period. Check this web site at: <http://www.p12.nysed.gov/assessment/> and select the link “Scoring Information” for any recently posted information regarding this examination. This site should be checked before the rating process for this examination begins and several times throughout the Regents Examination period.

Beginning in June 2015, the Department is providing supplemental scoring guidance, the “Model Response Set,” for the Regents Examination in Geometry (Common Core). This guidance is intended to be part of the scorer training. Schools should use the Model Response Set along with the rubrics in the Scoring Key and Rating Guide to help guide scoring of student work. While not reflective of all scenarios, the Model Response Set illustrates how less common student responses to constructed-response questions may be scored. The Model Response Set will be available on the Department’s web site at: <http://www.nysedregents.org/Geometrycc/>.

General Rules for Applying Mathematics Rubrics

I. General Principles for Rating

The rubrics for the constructed-response questions on the Regents Examination in Geometry (Common Core) are designed to provide a systematic, consistent method for awarding credit. The rubrics are not to be considered all-inclusive; it is impossible to anticipate all the different methods that students might use to solve a given problem. Each response must be rated carefully using the teacher's professional judgment and knowledge of mathematics; all calculations must be checked. The specific rubrics for each question must be applied consistently to all responses. In cases that are not specifically addressed in the rubrics, raters must follow the general rating guidelines in the publication *Information Booklet for Scoring the Regents Examination in Geometry (Common Core)*, use their own professional judgment, confer with other mathematics teachers, and/or contact the State Education Department for guidance. During each Regents Examination administration period, rating questions may be referred directly to the Education Department. The contact numbers are sent to all schools before each administration period.

II. Full-Credit Responses

A full-credit response provides a complete and correct answer to all parts of the question. Sufficient work is shown to enable the rater to determine how the student arrived at the correct answer.

When the rubric for the full-credit response includes one or more examples of an acceptable method for solving the question (usually introduced by the phrase “such as”), it does not mean that there are no additional acceptable methods of arriving at the correct answer. Unless otherwise specified, mathematically correct alternative solutions should be awarded credit. The only exceptions are those questions that specify the type of solution that must be used; e.g., an algebraic solution or a graphic solution. A correct solution using a method other than the one specified is awarded half the credit of a correct solution using the specified method.

III. Appropriate Work

Full-Credit Responses: The directions in the examination booklet for all the constructed-response questions state: “Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc.” The student has the responsibility of providing the correct answer **and** showing how that answer was obtained. The student must “construct” the response; the teacher should not have to search through a group of seemingly random calculations scribbled on the student paper to ascertain what method the student may have used.

Responses With Errors: Rubrics that state “Appropriate work is shown, but...” are intended to be used with solutions that show an essentially complete response to the question but contain certain types of errors, whether computational, rounding, graphing, or conceptual. If the response is incomplete; i.e., an equation is written but not solved or an equation is solved but not all of the parts of the question are answered, appropriate work has **not** been shown. Other rubrics address incomplete responses.

IV. Multiple Errors

Computational Errors, Graphing Errors, and Rounding Errors: Each of these types of errors results in a 1-credit deduction. Any combination of two of these types of errors results in a 2-credit deduction. No more than 2 credits should be deducted for such mechanical errors in any response. The teacher must carefully review the student's work to determine what errors were made and what type of errors they were.

Conceptual Errors: A conceptual error involves a more serious lack of knowledge or procedure. Examples of conceptual errors include using the incorrect formula for the area of a figure, choosing the incorrect trigonometric function, or multiplying the exponents instead of adding them when multiplying terms with exponents.

If a response shows repeated occurrences of the same conceptual error, the student should not be penalized twice. If the same conceptual error is repeated in responses to other questions, credit should be deducted in each response.

For 4- and 6-credit questions, if a response shows one conceptual error and one computational, graphing, or rounding error, the teacher must award credit that takes into account both errors. Refer to the rubric for specific scoring guidelines.

Part II

For each question, use the specific criteria to award a maximum of 2 credits. Unless otherwise specified, mathematically correct alternative solutions should be awarded appropriate credit.

(25) [2] A correct construction is drawn showing all appropriate arcs.

[1] Appropriate work is shown, but one construction error is made.

[0] A drawing that is not an appropriate construction is shown.

or

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

(26) [2] A correct explanation is written.

[1] Appropriate work is shown, but one computational error is made.

or

[1] Appropriate work is shown, but an incomplete explanation is written.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

(27) [2] $(-2, -3)$, and correct work is shown.

[1] Appropriate work is shown, but one computational or graphing error is made.

or

[1] Appropriate work is shown, but one conceptual error is made.

or

[1] Appropriate work is shown to find -2 and -3 , but the answer is not written as coordinates.

or

[1] Appropriate work is shown, and point P is graphed correctly, but the coordinates are not stated or are stated incorrectly.

or

[1] $(-2, -3)$, but no work is shown.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

(28) [2] 23, and correct work is shown.

[1] Appropriate work is shown, but one computational or rounding error is made.

or

[1] Appropriate work is shown, but one conceptual error is made.

or

[1] $\sin x = \frac{4.5}{11.75}$ or an equivalent trigonometric equation is written, but no further correct work is shown.

or

[1] 23, but no work is shown.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

(29) [2] 120 or $\frac{2\pi}{3}$, and correct work is shown.

[1] Appropriate work is shown, but one computational error is made.

or

[1] Appropriate work is shown, but one conceptual error is made.

or

[1] 120 or $\frac{2\pi}{3}$, but no work is shown.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

(30) [2] A correct explanation is written.

[1] An incomplete explanation is written.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

(31) [2] 6.6, and correct work is shown.

[1] Appropriate work is shown, but one computational or rounding error is made.

or

[1] Appropriate work is shown, but one conceptual error is made.

or

[1] 6.6, but no work is shown.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

Part III

For each question, use the specific criteria to award a maximum of 4 credits. Unless otherwise specified, mathematically correct alternative solutions should be awarded appropriate credit.

- (32) [4] A complete and correct explanation is written.
- [3] Appropriate work is shown, but one computational error is made. An appropriate explanation is written.
- or*
- [3] At least five correct angles are stated, but the explanation is incomplete.
- [2] Appropriate work is shown, but two or more computational errors are made. An appropriate explanation is written.
- or*
- [2] Appropriate work is shown, but one conceptual error is made.
- or*
- [2] At least five correct angles are stated, but the explanation is incorrect or is missing.
- [1] Appropriate work is shown, but one conceptual error and one computational error are made.
- [0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

- (33) [4] A complete and correct proof is written that includes a concluding statement, and a correct single rigid motion is stated.
- [3] A proof is written that demonstrates a thorough understanding of the method of proof and contains no conceptual errors, but one statement and/or reason is missing or is incorrect. A correct single rigid motion is stated.
- or***
- [3] A complete and correct proof that includes a concluding statement is written, but the rigid motion is not stated or is stated incorrectly.
- [2] A proof is written that demonstrates a thorough understanding of the method of proof and contains no conceptual errors, but one statement and/or reason is missing or is incorrect. The rigid motion is stated incorrectly or is missing.
- or***
- [2] A proof is written that demonstrates a good understanding of the method of proof and contains no conceptual errors, but two statements and/or reasons are missing or are incorrect. A correct rigid motion is stated.
- or***
- [2] A proof is written that demonstrates a good understanding of the method of proof, but one conceptual error is made.
- [1] A correct rigid motion is stated, but no further correct work is shown.
- or***
- [1] Some correct relevant statements about the proof are made, but three or four statements and/or reasons are missing or are incorrect.
- [0] The “given” and/or “prove” statements are written, but no further correct relevant statements are written.
- or***
- [0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

- (34) [4] 0.49, and correct work is shown. No, with a correct justification, is written.
- [3] Appropriate work is shown, but one computational or rounding error is made.
- [2] Appropriate work is shown, but two or more computational or rounding errors are made.
- or*
- [2] Appropriate work is shown, but one conceptual error is made.
- or*
- [2] Correct work is shown to find 0.49 or No, with a correct justification. No further correct work is shown.
- [1] Appropriate work is shown, but one conceptual error and one computational error are made.
- or*
- [1] A correct equation is written to find the distance between the park ranger station and the lifeguard chair, or a correct equation is written to find the distance between the first aid station and the campground, but no further correct work is shown.
- or*
- [1] 0.49, but no work is shown.
- [0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.
-

Part IV

For each question, use the specific criteria to award a maximum of 6 credits. Unless otherwise specified, mathematically correct alternative solutions should be awarded appropriate credit.

- (35) [6] 7650 and No, and correct work is shown.
- [5] Appropriate work is shown, but one computational or rounding error is made.
- or*
- [5] Correct work is shown to find 7650 and 85% of the water tower's volume, but no further correct work is shown.
- [4] Appropriate work is shown, but two computational or rounding errors are made.
- or*
- [4] Appropriate work is shown, but one conceptual error is made.
- or*
- [4] Correct work is shown to find the total volume of the tower, 7650, but no further correct work is shown.
- [3] Appropriate work is shown, but three or more computational or rounding errors are made.
- or*
- [3] Appropriate work is shown, but one conceptual error and one computational or rounding error are made.
- or*
- [3] Correct work is shown to find the volume of each geometric component of the water tower, but no further correct work is shown.
- [2] Correct work is shown to find the volume of the cone, but no further correct work is shown.
- or*
- [2] Appropriate work is shown, but one conceptual error and two or more computational or rounding errors are made.
- or*
- [2] Appropriate work is shown, but two conceptual errors are made.
- [1] Appropriate work is shown, but two conceptual errors and one computational or rounding error are made.
- or*

[1] Correct work is shown to find the volume of the hemisphere and the volume of the cylinder or the height of the cone, but no further correct work is shown.

or

[1] 7650 and No, but no work is shown.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

- (36) [6] Correct work is shown to prove $\triangle RST$ is a right triangle. $P(0,9)$ is stated, and correct work is shown to prove $RSTP$ is a rectangle.
- [5] Appropriate work is shown, but one computational or graphing error is made.
- or**
- [5] Appropriate work is shown, but one concluding statement is missing or is incorrect.
- or**
- [5] Correct proofs are written, but $P(0,9)$ is not stated.
- [4] Appropriate work is shown, but two computational or graphing errors are made.
- or**
- [4] Appropriate work is shown, but one conceptual error is made.
- or**
- [4] Correct work is shown to find $P(0,9)$ and prove quadrilateral $RSTP$ is a rectangle.
- [3] Appropriate work is shown, but three or more computational or graphing errors are made.
- or**
- [3] Appropriate work is shown, but one conceptual error and one computational or graphing error are made.
- or**
- [3] Correct work is shown to prove $\triangle RST$ is a right triangle, and $P(0,9)$ is stated. No further correct work is shown.
- [2] Appropriate work is shown, but one conceptual error and two or more computational or graphing errors are made.
- or**
- [2] Appropriate work is shown, but two conceptual errors are made.
- or**
- [2] Correct work is shown to prove $\triangle RST$ is a right triangle, but no further correct work is shown.

[1] Appropriate work is shown, but two conceptual errors and one computational or graphing error are made.

or

[1] $P(0,9)$ is stated, but no further correct work is shown.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

**Map to the Common Core Learning Standards
Geometry (Common Core)
June 2015**

Question	Type	Credits	Cluster
1	Multiple Choice	2	G-GMD.B
2	Multiple Choice	2	G-CO.B
3	Multiple Choice	2	G-GPE.B
4	Multiple Choice	2	G-CO.A
5	Multiple Choice	2	G-SRT.C
6	Multiple Choice	2	G-GMD.B
7	Multiple Choice	2	G-MG.A
8	Multiple Choice	2	G-SRT.B
9	Multiple Choice	2	G-GPE.B
10	Multiple Choice	2	G-CO.A
11	Multiple Choice	2	G-SRT.B
12	Multiple Choice	2	G-SRT.C
13	Multiple Choice	2	G-CO.C
14	Multiple Choice	2	G-GPE.A
15	Multiple Choice	2	G-SRT.B
16	Multiple Choice	2	G-SRT.B
17	Multiple Choice	2	G-CO.C
18	Multiple Choice	2	G-SRT.A
19	Multiple Choice	2	G-MG.A
20	Multiple Choice	2	G-C.A
21	Multiple Choice	2	G-SRT.B
22	Multiple Choice	2	G-SRT.A
23	Multiple Choice	2	G-GMD.A
24	Multiple Choice	2	G-CO.B
25	Constructed Response	2	G-CO.D
26	Constructed Response	2	G-CO.C

27	Constructed Response	2	G-GPE.B
28	Constructed Response	2	G-SRT.C
29	Constructed Response	2	G-C.B
30	Constructed Response	2	G-CO.B
31	Constructed Response	2	G-SRT.B
32	Constructed Response	4	G-CO.C
33	Constructed Response	4	G-CO.C
34	Constructed Response	4	G-SRT.C
35	Constructed Response	6	G-MG.A
36	Constructed Response	6	G-GPE.B

Regents Examination in Geometry (Common Core)

June 2015

Chart for Converting Total Test Raw Scores to Final Examination Scores (Scale Scores)

The Chart for Determining the Final Examination Score for the June 2015 Regents Examination in Geometry (Common Core) will be posted on the Department's web site at: <http://www.p12.nysed.gov/assessment/> no later than Thursday, June 25, 2015.

Online Submission of Teacher Evaluations of the Test to the Department

Suggestions and feedback from teachers provide an important contribution to the test development process. The Department provides an online evaluation form for State assessments. It contains spaces for teachers to respond to several specific questions and to make suggestions. Instructions for completing the evaluation form are as follows:

1. Go to <http://www.forms2.nysed.gov/emsc/osa/exameval/reexameval.cfm>.
2. Select the test title.
3. Complete the required demographic fields.
4. Complete each evaluation question and provide comments in the space provided.
5. Click the SUBMIT button at the bottom of the page to submit the completed form.

The University of the State of New York
REGENTS HIGH SCHOOL EXAMINATION

GEOMETRY (Common Core)

Tuesday, June 2, 2015 — 1:15 to 4:15 p.m.

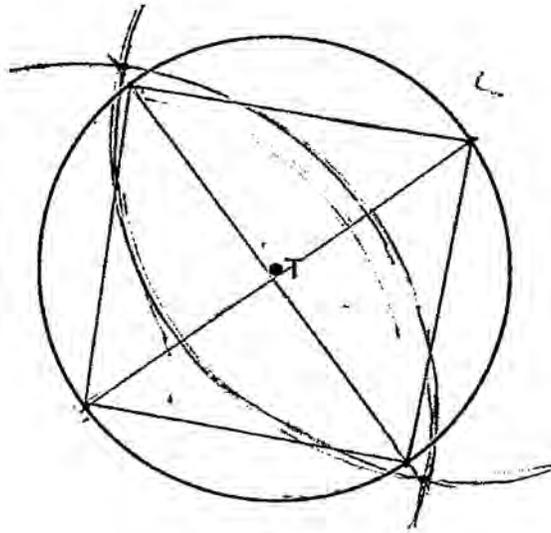
MODEL RESPONSE SET

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Question 25

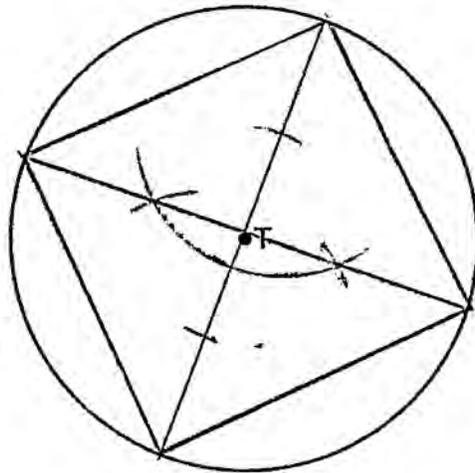
25 Use a compass and straightedge to construct an inscribed square in circle T shown below.
[Leave all construction marks.]



Score 2: The student drew a correct construction showing all appropriate construction marks and the square was drawn.

Question 25

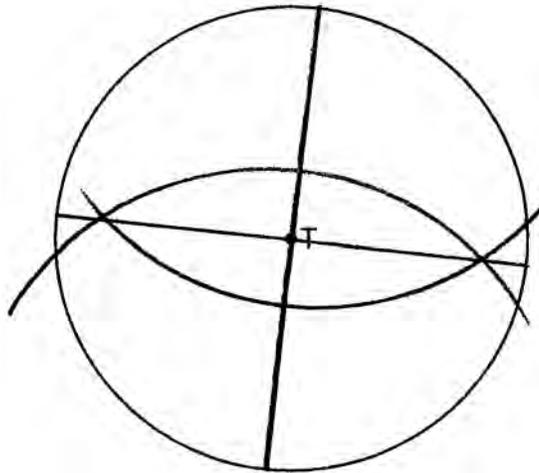
25 Use a compass and straightedge to construct an inscribed square in circle T shown below.
[Leave all construction marks.]



Score 2: The student drew a correct construction showing all appropriate construction marks and the square was drawn.

Question 25

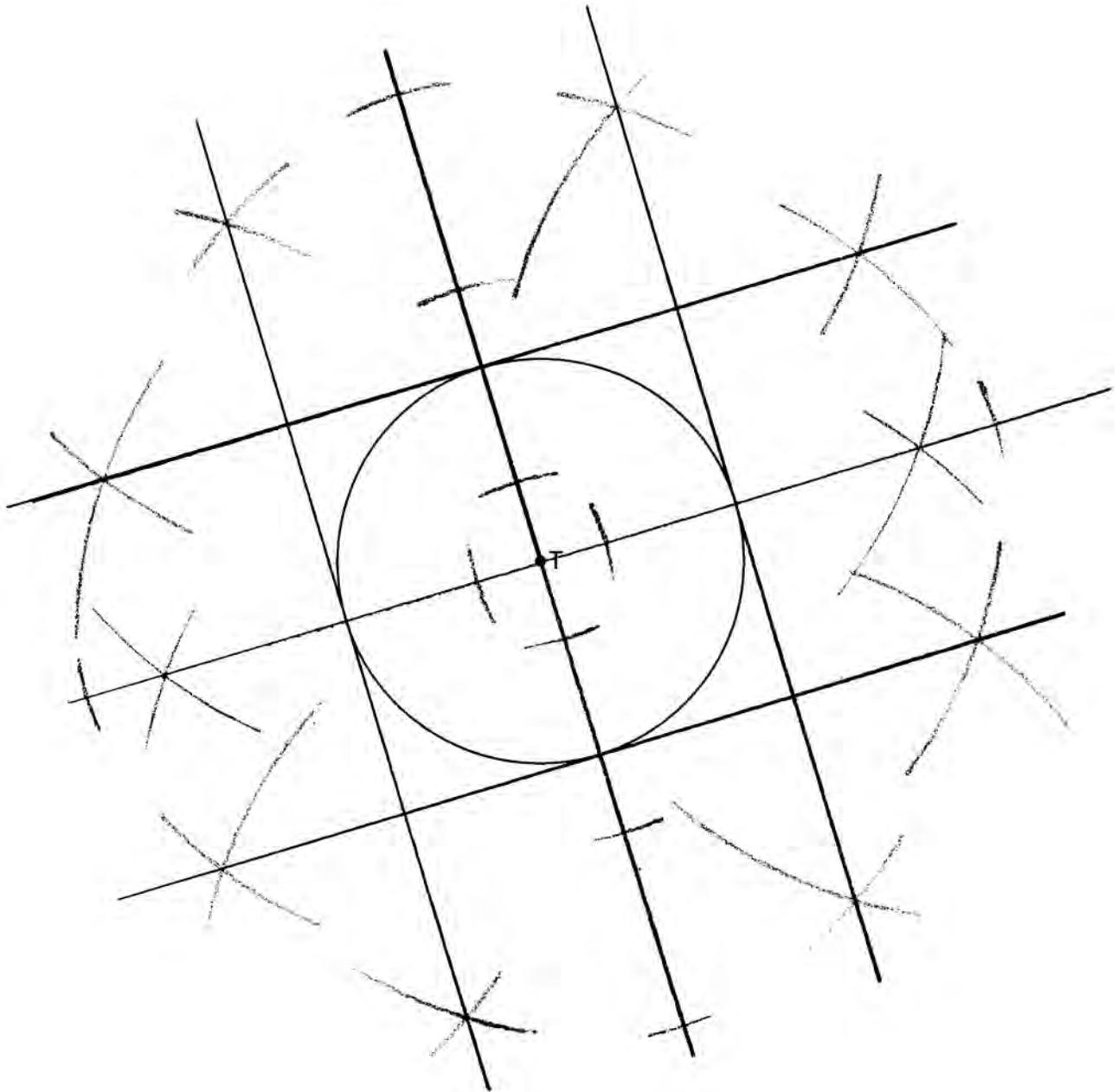
25 Use a compass and straightedge to construct an inscribed square in circle T shown below.
[Leave all construction marks.]



Score 1: The student drew a correct construction showing all appropriate construction marks, but the square was not drawn.

Question 25

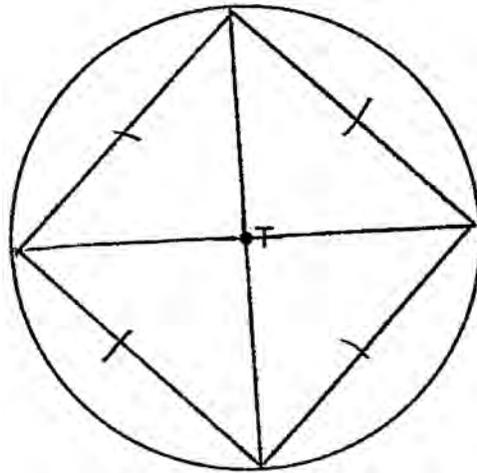
25 Use a compass and straightedge to construct an inscribed square in circle T shown below.
[Leave all construction marks.]



Score 1: The student made an error by correctly constructing a circumscribed square around circle T .

Question 25

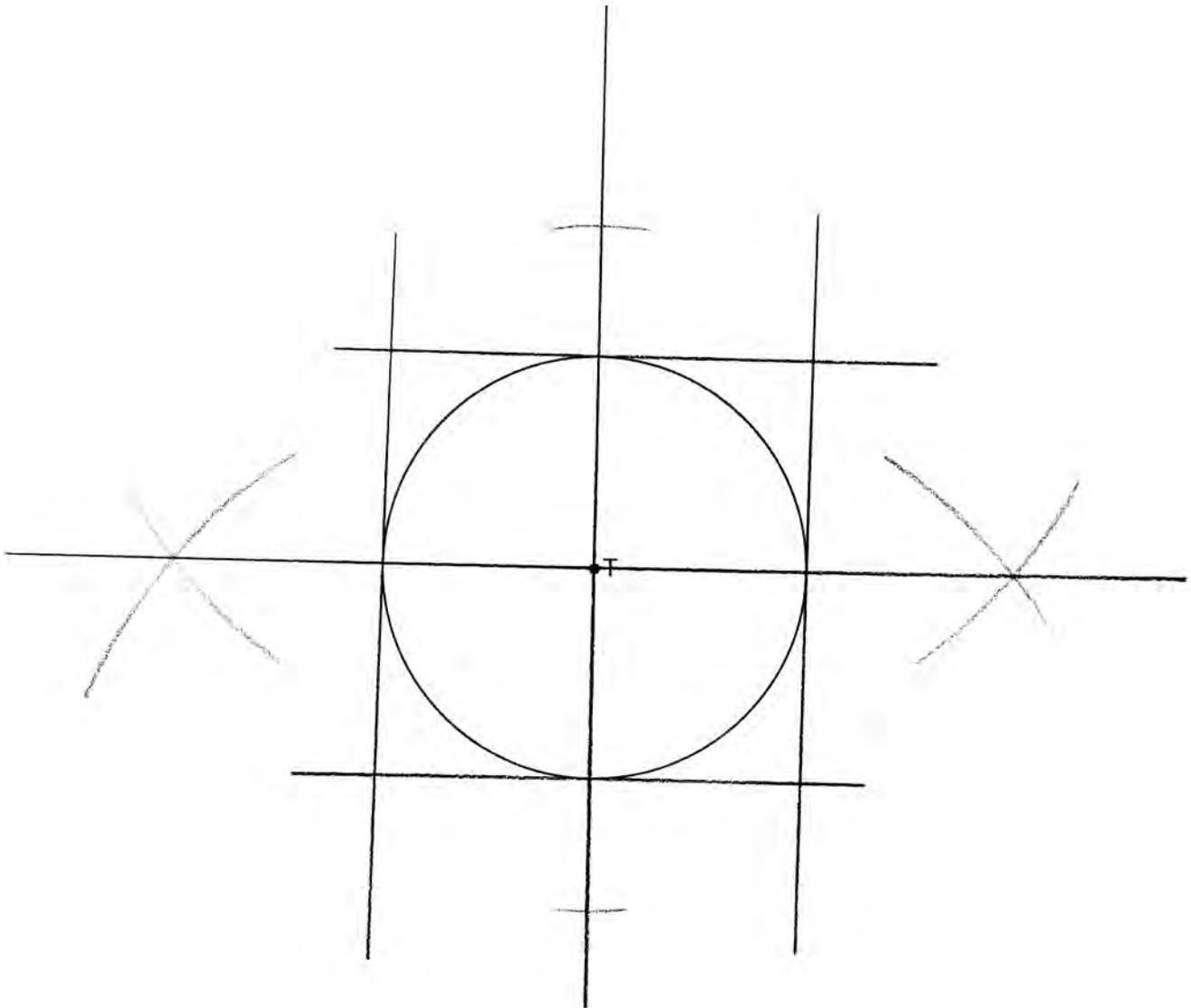
25 Use a compass and straightedge to construct an inscribed square in circle T shown below.
[Leave all construction marks.]



Score 0: The student made a drawing that is not a construction.

Question 25

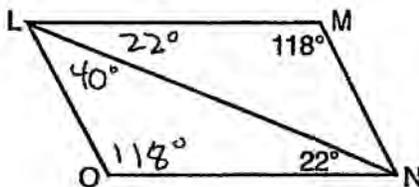
25 Use a compass and straightedge to construct an inscribed square in circle T shown below.
[Leave all construction marks.]



Score 0: The student incorrectly drew a circumscribed square around circle T .

Question 26

26 The diagram below shows parallelogram $LMNO$ with diagonal \overline{LN} , $m\angle M = 118^\circ$, and $m\angle LNO = 22^\circ$.



Explain why $m\angle LNO$ is 40 degrees.

$\angle LON$ is 118° b/c opposite \angle 's of a \square are \cong .

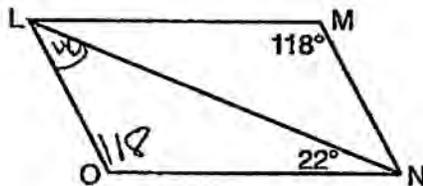
A \triangle 's \angle measures add up to 180° .

$118 + 22 = 140$ so $\angle LNO$ must be 40° .

Score 2: The student has a complete and correct response.

Question 26

26 The diagram below shows parallelogram $LMNO$ with diagonal \overline{LN} , $m\angle M = 118^\circ$, and $m\angle LNO = 22^\circ$.



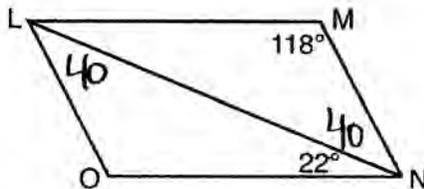
Explain why $m\angle LNO$ is 40 degrees.

They split the parallelogram in half so the two triangles are congruent to each other so $\angle LON$ is 118° . If you add 22° and 40° you get 140° then subtract that by 118° you get 40° and that is the angle for $\angle LNO$.

Score 2: The student has a complete and correct response.

Question 26

26 The diagram below shows parallelogram $LMNO$ with diagonal \overline{LN} , $m\angle M = 118^\circ$, and $m\angle LNO = 22^\circ$.



Explain why $m\angle NLO$ is 40 degrees.

$m\angle M + m\angle N = 180^\circ$ (In a p-gram, consecutive \angle 's are supp)

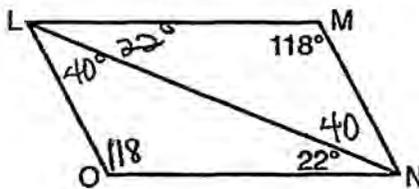
$$118 + 22 = 140 \text{ for } m\angle LNM$$

$m\angle NLO = 40^\circ$ because since $\overline{LO} \parallel \overline{MN}$, alternate interior \angle 's are congruent.

Score 2: The student has a complete and correct response.

Question 26

26 The diagram below shows parallelogram $LMNO$ with diagonal \overline{LN} , $m\angle M = 118^\circ$, and $m\angle LNO = 22^\circ$.



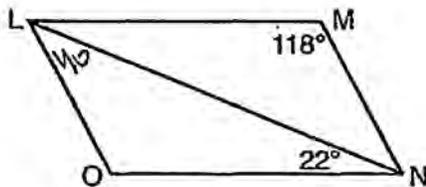
Explain why $m\angle NLO$ is 40 degrees.

$$\begin{aligned} 118^\circ + 118^\circ &= 236^\circ \\ 360 - 236 &= 124^\circ / 2 = 62^\circ \\ 62^\circ - 22^\circ &= 40 \\ 118 + 40 + x &= 180 \\ 158 + x &= 180 \\ x &= 22^\circ \\ \downarrow \\ \text{therefore, } m\angle NLO &= 40^\circ \end{aligned}$$

Score 1: The student mathematically justified the angle measure, but did not provide an explanation in words.

Question 26

26 The diagram below shows parallelogram $LMNO$ with diagonal \overline{LN} , $m\angle M = 118^\circ$, and $m\angle LNO = 22^\circ$.



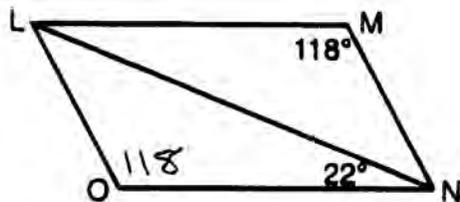
Explain why $m\angle NLO$ is 40 degrees.

because if you add 118 and 22
you get 140 and every triangle
equals 180 so you subtract
140 from 180 to get 40.

Score 1: The student gave an incomplete explanation, because a geometric relationship between 118° and 22° was not established.

Question 26

26 The diagram below shows parallelogram $LMNO$ with diagonal \overline{LN} , $m\angle M = 118^\circ$, and $m\angle LNO = 22^\circ$.



Explain why $m\angle NLO$ is 40 degrees.

Opposite angles of parallelogram are congruent
The angles of a triangle add to 180.

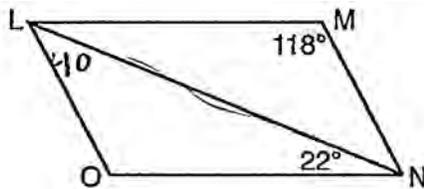
$$\begin{array}{r} 118 \\ + 22 \\ \hline 130 \end{array} \qquad \begin{array}{r} 180 \\ - 130 \\ \hline 50 \end{array}$$

So $m\angle NLO = 50^\circ$ not 40°

Score 1: The student had one computational error with an appropriate explanation.

Question 26

26 The diagram below shows parallelogram $LMNO$ with diagonal \overline{LN} , $m\angle M = 118^\circ$, and $m\angle LNO = 22^\circ$.



Explain why $m\angle LNO$ is 40 degrees.

because $\angle M$ & $\angle N$ are complementary angles
So when you add them up and equal it
to 180 you get 140 then subtract that from
180 and you get 40°

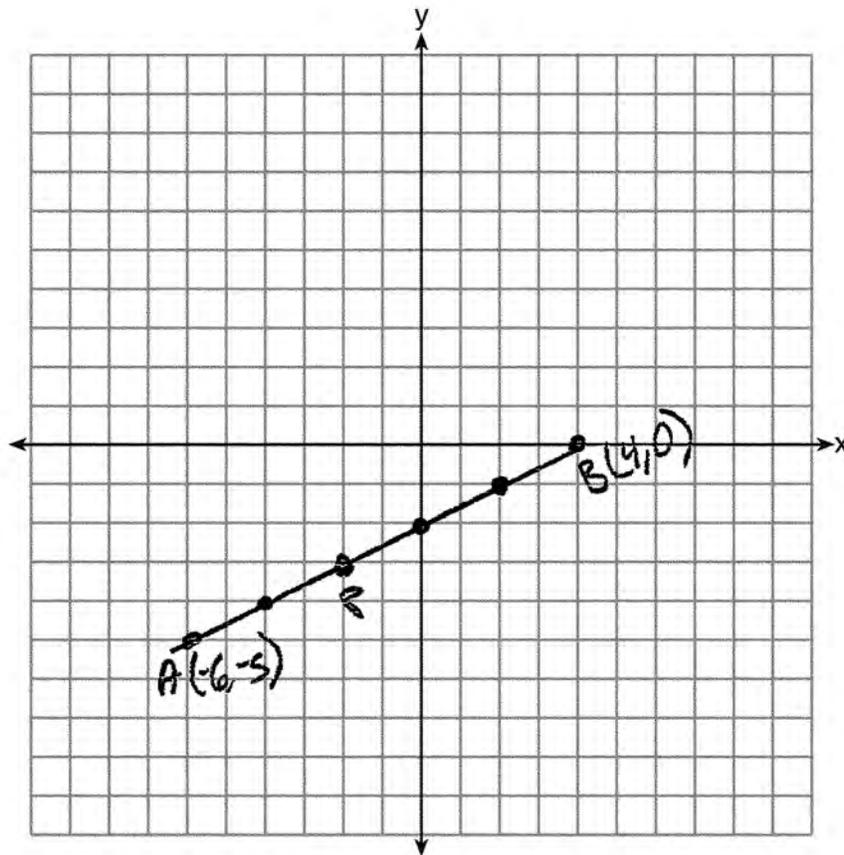
Score 0: The student gave a completely incorrect explanation.

Question 27

27 The coordinates of the endpoints of \overline{AB} are $A(-6,-5)$ and $B(4,0)$. Point P is on \overline{AB} . Determine and state the coordinates of point P , such that $AP:PB$ is $2:3$.
[The use of the set of axes below is optional.]

$$\begin{aligned}d_{AB} &= \sqrt{(x-x)^2 + (y-y)^2} \\ &= \sqrt{(-6-4)^2 + (-5-0)^2} \\ &= \sqrt{(-10)^2 + (-5)^2} \\ &= \sqrt{100+25} \\ &= \sqrt{125} \\ &= \sqrt{25} \cdot \sqrt{5} \\ &= 5\sqrt{5}\end{aligned}$$

$P(-2, -3)$



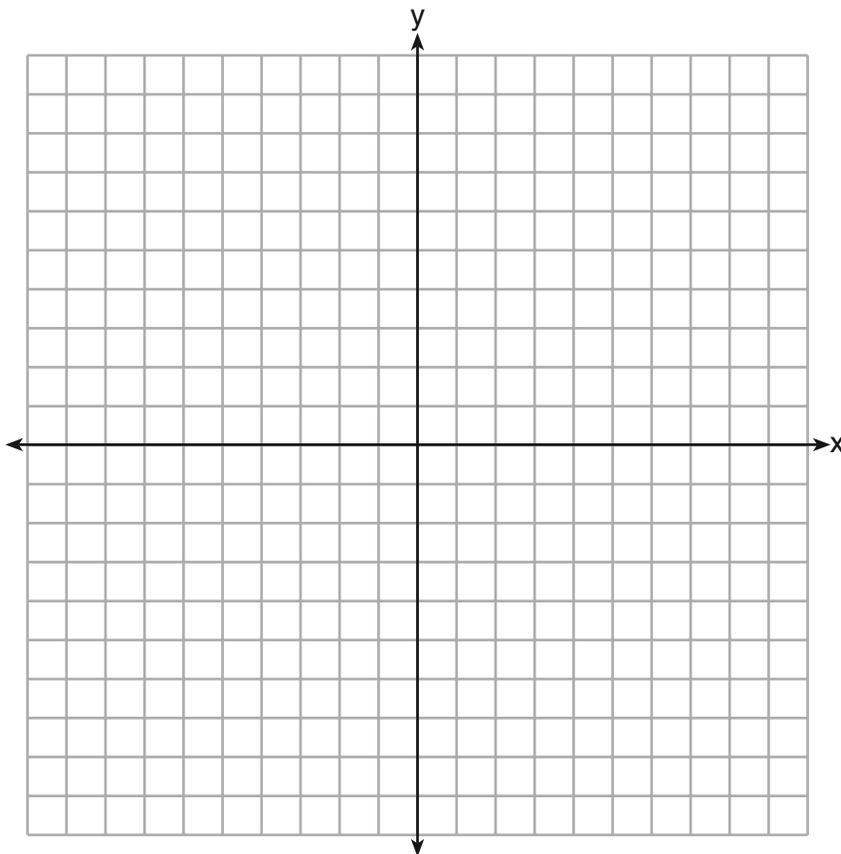
Score 2: The student has a complete and correct response. The student showed correct work that was not necessary.

Question 27

27 The coordinates of the endpoints of \overline{AB} are $A(-6, -5)$ and $B(4, 0)$. Point P is on \overline{AB} . Determine and state the coordinates of point P , such that $AP:PB$ is 2:3.

[The use of the set of axes below is optional.]

<u>X-value</u>	<u>y-value</u>
$-6 + \frac{2}{5}(4 - (-6))$	$-5 + \frac{2}{5}(0 - (-5))$
$-6 + \frac{2}{5}(10)$	$-5 + \frac{2}{5}(5)$
$-6 + 4$	$-5 + 2$
-2	-3
$(-2, -3)$	

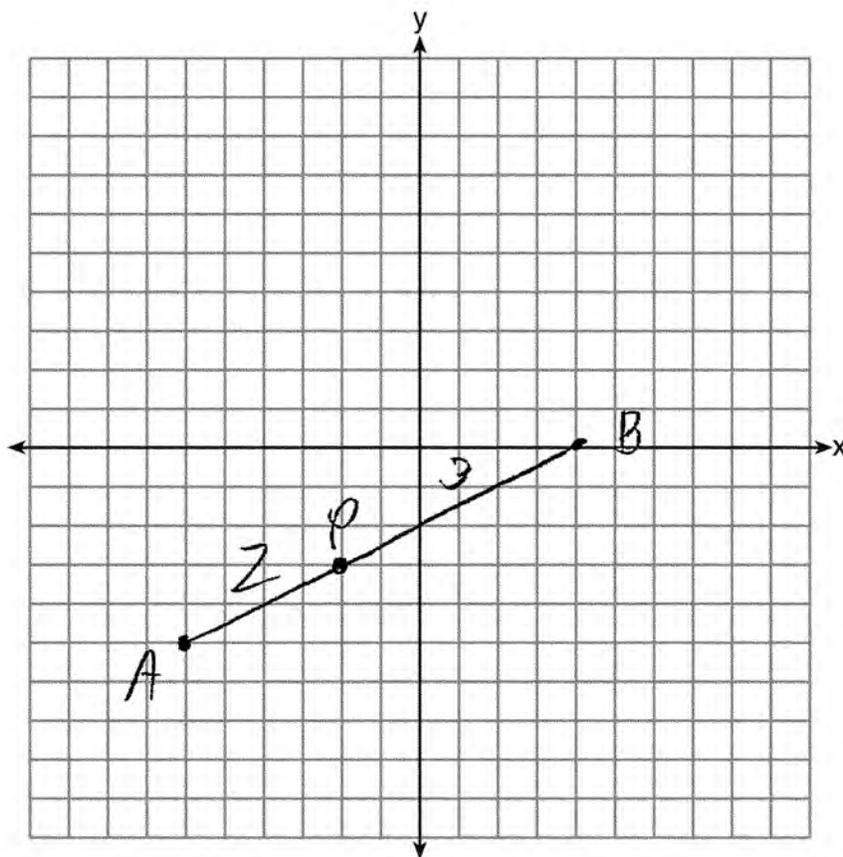


Score 2: The student has a complete and correct response.

Question 27

27 The coordinates of the endpoints of \overline{AB} are $A(-6, -5)$ and $B(4, 0)$. Point P is on \overline{AB} . Determine and state the coordinates of point P , such that $AP:PB$ is $2:3$.
[The use of the set of axes below is optional.]

$-2, -3$



Score 1: The coordinates of P were not stated as a point.

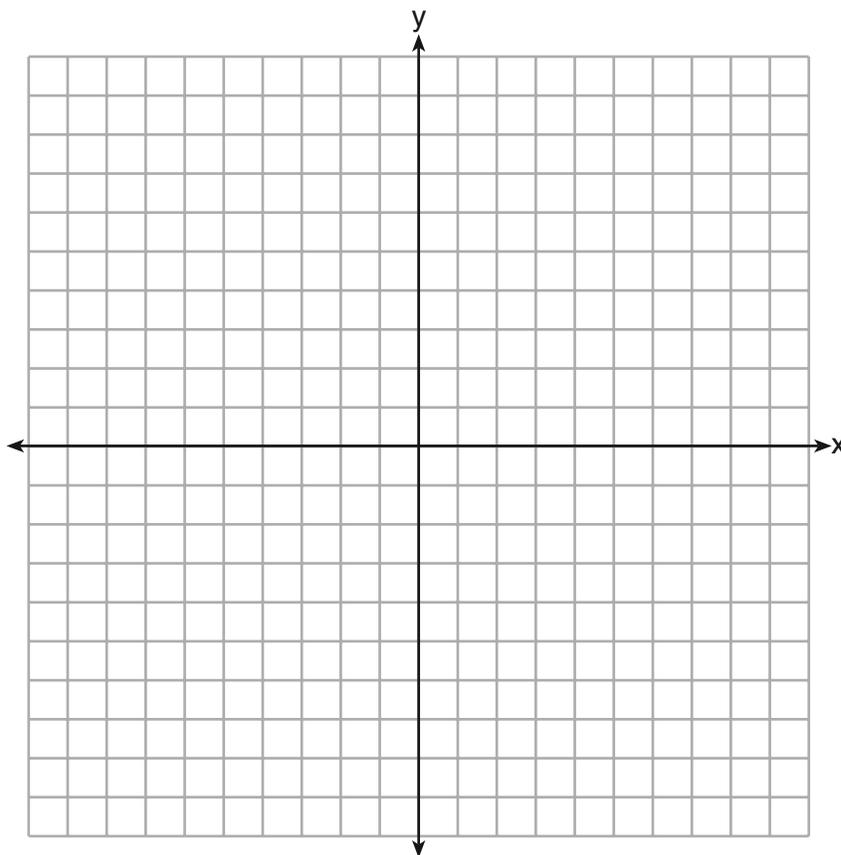
Question 27

- 27 The coordinates of the endpoints of \overline{AB} are $A(-6, -5)$ and $B(4, 0)$. Point P is on \overline{AB} . Determine and state the coordinates of point P , such that $AP:PB$ is 2:3.
[The use of the set of axes below is optional.]

$$P_x = \frac{2}{5}(4 - (-6)) - 6 = 4 - 6 = -2$$

$$P_y = \frac{2}{5}(0 - (-5)) - 5 = -2 - 5 = -7$$

$$P(-2, -7)$$

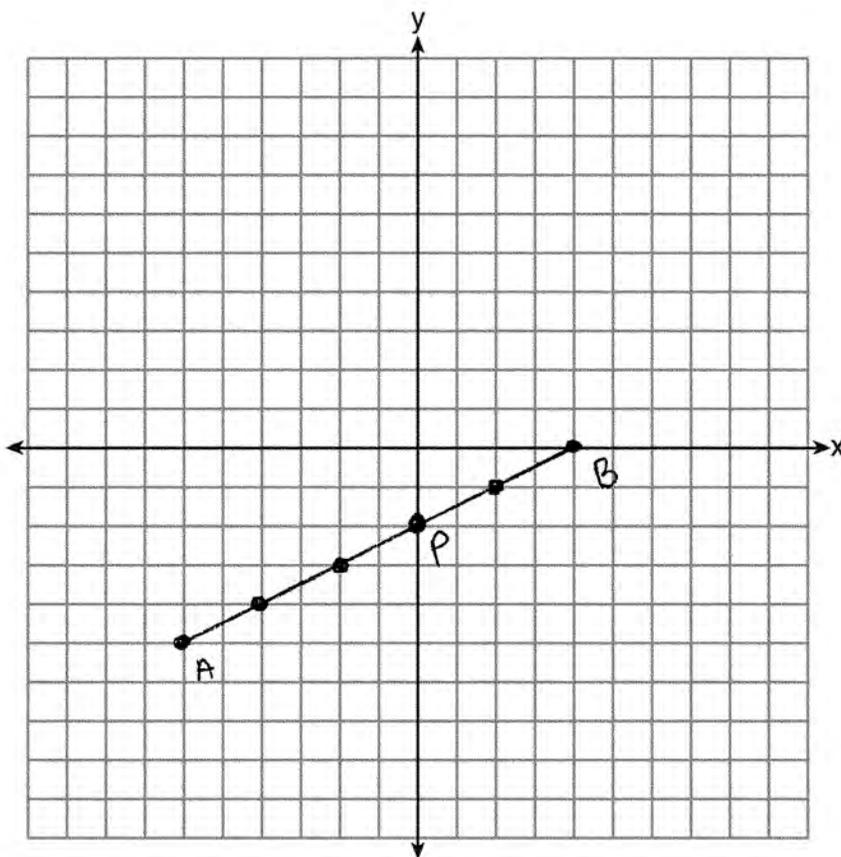


Score 1: The student made an error in determining the y -coordinate.

Question 27

27 The coordinates of the endpoints of \overline{AB} are $A(-6, -5)$ and $B(4, 0)$. Point P is on \overline{AB} . Determine and state the coordinates of point P , such that $AP:PB$ is 2:3.
[The use of the set of axes below is optional.]

$(0, -2)$



Score 1: The student determined the coordinates of P such that $AP:PB$ is in a 3:2 ratio.

Question 27

27 The coordinates of the endpoints of \overline{AB} are $A(-6, -5)$ and $B(4, 0)$. Point P is on \overline{AB} . Determine and state the coordinates of point P , such that $AP:PB$ is 2:3.

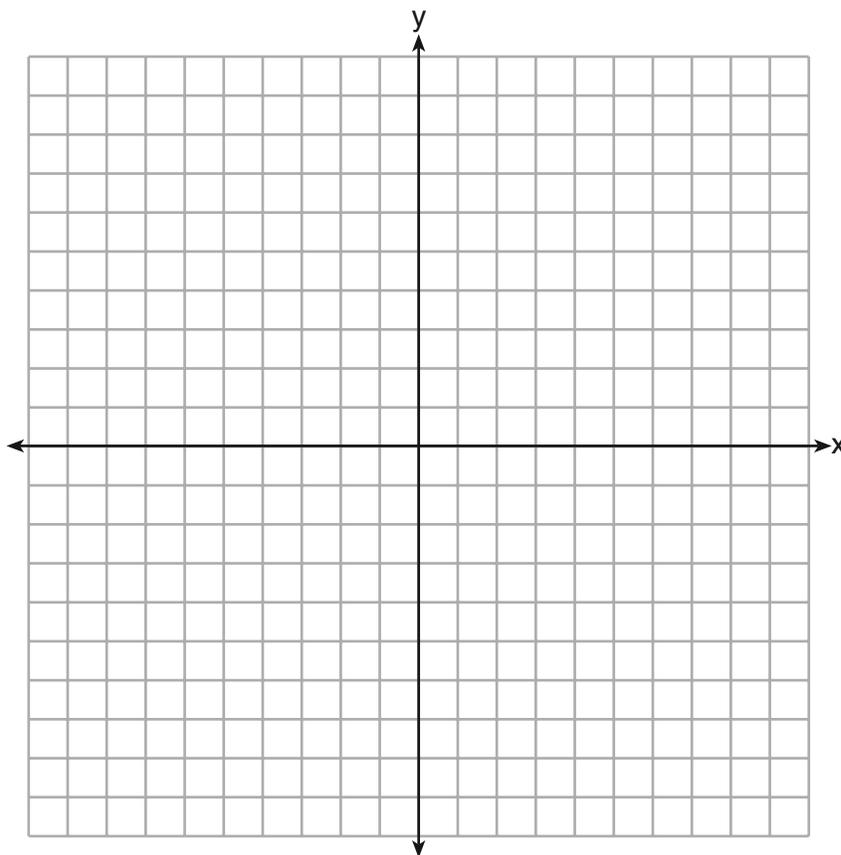
[The use of the set of axes below is optional.]

$$P \left(-6 + \frac{2}{3}(10), -5 + \frac{2}{3}(5) \right)$$

$$P \left(-6 + \frac{20}{3}, -5 + \frac{10}{3} \right)$$

$$P \left(-6 + 6\frac{2}{3}, -5 + 3\frac{1}{3} \right)$$

$$P \left(2\frac{2}{3}, -1\frac{2}{3} \right)$$

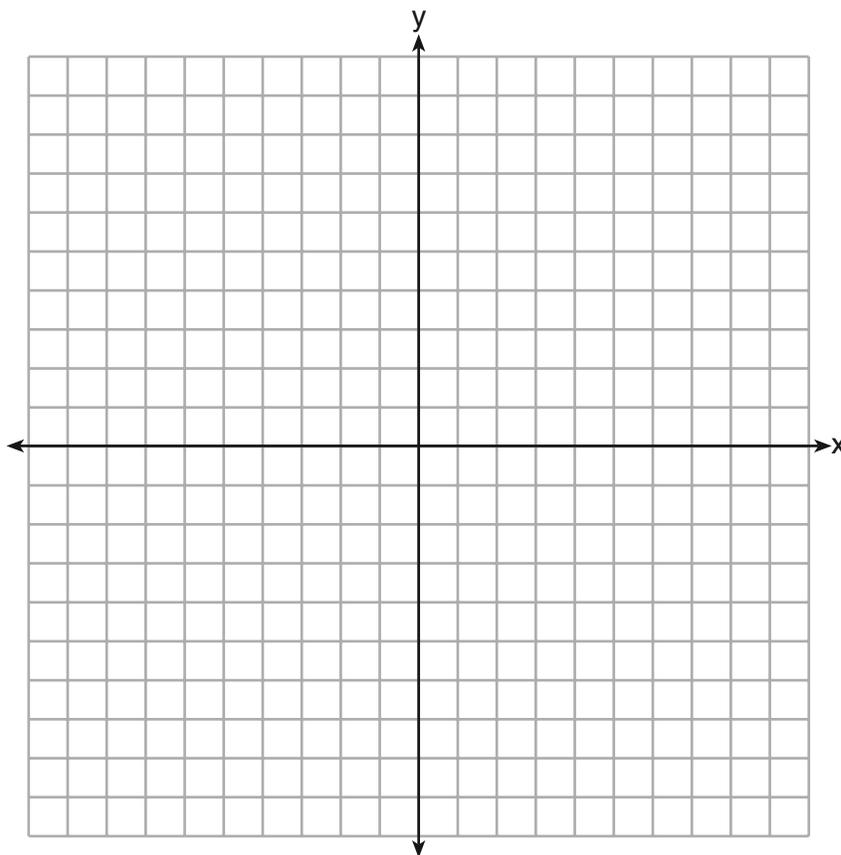


Score 1: The student made an error by multiplying by $\frac{2}{3}$ instead of $\frac{2}{5}$.

Question 27

27 The coordinates of the endpoints of \overline{AB} are $A(-6,-5)$ and $B(4,0)$. Point P is on \overline{AB} . Determine and state the coordinates of point P , such that $AP:PB$ is 2:3.
[The use of the set of axes below is optional.]

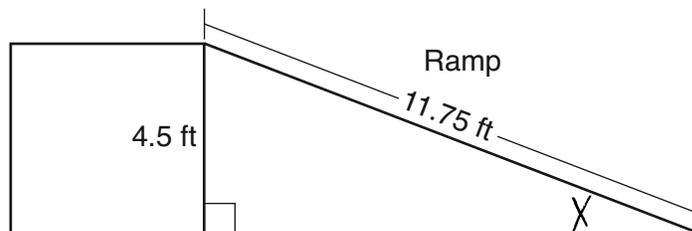
$$\begin{aligned} & \frac{-6+4}{2}, \frac{-5+0}{2} \\ & \frac{-2}{2}, \frac{-5}{2} \\ & (-1, -\frac{5}{2}) \end{aligned}$$



Score 0: The student's use of the midpoint formula was irrelevant to the question.

Question 28

28 The diagram below shows a ramp connecting the ground to a loading platform 4.5 feet above the ground. The ramp measures 11.75 feet from the ground to the top of the loading platform.



Determine and state, to the *nearest degree*, the angle of elevation formed by the ramp and the ground.

$$X = \sin^{-1}\left(\frac{4.5}{11.75}\right)$$

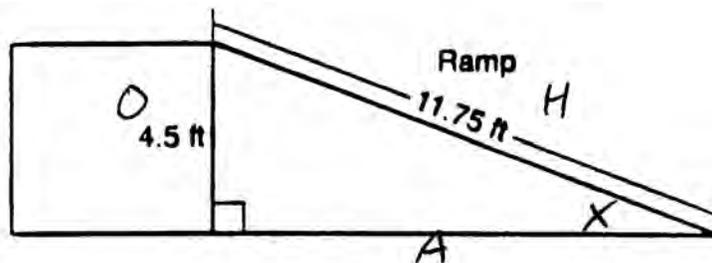
$$X = 22.518$$

$$X = 23^\circ$$

Score 2: The student has a complete and correct response.

Question 28

28 The diagram below shows a ramp connecting the ground to a loading platform 4.5 feet above the ground. The ramp measures 11.75 feet from the ground to the top of the loading platform.



Determine and state, to the *nearest degree*, the angle of elevation formed by the ramp and the ground.

SOH CAH TOA

$$\sin X = \frac{4.5}{11.75}$$

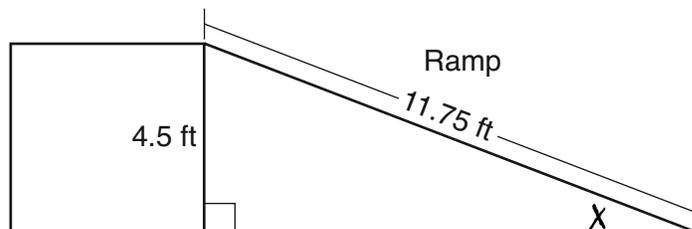
$$\sin X = .3829787234$$

$$38^\circ$$

Score 1: The student wrote a correct equation, but the angle of elevation was found incorrectly.

Question 28

28 The diagram below shows a ramp connecting the ground to a loading platform 4.5 feet above the ground. The ramp measures 11.75 feet from the ground to the top of the loading platform.



Determine and state, to the *nearest degree*, the angle of elevation formed by the ramp and the ground.

$$\tan x = \frac{4.5}{11.75}$$

$$x = \tan^{-1} \frac{4.5}{11.75}$$

$$x = 20.9557767306$$

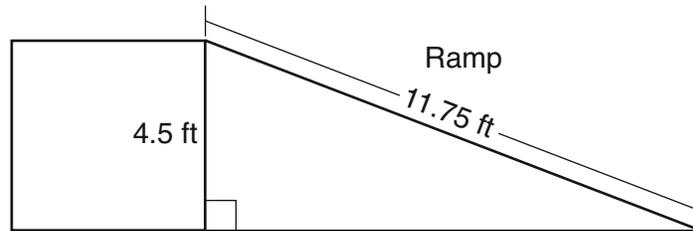
$$x = 21$$

$$\textcircled{21}$$

Score 1: The student made an error by using the wrong trigonometric function, but found an appropriate angle of elevation.

Question 28

28 The diagram below shows a ramp connecting the ground to a loading platform 4.5 feet above the ground. The ramp measures 11.75 feet from the ground to the top of the loading platform.



Determine and state, to the *nearest degree*, the angle of elevation formed by the ramp and the ground.

$$\sin x = \frac{4.5}{11.75}$$

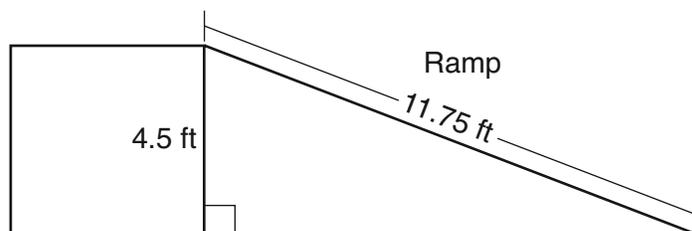
$$.393$$

$$1^\circ$$

Score 1: The student wrote a correct equation, but no further correct work was shown.

Question 28

28 The diagram below shows a ramp connecting the ground to a loading platform 4.5 feet above the ground. The ramp measures 11.75 feet from the ground to the top of the loading platform.



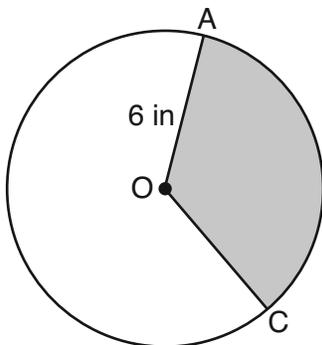
Determine and state, to the *nearest degree*, the angle of elevation formed by the ramp and the ground.

$$\begin{aligned} a^2 + b^2 &= c^2 \\ (4.5)^2 + b^2 &= (11.75)^2 \\ 20.25 + b^2 &= 138.0625 \\ -20.25 & \quad -20.25 \\ \hline \sqrt{b^2} &= \sqrt{117.8125} \\ b &= 10.854146673 \\ \boxed{11^\circ} \end{aligned}$$

Score 0: The student had a completely incorrect response.

Question 29

29 In the diagram below of circle O , the area of the shaded sector AOC is $12\pi \text{ in}^2$ and the length of \overline{OA} is 6 inches. Determine and state $m\angle AOC$.



$$\begin{aligned} A &= \pi r^2 \\ &= 6^2 \cdot \pi \\ &= 36\pi \end{aligned}$$

$$\frac{12\pi}{36\pi} = \frac{1}{3}$$

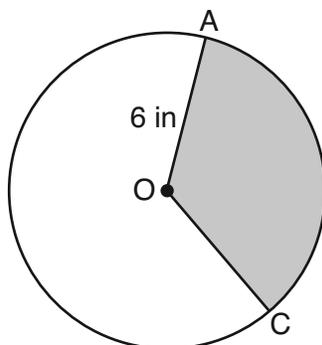
$$\frac{1}{3} \cdot 360$$

120°

Score 2: The student has a complete and correct response.

Question 29

29 In the diagram below of circle O , the area of the shaded sector AOC is 12π in² and the length of OA is 6 inches. Determine and state $m\angle AOC$.



$$A = \frac{rs}{2}$$

$$2 \quad 12\pi = \frac{6 \cdot s}{2} \cdot 2$$

$$\frac{24\pi}{6} = \frac{6s}{6}$$

$$s = 4\pi$$

$$s = r\theta$$

$$\frac{4\pi}{6} = \frac{6\theta}{6}$$

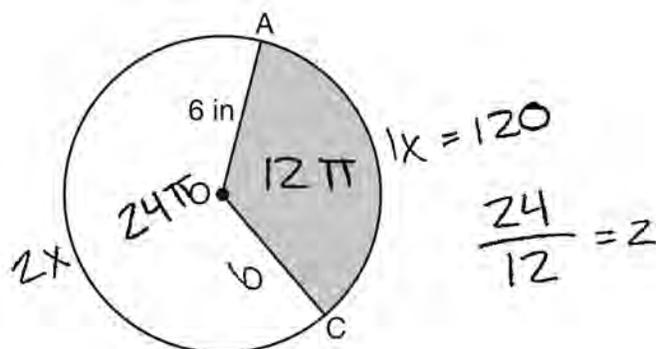
$$\theta = \frac{4\pi}{6} = \frac{2\pi}{3}$$

$$m\angle AOC = \frac{2\pi}{3}$$

Score 2: The student has a complete and correct response.

Question 29

29 In the diagram below of circle O , the area of the shaded sector AOC is 12π in² and the length of OA is 6 inches. Determine and state $m\angle AOC$.



$$A = \pi r^2$$

$$A = \pi (6)^2$$

$$A = 36\pi$$

$$36\pi - 12\pi = 24\pi$$

$$m\angle AOC = 120^\circ$$

$$2x + 1x = 360$$

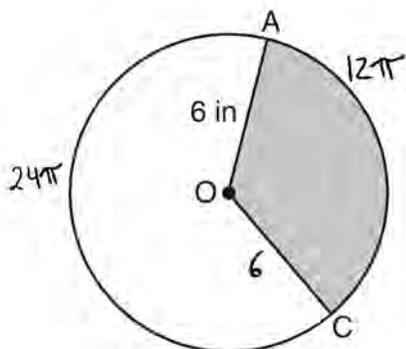
$$3x = 360$$

$$x = 120$$

Score 2: The student has a complete and correct response.

Question 29

29 In the diagram below of circle O , the area of the shaded sector AOC is 12π in² and the length of OA is 6 inches. Determine and state $m\angle AOC$.



$$A = \pi r^2$$

$$A = \pi 6^2$$

$$A = 36\pi$$

$$36\pi - 12\pi = 24\pi$$

$$\frac{24\pi}{36\pi} = \frac{x}{360}$$

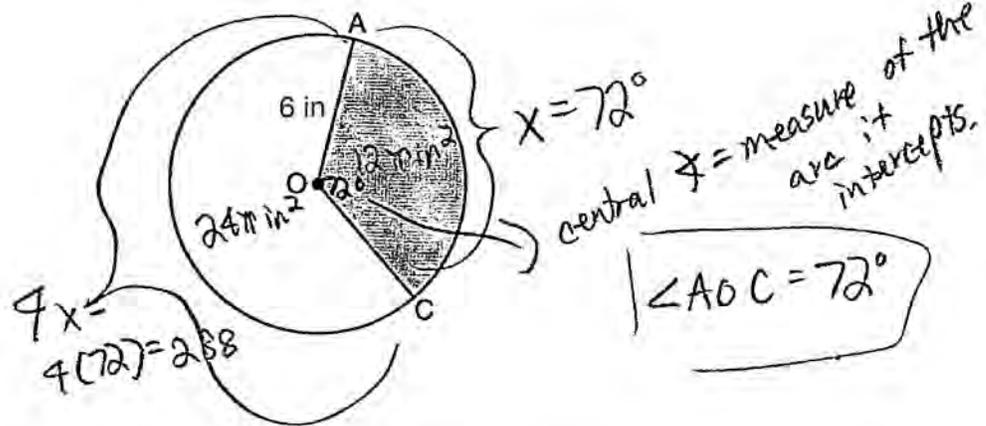
$$8640 = 36x$$

$$240^\circ = x$$

Score 1: The student made an error by finding the central angle for the unshaded sector.

Question 29

29 In the diagram below of circle O , the area of the shaded sector AOC is $12\pi \text{ in}^2$ and the length of OA is 6 inches. Determine and state $m\angle AOC$.



$$A = \pi r^2$$

$$A = (6)^2 \pi = 36\pi \text{ in}^2$$

$36\pi \text{ in}^2$	
$- 12\pi \text{ in}^2$	
<hr/>	
$24\pi \text{ in}^2$	

$$\frac{12}{24} = \frac{1}{2}$$

$$1x + 4x = 360$$

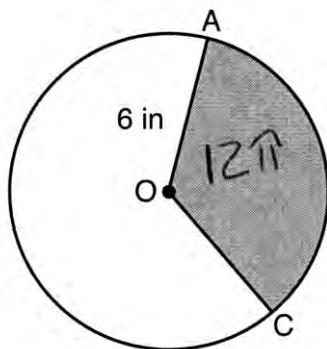
$$\frac{5x}{5} = \frac{360}{5}$$

$$x = 72$$

Score 1: The student made an error when reducing $\frac{12}{24}$.

Question 29

29 In the diagram below of circle O , the area of the shaded sector AOC is 12π in² and the length of OA is 6 inches. Determine and state $m\angle AOC$.



$$\text{Sector} = \theta r$$

$$\frac{12\pi}{6} = \frac{\theta(6)}{6}$$

$$2\pi = \theta$$

$$m\angle AOC = 2\pi$$

Score 0: The student had a completely incorrect response.

Question 30

30 After a reflection over a line, $\triangle A'B'C'$ is the image of $\triangle ABC$. Explain why triangle ABC is congruent to triangle $A'B'C'$.

Reflections are rigid motions and Rigid
Motions ~~do~~ keep distances the same.
So $\overline{AB} \cong \overline{A'B'}$ and $\overline{BC} \cong \overline{B'C'}$ and
 $\overline{AC} \cong \overline{A'C'}$, so $\triangle's \cong$ SSS

Score 2: The student has a complete and correct response.

Question 30

30 After a reflection over a line, $\triangle A'B'C'$ is the image of $\triangle ABC$. Explain why triangle ABC is congruent to triangle $A'B'C'$.

Two triangles are congruent if rigid motions can map one onto another. A reflection is a rigid motion.
So $\triangle ABC \cong \triangle A'B'C'$ after a reflection.

Score 2: The student has a complete and correct response.

Question 30

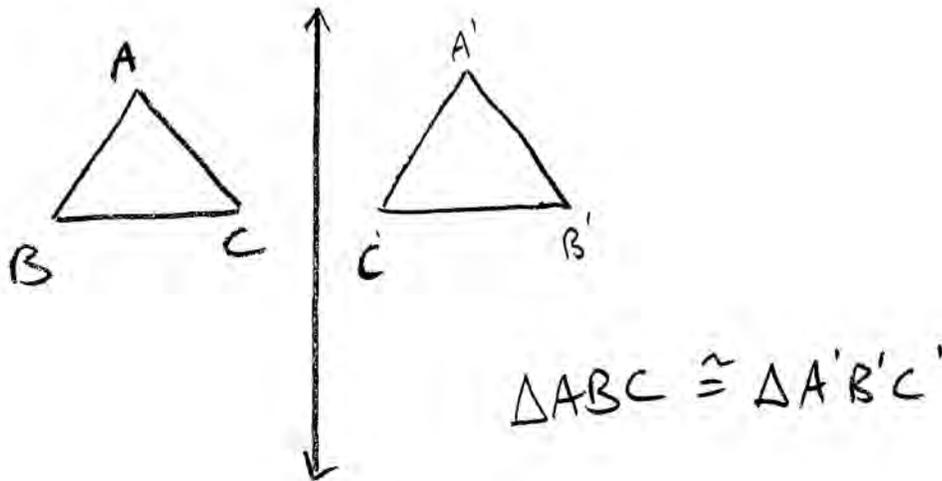
30 After a reflection over a line, $\triangle A'B'C'$ is the image of $\triangle ABC$. Explain why triangle ABC is congruent to triangle $A'B'C'$.

Because reflections are rigid motions.

Score 1: The student wrote an incomplete explanation.

Question 30

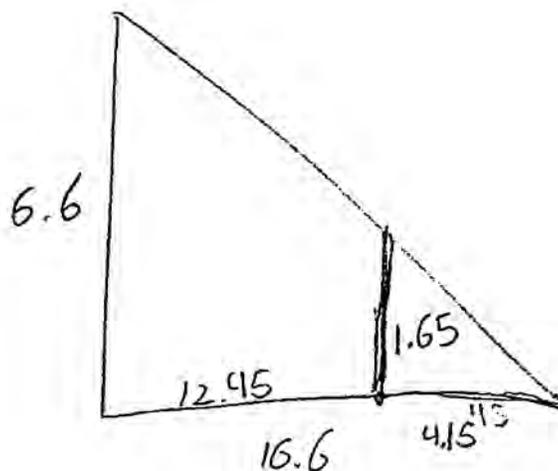
30 After a reflection over a line, $\triangle A'B'C'$ is the image of $\triangle ABC$. Explain why triangle ABC is congruent to triangle $A'B'C'$.



Score 0: The student did not provide an explanation.

Question 31

- 31 A flagpole casts a shadow 16.60 meters long. Tim stands at a distance of 12.45 meters from the base of the flagpole, such that the end of Tim's shadow meets the end of the flagpole's shadow. If Tim is 1.65 meters tall, determine and state the height of the flagpole to the *nearest tenth of a meter*.



$$\frac{1.65}{4.15} = \frac{x}{16.6}$$

$$4.15x = 27.39$$

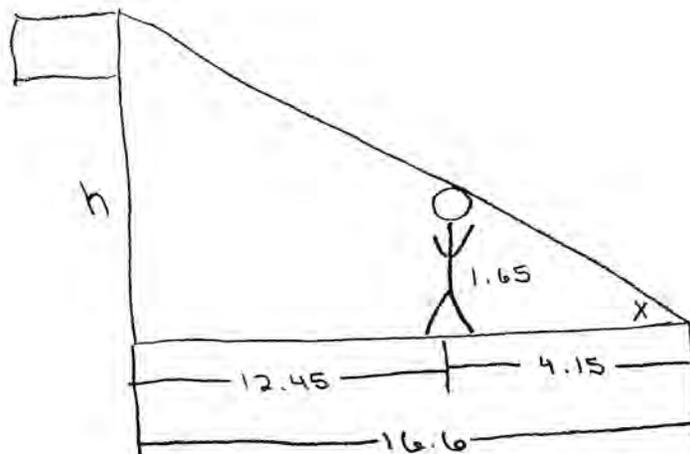
$$x = 6.6$$

6.6 m

Score 2: The student has a complete and correct response.

Question 31

- 31 A flagpole casts a shadow 16.60 meters long. Tim stands at a distance of 12.45 meters from the base of the flagpole, such that the end of Tim's shadow meets the end of the flagpole's shadow. If Tim is 1.65 meters tall, determine and state the height of the flagpole to the *nearest tenth of a meter*.



$$\tan x = \frac{1.65}{4.15}$$

$$x = 21.7$$

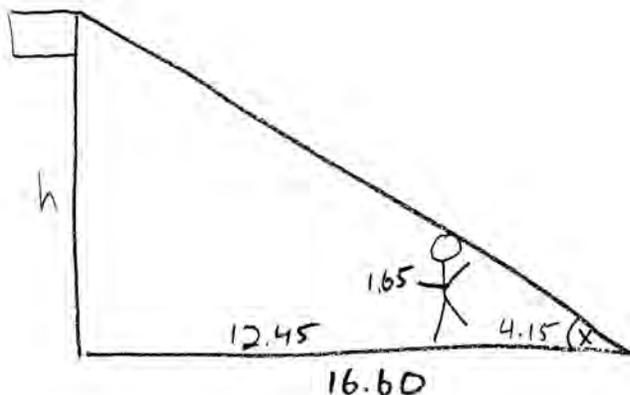
$$\tan 21.7 = \frac{h}{16.6}$$

$$h = 6.6$$

Score 2: The student has a complete and correct response.

Question 31

- 31 A flagpole casts a shadow 16.60 meters long. Tim stands at a distance of 12.45 meters from the base of the flagpole, such that the end of Tim's shadow meets the end of the flagpole's shadow. If Tim is 1.65 meters tall, determine and state the height of the flagpole to the *nearest tenth of a meter*.



$$\begin{array}{r} 16.60 \\ - 12.45 \\ \hline 4.15 \end{array}$$

$$\tan X = \frac{1.65}{4.15}$$

$$\tan(.378427378) = \frac{h}{16.60}$$

$$X = \tan^{-1}\left(\frac{1.65}{4.15}\right)$$

$$h = 16.60 \cdot \tan(.378427378)$$

$$X = .378427378$$

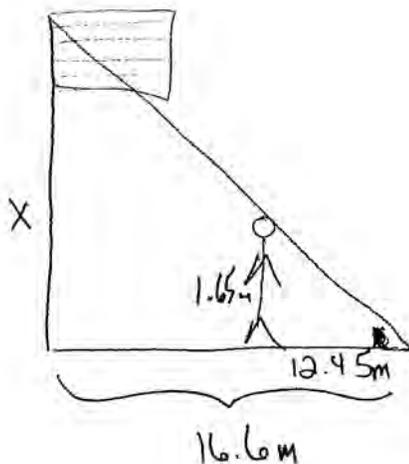
$$h = 6.6$$

6.6 meters

Score 2: The student has a complete and correct response.

Question 31

- 31 A flagpole casts a shadow 16.60 meters long. Tim stands at a distance of 12.45 meters from the base of the flagpole, such that the end of Tim's shadow meets the end of the flagpole's shadow. If Tim is 1.65 meters tall, determine and state the height of the flagpole to the *nearest tenth of a meter*.

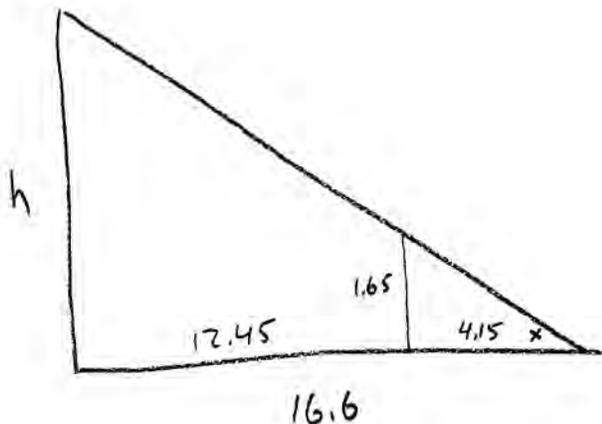


$$\frac{x}{16.60} = \frac{1.65}{12.45}$$
$$12.45x = 27.39$$
$$x = 2.2\text{ m}$$

Score 1: The student wrote an incorrect equation based on an incorrectly labeled diagram, but solved it appropriately.

Question 31

- 31 A flagpole casts a shadow 16.60 meters long. Tim stands at a distance of 12.45 meters from the base of the flagpole, such that the end of Tim's shadow meets the end of the flagpole's shadow. If Tim is 1.65 meters tall, determine and state the height of the flagpole to the *nearest tenth of a meter*.



$$\sin x = \frac{1.65}{4.15}$$

$$x = \sin^{-1}\left(\frac{1.65}{4.15}\right)$$

$$x = 23.427626509$$

$$16.6 \cdot \sin(23.427626509) = \frac{h}{16.6} \cdot 16.6$$

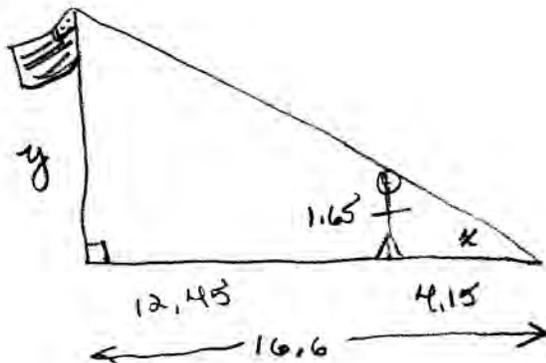
$$6.6 = h$$

6.6 meters

Score 1: The student made an error using the incorrect trigonometric function, and found an incorrect angle measure for x . The student made the same error in finding the height.

Question 31

- 31 A flagpole casts a shadow 16.60 meters long. Tim stands at a distance of 12.45 meters from the base of the flagpole, such that the end of Tim's shadow meets the end of the flagpole's shadow. If Tim is 1.65 meters tall, determine and state the height of the flagpole to the *nearest tenth of a meter*.



$$\tan x = \frac{1.65}{4.15}$$

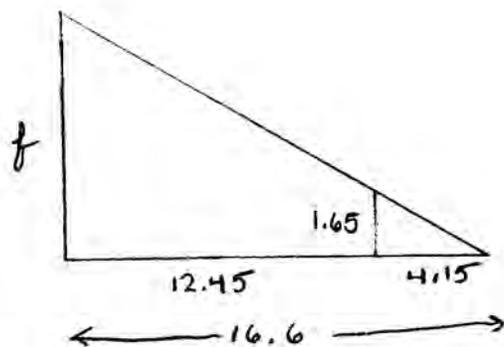
$$\tan x = \frac{y}{16.6}$$

?

Score 1: The student wrote a correct system of equations, but no further work was shown.

Question 31

- 31 A flagpole casts a shadow 16.60 meters long. Tim stands at a distance of 12.45 meters from the base of the flagpole, such that the end of Tim's shadow meets the end of the flagpole's shadow. If Tim is 1.65 meters tall, determine and state the height of the flagpole to the *nearest tenth of a meter*.



$$\frac{f}{16.6} = \frac{1.65}{4.15}$$

$$\frac{f}{16.6} = \frac{1.7}{4.2}$$

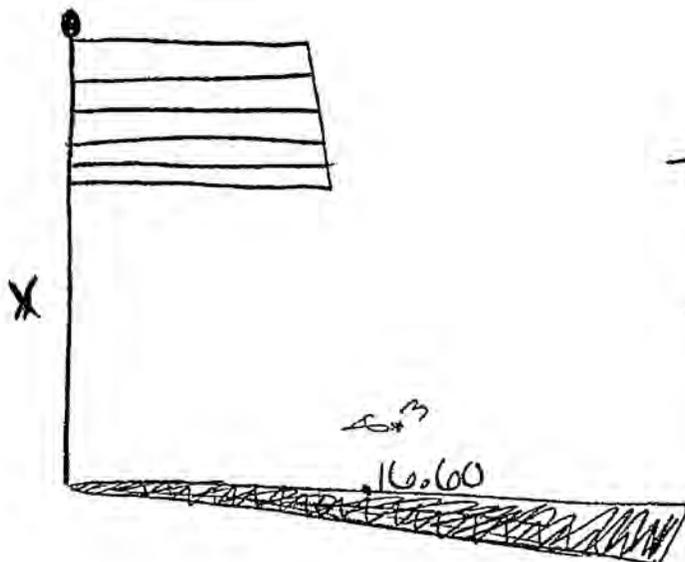
$$f = 6.719047619$$

$$f = 6.7$$

Score 1: The student wrote a correct proportion, but no further correct work was shown.

Question 31

- 31 A flagpole casts a shadow 16.60 meters long. Tim stands at a distance of 12.45 meters from the base of the flagpole, such that the end of Tim's shadow meets the end of the flagpole's shadow. If Tim is 1.65 meters tall, determine and state the height of the flagpole to the *nearest tenth of a meter*.



$$\frac{x}{1.65} = \frac{12.45}{16.60}$$

$$\frac{20.5425}{16.60} = \frac{16.60x}{16.60}$$

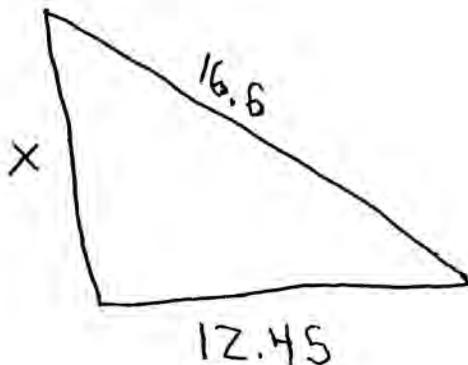
$$1.2375 = x$$

height of Flagpole = 1.2 meters long

Score 0: The student did not subtract 12.45 from 16.60. The student also wrote an incorrect proportion.

Question 31

- 31 A flagpole casts a shadow 16.60 meters long. Tim stands at a distance of 12.45 meters from the base of the flagpole, such that the end of Tim's shadow meets the end of the flagpole's shadow. If Tim is 1.65 meters tall, determine and state the height of the flagpole to the *nearest tenth of a meter*.



$$x^2 + 12.45^2 = 16.6$$

$$x^2 + 155.0025 = 275.56$$

$$x^2 = 120.5575$$

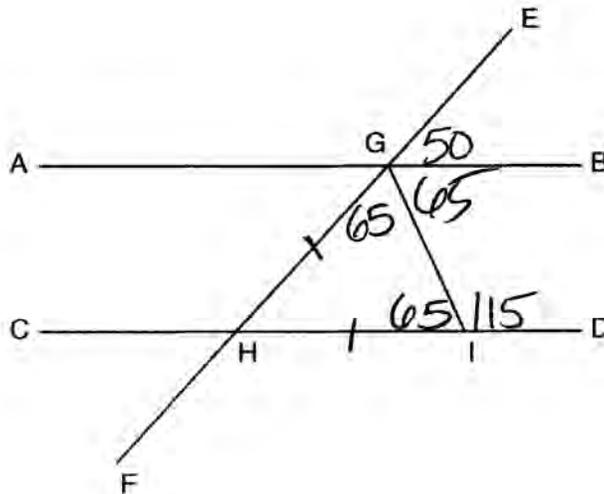
$$x = 10.979$$

(11)

Score 0: The student had a completely incorrect response.

Question 32

32 In the diagram below, \overline{EF} intersects \overline{AB} and \overline{CD} at G and H , respectively, and \overline{GI} is drawn such that $\overline{GH} \cong \overline{IH}$.



If $m\angle EGB = 50^\circ$ and $m\angle DIG = 115^\circ$, explain why $\overline{AB} \parallel \overline{CD}$.

$m\angle GIH = 65$ linear pairs are supplementary

$m\angle HGI = 65$ - Base angles of an isosceles triangle are equal

$$m\angle EGB + m\angle BGI + m\angle HGI = 180$$

$$50 + m\angle BGI + 65 = 180$$

$$115 + m\angle BGI = 180$$

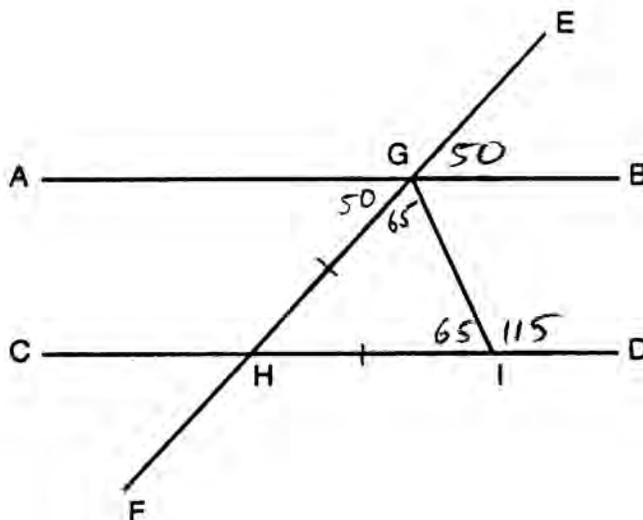
$$\begin{array}{r} 115 + m\angle BGI = 180 \\ -115 \qquad \qquad -115 \\ \hline m\angle BGI = 65 \end{array}$$

$\angle BGI$ and $\angle DIG$ are same-side interior \angle 's,
and since they are supplementary, $\overline{AB} \parallel \overline{CD}$.

Score 4: The student has a complete and correct response.

Question 32

32 In the diagram below, \overline{EF} intersects \overline{AB} and \overline{CD} at G and H , respectively, and \overline{GI} is drawn such that $\overline{GH} \cong \overline{IH}$.



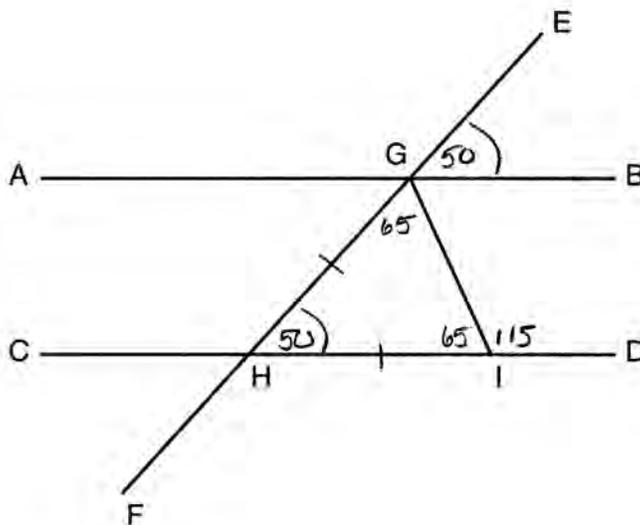
If $m\angle EGB = 50^\circ$ and $m\angle DIG = 115^\circ$, explain why $\overline{AB} \parallel \overline{CD}$.

$\angle AGH = 50$, and $\angle GIH = 65$, $\triangle GHI$ is isosceles
 so $\angle GIH \cong \angle IGH$. This makes $\angle AGI = 115$,
 and since alternate interior angles $\angle AGI$ and
 $\angle DIG$ are congruent, $\overline{AB} \parallel \overline{CD}$.

Score 3: The student stated correct angle measures, but did not have an explanation for $m\angle AGH$ and $m\angle GIH$.

Question 32

32 In the diagram below, \overline{EF} intersects \overline{AB} and \overline{CD} at G and H , respectively, and \overline{GI} is drawn such that $\overline{GH} \cong \overline{HI}$.



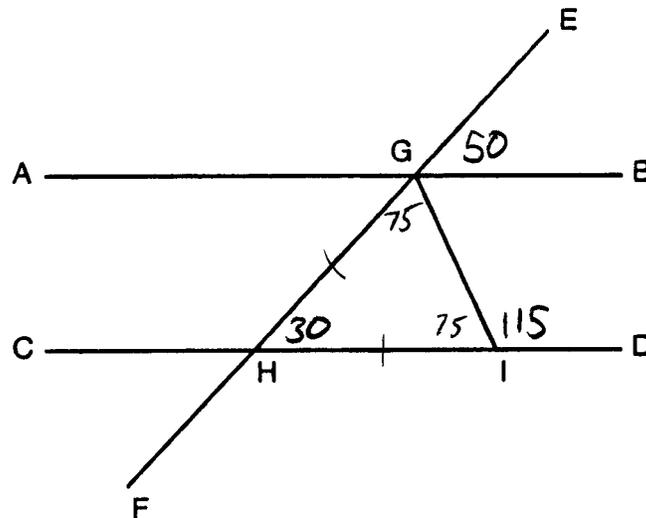
If $m\angle EGB = 50^\circ$ and $m\angle DIG = 115^\circ$, explain why $\overline{AB} \parallel \overline{CD}$.

$\angle DIG$ is supplementary to $\angle HIG$, so $m\angle HIG = 65$.
 $\angle HIG = \angle HGI$ because angles opposite equal sides are equal.
 The sum of angles of a triangle is 180° so $\angle GHI$ is 50 .
 So, $\overline{AB} \parallel \overline{CD}$.

Score 3: The student stated correct angle measures with explanations, but did not explain why $\overline{AB} \parallel \overline{CD}$.

Question 32

32 In the diagram below, \overline{EF} intersects \overline{AB} and \overline{CD} at G and H , respectively, and \overline{GI} is drawn such that $\overline{GH} \cong \overline{HI}$.



$$\begin{array}{r} 180 \\ - 115 \\ \hline 75 \end{array}$$

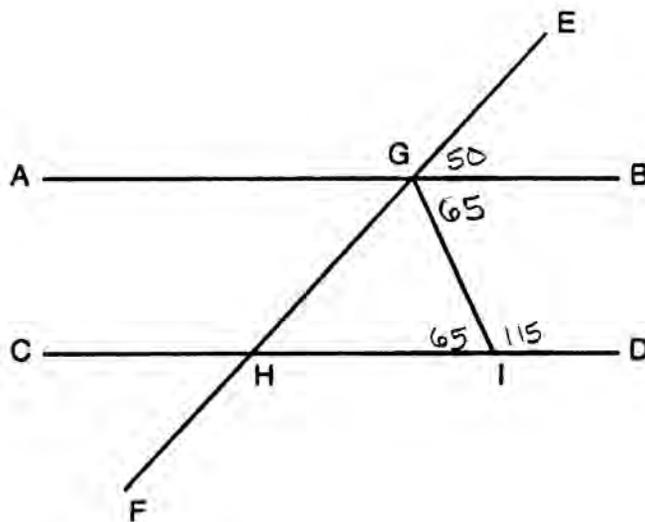
If $m\angle EGB = 50^\circ$ and $m\angle DIG = 115^\circ$, explain why $\overline{AB} \parallel \overline{CD}$.

$\angle HIG$ is supplementary to $\angle DIG$, so $\angle HIG = 75$.
 $\triangle GHI$ is isosceles, so $\angle HGI = 75$ too. The angles of a triangle add to 180, so $\angle GHI = 30$.
 Since alternate interior angles $\angle EGB$ and $\angle GHI$ are not equal, \overline{AB} is not parallel to \overline{CD} .

Score 2: The student made one computational error in finding $m\angle HIG$. The student made an error in the explanation by identifying $\angle EGB$ and $\angle GHI$ as alternate interior angles.

Question 32

32 In the diagram below, \overline{EF} intersects \overline{AB} and \overline{CD} at G and H , respectively, and \overline{GI} is drawn such that $\overline{GH} \cong \overline{IH}$.



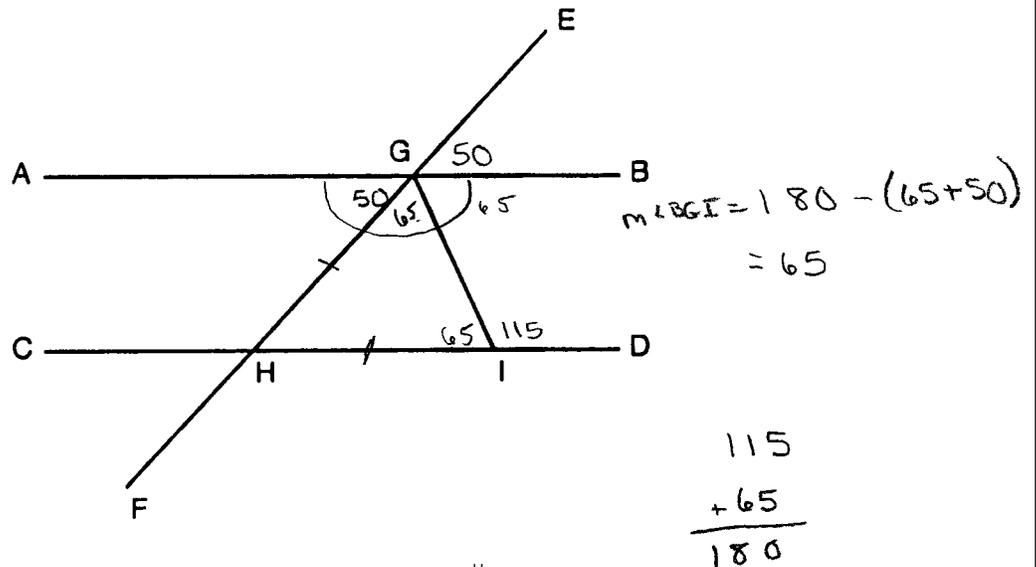
If $m\angle EGB = 50^\circ$ and $m\angle DIG = 115^\circ$, explain why $\overline{AB} \parallel \overline{CD}$.

$m\angle DIG + m\angle HIG = 180$ supplementary
 $m\angle HIG \cong m\angle BGI$ alternate interior
 $m\angle BGI + m\angle DIG = 180$ same side interior
 $65 + 115 = 180$
 $180 = 180$ lines parallel when
 same side interior
 angles add up to 180.

Score 2: The student made one conceptual error using alternate interior angles of parallel lines to prove the same lines parallel.

Question 32

32 In the diagram below, \overline{EF} intersects \overline{AB} and \overline{CD} at G and H , respectively, and \overline{GI} is drawn such that $\overline{GH} \cong \overline{IH}$.



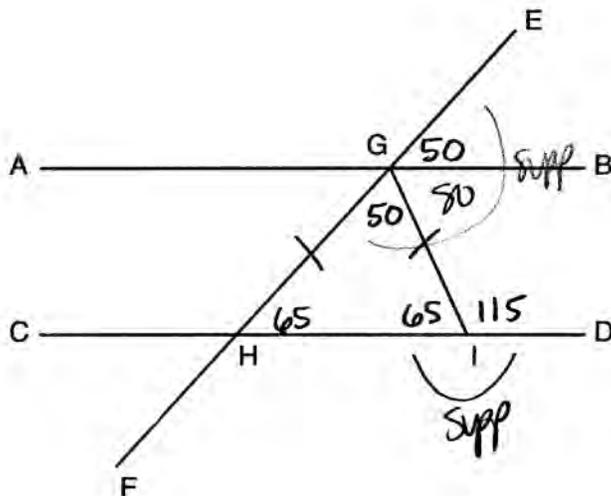
If $m\angle EGB = 50^\circ$ and $m\angle DIG = 115^\circ$, explain why $\overline{AB} \parallel \overline{CD}$.

$$\begin{aligned}
 m\angle BGI + m\angle DIG &= 180 \\
 65 + 115 &= 180 \\
 180 &= 180 \rightarrow \text{only when lines are } \parallel
 \end{aligned}$$

Score 2: The student had appropriate angle measures stated correctly, but was missing the explanation.

Question 32

32 In the diagram below, \overline{EF} intersects \overline{AB} and \overline{CD} at G and H , respectively, and \overline{GI} is drawn such that $\overline{GH} \cong \overline{IH}$.

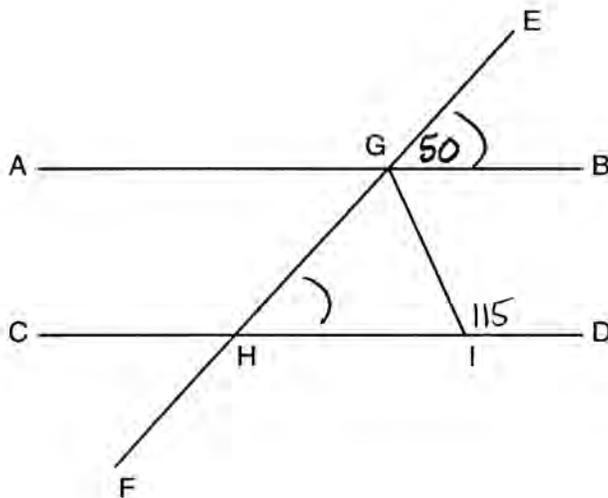


If $m\angle EGB = 50^\circ$ and $m\angle DIG = 115^\circ$, explain why $\overline{AB} \parallel \overline{CD}$.

Score 1: The student found appropriate angle measures based on a mislabeled diagram, and the explanation was missing.

Question 32

32 In the diagram below, \overline{EF} intersects \overline{AB} and \overline{CD} at G and H , respectively, and \overline{GI} is drawn such that $\overline{GH} \cong \overline{IH}$.



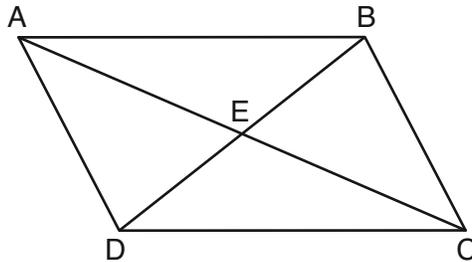
If $m\angle EGB = 50^\circ$ and $m\angle DIG = 115^\circ$, explain why $\overline{AB} \parallel \overline{CD}$.

Corresponding \angle s are \cong ,
so $AB \parallel CD$.

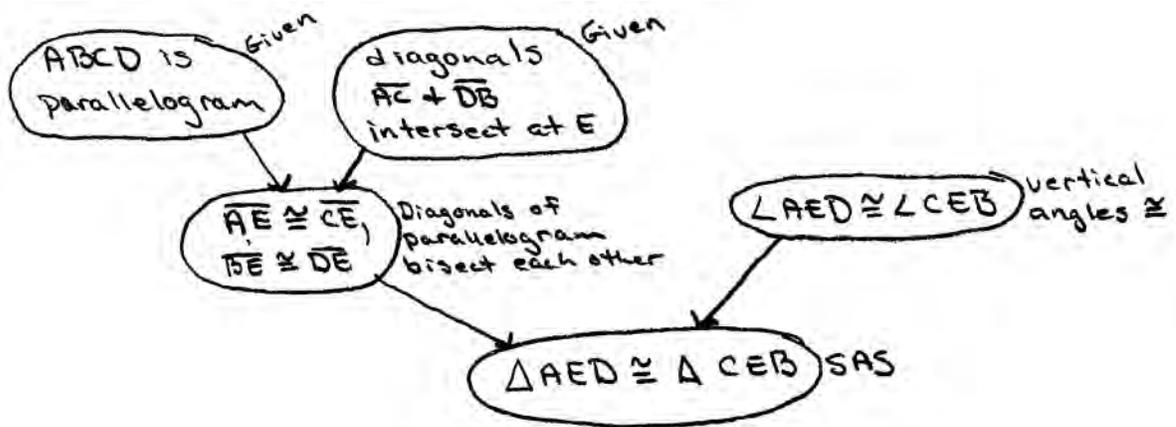
Score 0: The student did not show enough work on which to base the explanation.

Question 33

33 Given: Quadrilateral $ABCD$ is a parallelogram with diagonals \overline{AC} and \overline{BD} intersecting at E



Prove: $\triangle AED \cong \triangle CEB$



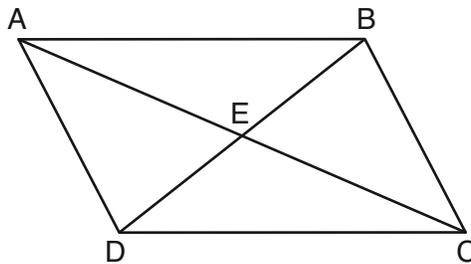
Describe a single rigid motion that maps $\triangle AED$ onto $\triangle CEB$.

rotation of 180° at point E .

Score 4: The student has a complete and correct proof, and a correct rigid motion is stated.

Question 33

33 Given: Quadrilateral $ABCD$ is a parallelogram with diagonals \overline{AC} and \overline{BD} intersecting at E



Prove: $\triangle AED \cong \triangle CEB$

Statement	Reason
1. Quad $ABCD$ is a parallelogram	1. given
2. $\overline{AD} \cong \overline{CB}$	2. opposite sides of parallelogram are congruent
3. \overline{AC} and \overline{DB} intersect at E	3. given
4. $\angle AED \cong \angle CEB$	4. vertical angles are congruent
5. $\overline{BC} \parallel \overline{DA}$	5. def. of \square
6. $\angle DBC \cong \angle BDA$	6. alt. interior angles are \cong
7. $\triangle AED \cong \triangle CEB$	7. AAS \cong AAS

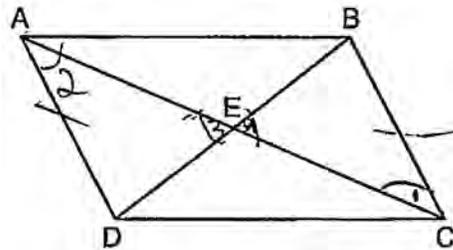
Describe a single rigid motion that maps $\triangle AED$ onto $\triangle CEB$.

Rotation of $\triangle AED$ around point E of 180°

Score 4: The student has a complete and correct proof, and a correct rigid motion is described.

Question 33

33 Given: Quadrilateral $ABCD$ is a parallelogram with diagonals \overline{AC} and \overline{BD} intersecting at E



Prove: $\triangle AED \cong \triangle CEB$

Statements	Reasons
1. Quadrilateral $ABCD$ is a \square with diagonals \overline{AC} & \overline{DB} intersecting at E	1. GN
2. $AD \cong BC$	2. Opposite sides of a \square are \cong
3. $AD \parallel BC$	3. Opposite sides of a \square are \parallel
4. $\angle 1 \cong \angle 2$	4. If 2 \parallel lines are cut by a transversal, the alternate interior \angle 's are \cong .
5. $\angle 3 \cong \angle 4$	5. Vertical \angle 's are \cong
6. $\triangle AED \cong \triangle BEC$	6. AAS \cong AAS

AAS

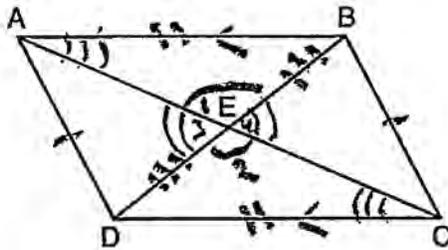
Describe a single rigid motion that maps $\triangle AED$ onto $\triangle CEB$.

Reflection

Score 3: The student wrote an incomplete description of the rigid motion.

Question 33

33 Given: Quadrilateral $ABCD$ is a parallelogram with diagonals \overline{AC} and \overline{BD} intersecting at E



Prove: $\triangle AED \cong \triangle CEB$

- | | |
|---|---|
| <p>1. AC and DB intersect at E,
ABCD is a \square</p> <p>2. $\overline{AB} \cong \overline{DC}$, $\overline{AD} \cong \overline{BC}$</p> <p>3. $\angle 1 \cong \angle 2$, $\angle 3 \cong \angle 4$</p> <p>4. $AB \parallel DC$</p> <p>5. $\angle DCA \cong \angle BAC$</p> <p>6. $\triangle DCE \cong \triangle BAE$</p> <p>7. $\overline{ED} \cong \overline{EB}$</p> <p>8. $\triangle AED \cong \triangle CEB$</p> | <p>1. Given</p> <p>2. opp. sides of a \square are \cong</p> <p>3. vertical \angle's are \cong</p> <p>4. opp. sides of a \square are \parallel</p> <p>5. IF 2 lines are \parallel their alt. int. \angle's are \cong</p> <p>6. AAS</p> <p>7. corresp sides of $\cong \Delta$'s are \cong</p> <p>8 SAS</p> |
|---|---|

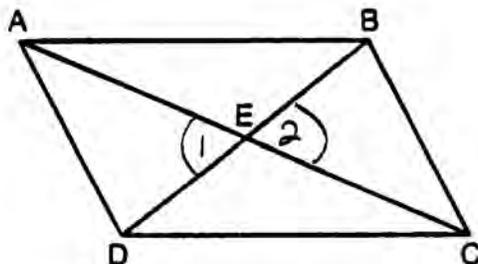
Describe a single rigid motion that maps $\triangle AED$ onto $\triangle CEB$.

reflection through E

Score 3: The student had an incorrect reason for the last step, but a correct rigid motion was stated.

Question 33

33 Given: Quadrilateral $ABCD$ is a parallelogram with diagonals \overline{AC} and \overline{BD} intersecting at E



Prove: $\triangle AED \cong \triangle CEB$

In a parallelogram, the diagonals bisect each other,
so $\overline{AE} \cong \overline{CE}$ and $\overline{BE} \cong \overline{DE}$. $\angle 1 \cong \angle 2$. So
 $\triangle AED \cong \triangle CEB$ by SAS.

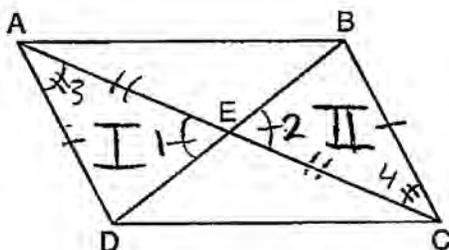
Describe a single rigid motion that maps $\triangle AED$ onto $\triangle CEB$.

180° rotation

Score 2: The student was missing the reason $\angle 1 \cong \angle 2$ and wrote an incomplete description of the rigid motion.

Question 33

33 Given: Quadrilateral $ABCD$ is a parallelogram with diagonals \overline{AC} and \overline{BD} intersecting at E



Prove: $\triangle AED \cong \triangle CEB$

Statement	Reason
1. $ABCD$ is a Parallelogram	1. Given
\overline{AC} + \overline{BD} intersect at E	2. verticle \angle s are \cong
2. $\angle 2 \cong \angle 1$	3. Opposite inverse are \cong
3. $\angle 3 \cong \angle 4$	4. Diagonals of a parallelogram are \cong
4. $\overline{AE} \cong \overline{EC}$	5. ASA
5. $\triangle I \cong \triangle II$	

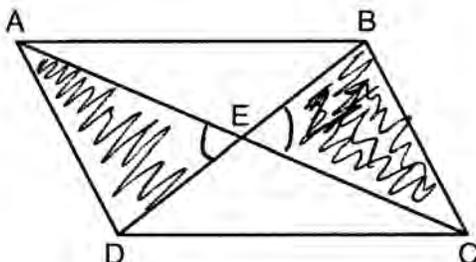
Describe a single rigid motion that maps $\triangle AED$ onto $\triangle CEB$.

Rotation 180°

Score 1: The student had some correct statements about the proof. The description of the rigid motion was incomplete.

Question 33

33 Given: Quadrilateral $ABCD$ is a parallelogram with diagonals \overline{AC} and \overline{BD} intersecting at E



Prove: $\triangle AED \cong \triangle CEB$

1) parallelogram $ABCD$ 1) given
diagonals \overline{AC} and \overline{BD}
intersecting at E

Prove: $\triangle AED \cong \triangle CEB$

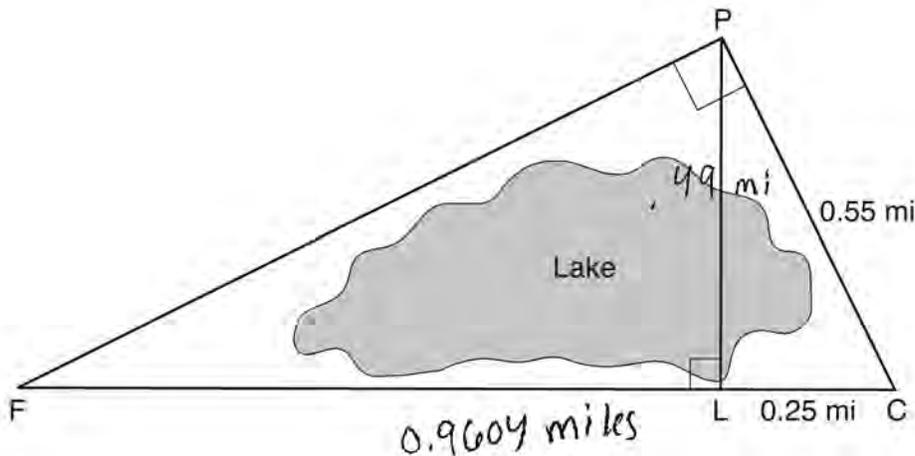
Describe a single rigid motion that maps $\triangle AED$ onto $\triangle CEB$.

Rotate 180°

Score 0: The student wrote only the “given” information, and an incomplete description of the rigid motion.

Question 34

34 In the diagram below, the line of sight from the park ranger station, P , to the lifeguard chair, L , on the beach of a lake is perpendicular to the path joining the campground, C , and the first aid station, F . The campground is 0.25 mile from the lifeguard chair. The straight paths from both the campground and first aid station to the park ranger station are perpendicular.



If the path from the park ranger station to the campground is 0.55 mile, determine and state, to the nearest hundredth of a mile, the distance between the park ranger station and the lifeguard chair.

$$a^2 + b^2 = c^2$$

$$a^2 + 0.0625 = 0.3025$$

$$a^2 = 0.24$$

$$a = 0.489897...$$

The distance is 0.49 miles

Gerald believes the distance from the first aid station to the campground is at least 1.5 miles. Is Gerald correct? Justify your answer.

Altitude = $\frac{x}{h} = \frac{h}{y}$

$$0.25y = 0.2401$$

$$y = 0.9604$$

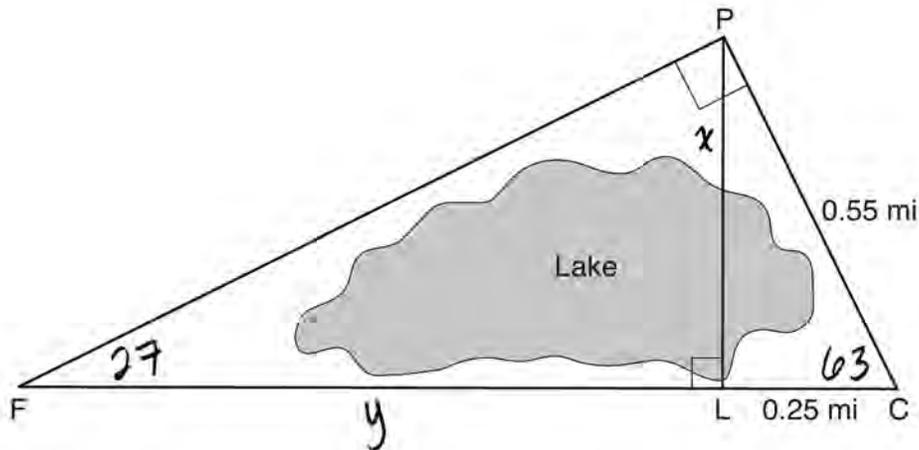
$$\frac{0.25}{0.49} = \frac{0.49}{y}$$

NO, the distance from F to L is 0.9604 miles: when added to the distance from L to C, it's only around 1.2 miles, not 1.5 miles.

Score 4: The student has a complete and correct response.

Question 34

34 In the diagram below, the line of sight from the park ranger station, P , to the lifeguard chair, L , on the beach of a lake is perpendicular to the path joining the campground, C , and the first aid station, F . The campground is 0.25 mile from the lifeguard chair. The straight paths from both the campground and first aid station to the park ranger station are perpendicular.



If the path from the park ranger station to the campground is 0.55 mile, determine and state, to the *nearest hundredth of a mile*, the distance between the park ranger station and the lifeguard chair.

$$\cos C = \frac{.25}{.55}$$

$$\tan 63 = \frac{x}{.25}$$

$$\angle C = 63^\circ$$

$$x = .4906$$

$$x = .49$$

Gerald believes the distance from the first aid station to the campground is at least 1.5 miles. Is Gerald correct? Justify your answer.

$$180 - 90 - 63 = 27$$

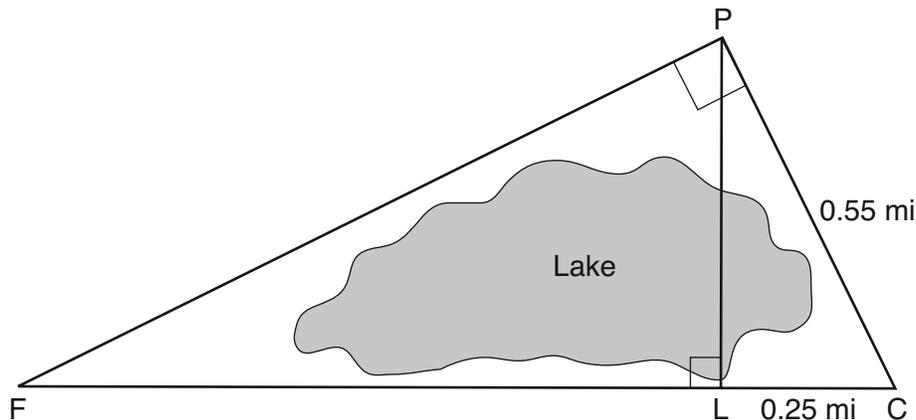
$$\begin{array}{r} y = .96 \\ + .25 \\ \hline 1.21 \end{array}$$

$$\tan 27 = \frac{.49}{y}$$

Score 3: The student did not state if Gerald is correct.

Question 34

34 In the diagram below, the line of sight from the park ranger station, P , to the lifeguard chair, L , on the beach of a lake is perpendicular to the path joining the campground, C , and the first aid station, F . The campground is 0.25 mile from the lifeguard chair. The straight paths from both the campground and first aid station to the park ranger station are perpendicular.



If the path from the park ranger station to the campground is 0.55 mile, determine and state, to the nearest hundredth of a mile, the distance between the park ranger station and the lifeguard chair.

$$\frac{0.55}{FC} = \frac{0.25}{0.55}$$

$$\frac{x}{0.96} = \frac{0.25}{x}$$

$$0.25 FC = 0.3025$$

$$x^2 = 0.24$$

$$FC = 1.21$$

$$x = 0.55$$

Distance between P and L = 0.5 mi.

Gerald believes the distance from the first aid station to the campground is at least 1.5 miles. Is Gerald correct? Justify your answer.

let x be FC

No because it is 1.21 miles.

$$\frac{0.55}{x} = \frac{0.25}{0.55}$$

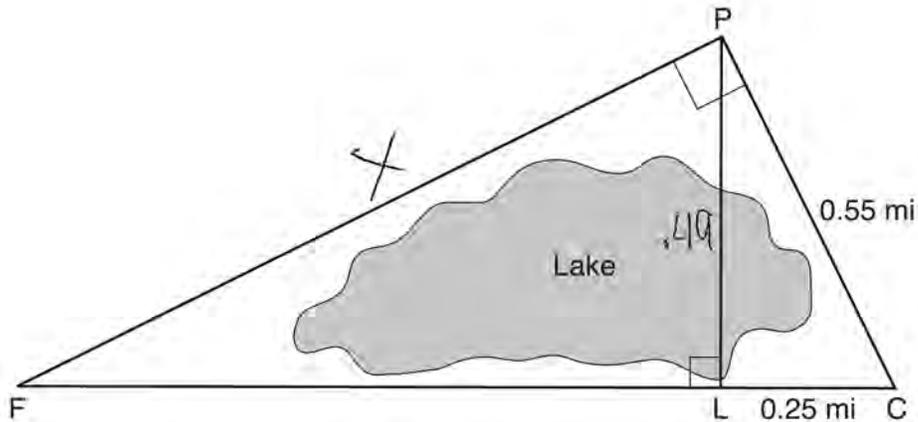
$$0.25 FC = 0.3025$$

$$FC = 1.21$$

Score 2: The student made one computational error and one rounding error in finding the distance between the park ranger station and the lifeguard chair.

Question 34

34 In the diagram below, the line of sight from the park ranger station, P , to the lifeguard chair, L , on the beach of a lake is perpendicular to the path joining the campground, C , and the first aid station, F . The campground is 0.25 mile from the lifeguard chair. The straight paths from both the campground and first aid station to the park ranger station are perpendicular.



If the path from the park ranger station to the campground is 0.55 mile, determine and state, to the nearest hundredth of a mile, the distance between the park ranger station and the lifeguard chair.

$$\begin{aligned}
 a^2 + b^2 &= c^2 \\
 0.25^2 + b^2 &= 0.55^2 \\
 0.0625 + b^2 &= 0.3025 \\
 -0.0625 & \quad -0.0625 \\
 \hline
 b^2 &= 0.24 \quad b = 0.49
 \end{aligned}$$

0.49 miles

Gerald believes the distance from the first aid station to the campground is at least 1.5 miles. Is Gerald correct? Justify your answer.

$$\begin{aligned}
 0.25^2 + x + 49^2 &= c^2 \\
 0.0625 + x + 2401 &= c^2 \\
 0.3025 + x &= c^2 \\
 \boxed{x = 0.55}
 \end{aligned}$$

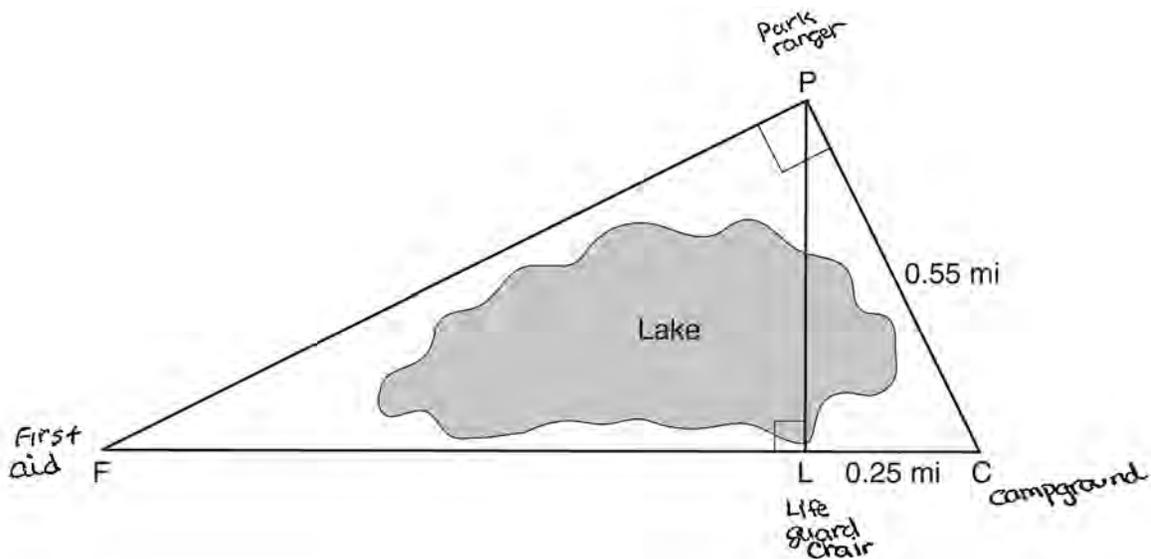
0.55 + 0.25 = 0.8

NO, Gerald is not correct it is about .8 miles from the first aid station to the campground

Score 2: The student showed correct work to find 0.49, but no further correct work was shown.

Question 34

34 In the diagram below, the line of sight from the park ranger station, P , to the lifeguard chair, L , on the beach of a lake is perpendicular to the path joining the campground, C , and the first aid station, F . The campground is 0.25 mile from the lifeguard chair. The straight paths from both the campground and first aid station to the park ranger station are perpendicular.



If the path from the park ranger station to the campground is 0.55 mile, determine and state, to the *nearest hundredth of a mile*, the distance between the park ranger station and the lifeguard chair.

$$\begin{aligned}
 (0.25)^2 + b^2 &= (0.55)^2 \\
 0.0625 + b^2 &= .3025 \\
 - .0625 &\quad - .0625 \\
 \hline
 b^2 &= \sqrt{.24} \\
 b &= 0.4898979486
 \end{aligned}$$

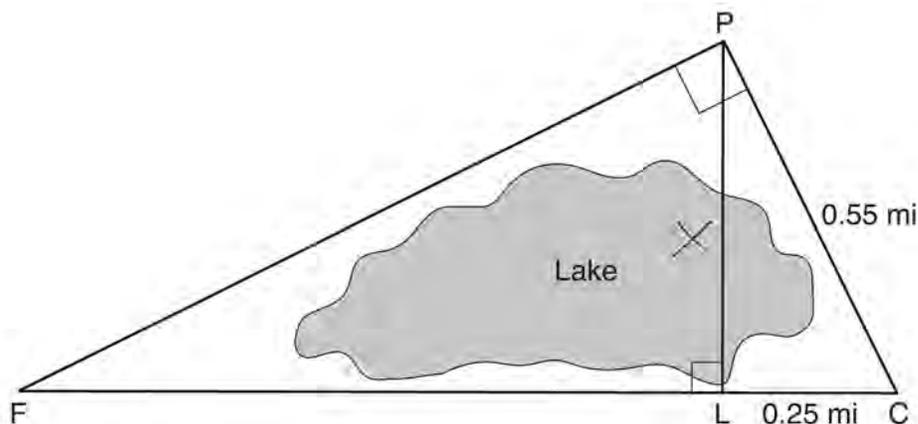
Distance \approx 0.5 miles

Gerald believes the distance from the first aid station to the campground is at least 1.5 miles. Is Gerald correct? Justify your answer.

Score 1: The student made one rounding error, and no further correct work was shown.

Question 34

- 34 In the diagram below, the line of sight from the park ranger station, P , to the lifeguard chair, L , on the beach of a lake is perpendicular to the path joining the campground, C , and the first aid station, F . The campground is 0.25 mile from the lifeguard chair. The straight paths from both the campground and first aid station to the park ranger station are perpendicular.



If the path from the park ranger station to the campground is 0.55 mile, determine and state, to the *nearest hundredth of a mile*, the distance between the park ranger station and the lifeguard chair.

$$\begin{aligned} .25^2 + .55^2 &= x^2 \\ .0625 + .3025 &= x^2 \\ .365 &= x^2 \\ \boxed{.6 = x} \end{aligned}$$

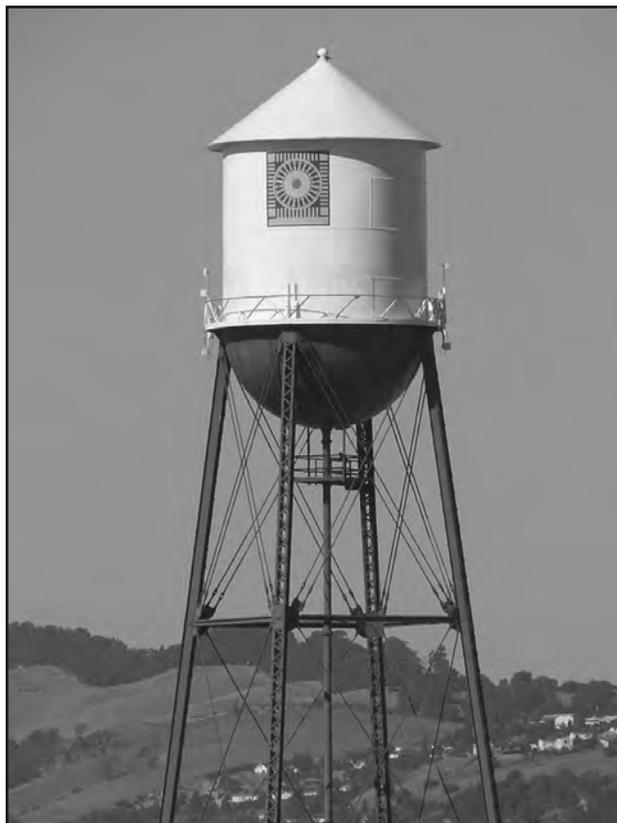
Gerald believes the distance from the first aid station to the campground is at least 1.5 miles. Is Gerald correct? Justify your answer.

Yes - its far away.

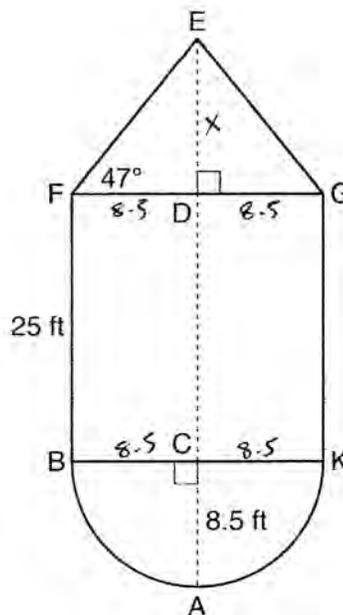
Score 0: The student had a completely incorrect response.

Question 35

35 The water tower in the picture below is modeled by the two-dimensional figure beside it. The water tower is composed of a hemisphere, a cylinder, and a cone. Let C be the center of the hemisphere and let D be the center of the base of the cone.



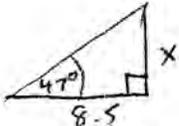
Source: <http://en.wikipedia.org>



Question 35 is continued on the next page.

Question 35 continued

If $AC = 8.5$ feet, $BF = 25$ feet, and $m\angle EFD = 47^\circ$, determine and state, to the nearest cubic foot, the volume of the water tower.



Volume cone	Volume cylinder	Volume Hemisphere
$V = \frac{1}{3}\pi r^2 h$	$V = \pi r^2 h$	$V = \frac{1}{2} \left(\frac{4}{3} \pi r^3 \right)$
$= \frac{1}{3} \pi (8.5)^2 (9.11513)$	$V = \pi (8.5)^2 (25)$	$= \frac{2}{3} \pi (8.5)^3$
$V = 689.65125$	$V = 5674.50173$	$= 1286.22039$

$$V = 689.65125 + 5674.50173 + 1286.22039 = 7650.37337$$

$$= \boxed{7650 \text{ ft}^3}$$

The water tower was constructed to hold a maximum of 400,000 pounds of water. If water weighs 62.4 pounds per cubic foot, can the water tower be filled to 85% of its volume and *not* exceed the weight limit? Justify your answer.

$$7650 \times 62.4 = 477,360 \text{ lbs}$$

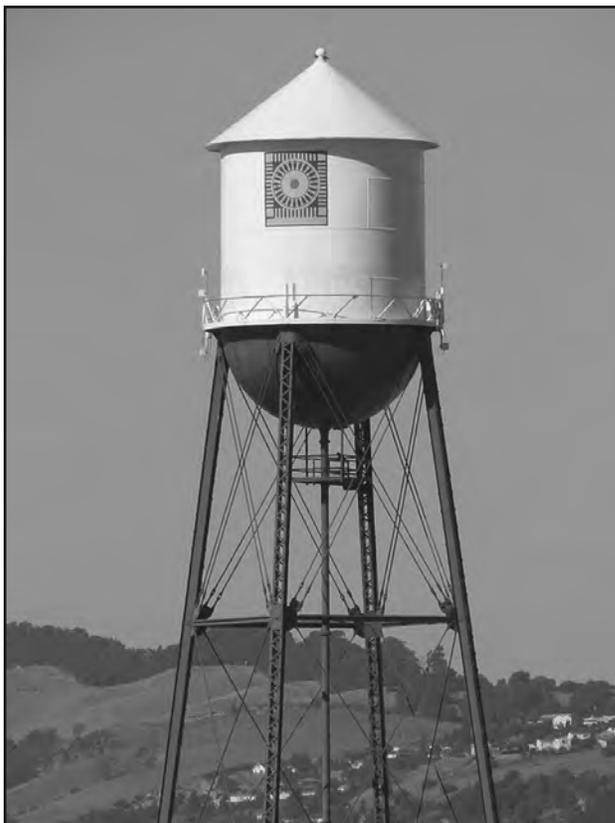
$$.85 \times 477,360 = \boxed{405,756 \text{ lbs}}$$

No - the weight would exceed 400,000 lbs

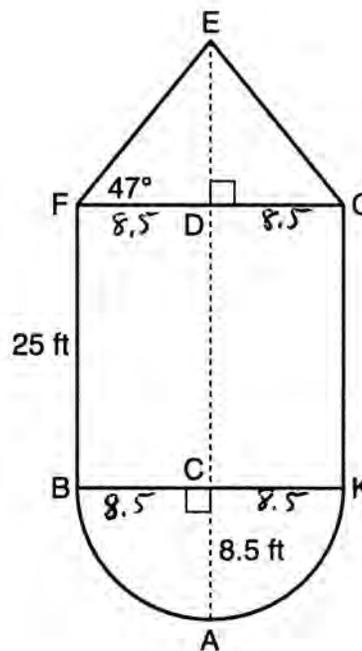
Score 6: The student had a complete and correct response.

Question 35

35 The water tower in the picture below is modeled by the two-dimensional figure beside it. The water tower is composed of a hemisphere, a cylinder, and a cone. Let C be the center of the hemisphere and let D be the center of the base of the cone.



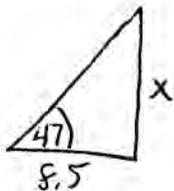
Source: <http://en.wikipedia.org>



Question 35 is continued on the next page.

Question 35 continued

If $AC = 8.5$ feet, $BF = 25$ feet, and $m\angle EFD = 47^\circ$, determine and state, to the nearest cubic foot, the volume of the water tower.



$$\tan 47 = \frac{x}{8.5}$$

$$x = 9.115$$

$$V = \frac{1}{3}\pi r^2 h + \pi r^2 h + \frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$$

$$V = \frac{1}{3}(3.14)(8.5)^2(9.115) + 3.14(8.5)^2(25) + \frac{1}{2} \cdot \frac{4}{3}(3.14)(8.5)^3$$

$$= 689.2914917 + 5671.625 + 1285.568333$$

$$= 7646.484825$$

$$V = 7646$$

The water tower was constructed to hold a maximum of 400,000 pounds of water. If water weighs 62.4 pounds per cubic foot, can the water tower be filled to 85% of its volume and *not* exceed the weight limit? Justify your answer.

$$7646(62.4) = 477,110.4 \text{ pounds}$$

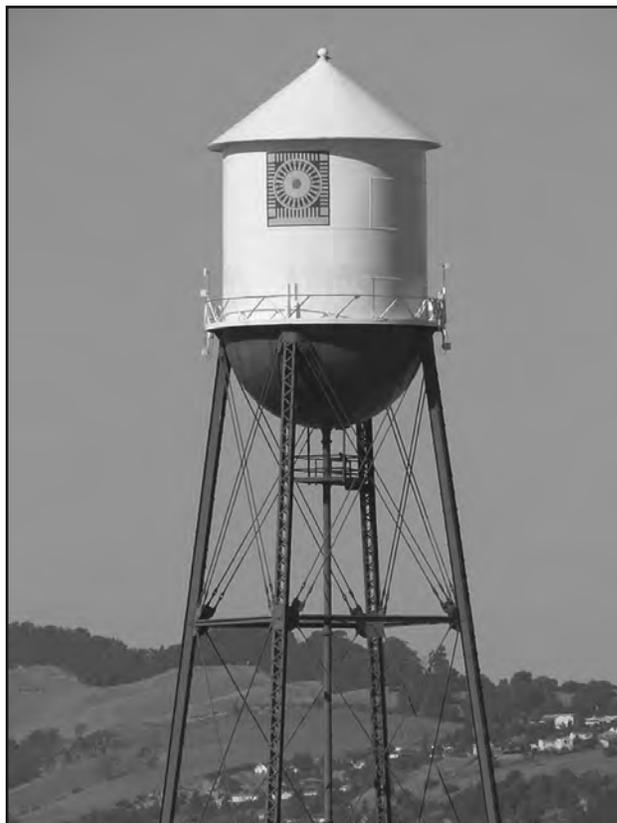
$$477,110.4(.85) = 405,543.84 \text{ pounds}$$

No because it would exceed 400,000 pounds

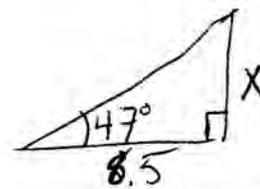
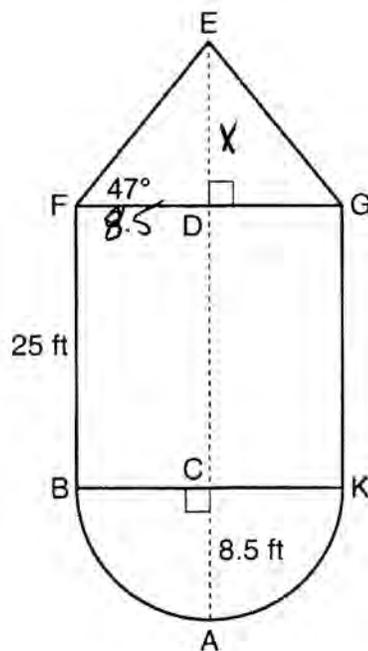
Score 5: The student used 3.14 instead of π to calculate the volume.

Question 35

35 The water tower in the picture below is modeled by the two-dimensional figure beside it. The water tower is composed of a hemisphere, a cylinder, and a cone. Let C be the center of the hemisphere and let D be the center of the base of the cone.



Source: <http://en.wikipedia.org>



$$\tan 47^\circ = \frac{X}{8.5}$$
$$X = 8.5 \tan 47^\circ$$
$$X = 9.11513$$

Question 35 is continued on the next page.

Question 35 continued

If $AC = 8.5$ feet, $BF = 25$ feet, and $m\angle EFD = 47^\circ$, determine and state, to the nearest cubic foot, the volume of the water tower.

Cone

$$V = \frac{1}{3}\pi r^2 h$$

$$V = \frac{1}{3}\pi (8.5)^2 (9.11513)$$

$$V = 689.65125$$

Cylinder

$$V = \pi r^2 h$$

$$V = \pi (8.5)^2 (25)$$

$$V = 5674.50173$$

Hemisphere

$$V = \frac{2}{3} \left(\frac{4}{3} \pi r^3 \right)$$

$$V = \frac{8}{6} \pi (8.5)^3$$

$$V = 1286.22039$$

$$V = 689.65125 + 1286.22039 + 5674.50173$$

$$V = 7650 \text{ ft}^3$$

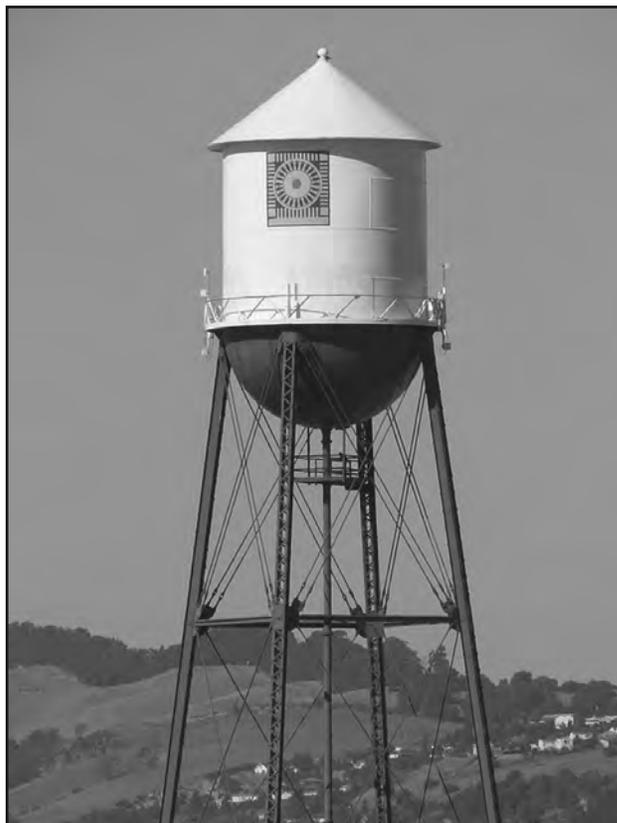
The water tower was constructed to hold a maximum of 400,000 pounds of water. If water weighs 62.4 pounds per cubic foot, can the water tower be filled to 85% of its volume and *not* exceed the weight limit? Justify your answer.

No

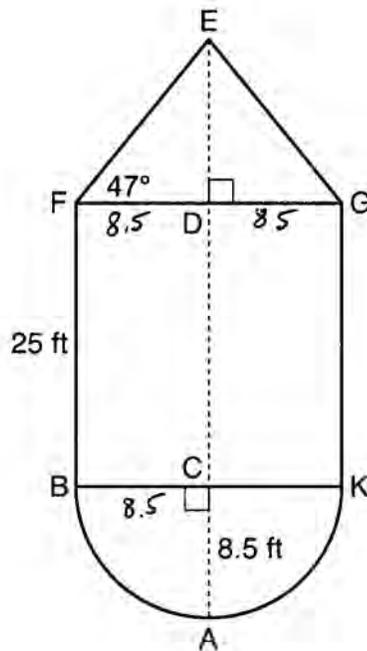
Score 4: The student found the correct volume, but did not justify the answer 'No.'

Question 35

35 The water tower in the picture below is modeled by the two-dimensional figure beside it. The water tower is composed of a hemisphere, a cylinder, and a cone. Let C be the center of the hemisphere and let D be the center of the base of the cone.



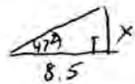
Source: <http://en.wikipedia.org>



Question 35 is continued on the next page.

Question 35 continued

If $AC = 8.5$ feet, $BF = 25$ feet, and $m\angle EFD = 47^\circ$, determine and state, to the nearest cubic foot, the volume of the water tower.



$$\tan 47 = \frac{x}{8.5}$$

$$x = 8.5 \tan 47$$

$$x = 9.1151$$

$$\boxed{x = 9.1}$$

Volume Hemisphere

$$V = \frac{1}{2} \left(\frac{4}{3} \pi r^3 \right)$$

$$= \frac{2}{3} \pi (8.5)^3$$

$$V = 1286.2209$$

Volume Cylinder

$$V = \pi r^2 h$$

$$V = \pi (8.5^2)(25)$$

$$V = 5674.5017$$

Volume Cone

$$V = \frac{1}{3} \pi r^2 h$$

$$V = \frac{1}{3} \pi (8.5)^2 (9.1)$$

$$V = 688.5062$$

$$V = 1286.2704 + 5674.5017 + 688.5062$$

$$= 7649.2183$$

$$V = \boxed{7649 \text{ ft}^3}$$

The water tower was constructed to hold a maximum of 400,000 pounds of water. If water weighs 62.4 pounds per cubic foot, can the water tower be filled to 85% of its volume and *not* exceed the weight limit? Justify your answer.

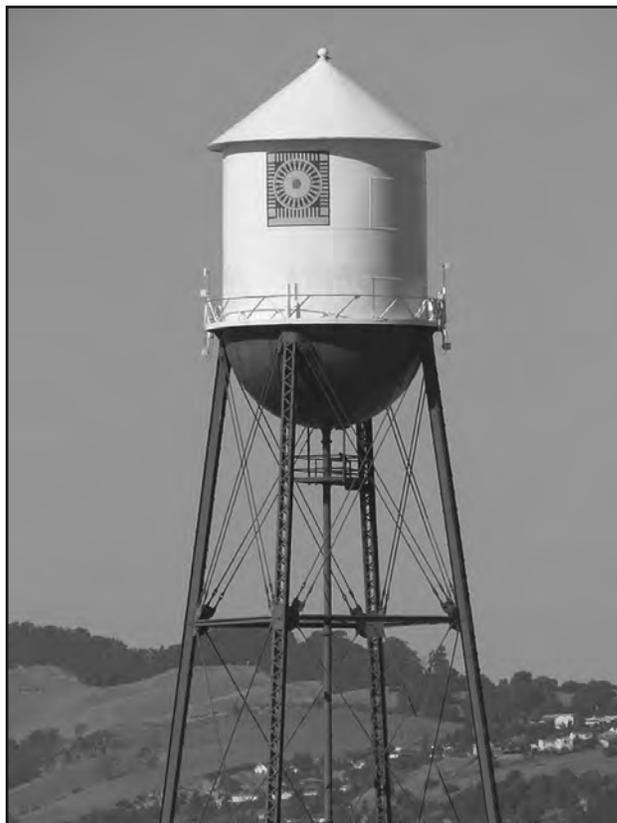
$$7649 \times 62.4 = 477,297.6 \text{ lbs.}$$

$$477,297.6 \times .85 = \boxed{405,702.96} \text{ lbs.}$$

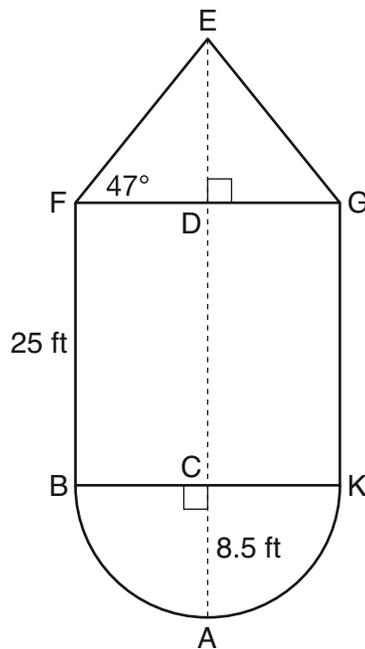
Score 4: The student rounded early with $x = 9.1$, and did not state if the water tower can be filled to 85%.

Question 35

35 The water tower in the picture below is modeled by the two-dimensional figure beside it. The water tower is composed of a hemisphere, a cylinder, and a cone. Let C be the center of the hemisphere and let D be the center of the base of the cone.



Source: <http://en.wikipedia.org>



Question 35 is continued on the next page.

Question 35 continued

If $AC = 8.5$ feet, $BF = 25$ feet, and $m\angle EFD = 47^\circ$, determine and state, to the nearest cubic foot, the volume of the water tower.

Cone

$\tan 47^\circ = \frac{h}{8.5} = 9.12$

$\frac{1}{3} \pi (8.5)^2 (9.12) = 219.64\pi$

cylinder

$\pi (8.5)^2 (25) = 1806.25\pi$

$\frac{1}{2} \text{ Circle}$

$\frac{1}{2} [\pi (8.5)^2] = 36.125\pi$

$219.64\pi + 1806.25\pi + 36.125\pi = 2062.015\pi$

6478.01

6478

The water tower was constructed to hold a maximum of 400,000 pounds of water. If water weighs 62.4 pounds per cubic foot, can the water tower be filled to 85% of its volume and *not* exceed the weight limit? Justify your answer.

$(.85)(6478) = 5506.3$

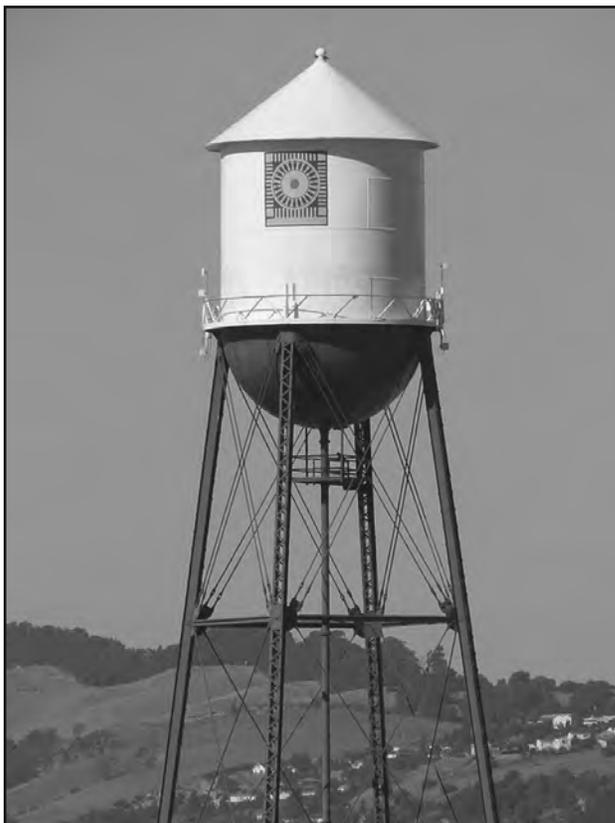
$(5506.3)(62.4) = 343,593.12$

yes because less than 400,000

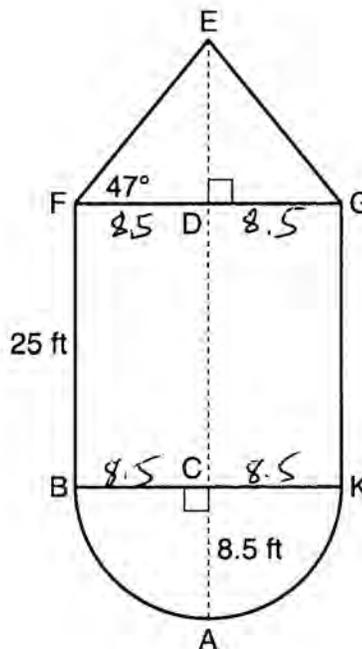
Score 3: The student made one conceptual error by finding the area of half of a circle instead of the volume of a hemisphere. The height of the cone was rounded incorrectly. The student used the answer from the first part to answer the second part appropriately.

Question 35

35 The water tower in the picture below is modeled by the two-dimensional figure beside it. The water tower is composed of a hemisphere, a cylinder, and a cone. Let C be the center of the hemisphere and let D be the center of the base of the cone.



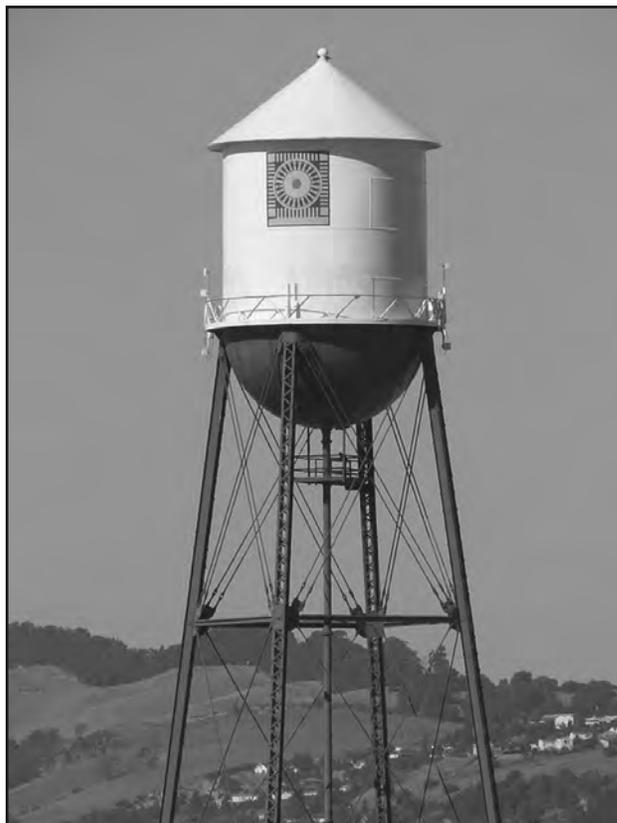
Source: <http://en.wikipedia.org>



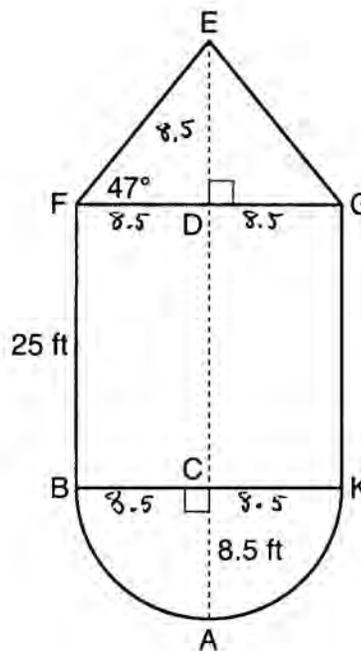
Question 35 is continued on the next page.

Question 35

35 The water tower in the picture below is modeled by the two-dimensional figure beside it. The water tower is composed of a hemisphere, a cylinder, and a cone. Let C be the center of the hemisphere and let D be the center of the base of the cone.



Source: <http://en.wikipedia.org>



Question 35 is continued on the next page.

Question 35 continued

If $AC = 8.5$ feet, $BF = 25$ feet, and $m\angle EFD = 47^\circ$, determine and state, to the nearest cubic foot, the volume of the water tower.

$$V = \frac{1}{3} \pi r^2 h + \pi r^2 h + \frac{4}{3} \pi r^3$$

$$V = \frac{1}{3} \pi (8.5)^2 (8.5) + \pi (8.5)^2 (25) + \frac{4}{3} \pi (8.5)^3$$

$$V = \frac{1}{3} \pi (614.125) + \pi (1806.25) + \frac{4}{3} \pi (72.25)$$

$$V = 643.1101961 + 5674.501731 + 302.6\overset{4}{0}0923$$

$$V = 6620.252019$$

6620

The water tower was constructed to hold a maximum of 400,000 pounds of water. If water weighs 62.4 pounds per cubic foot, can the water tower be filled to 85% of its volume and *not* exceed the weight limit? Justify your answer.

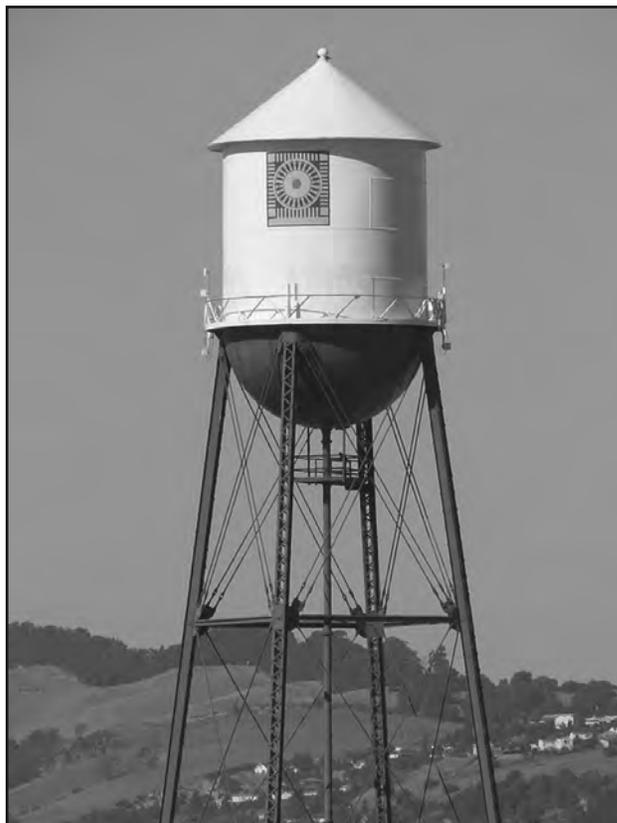
$$6620 (.85) = 5627$$

$$5627 (62.4) = 351,124.8$$

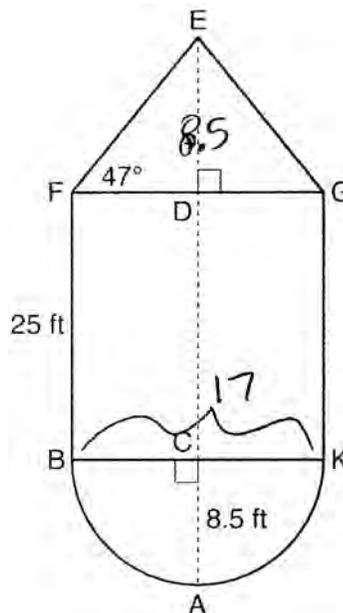
Score 2: The student made one conceptual error by using 8.5 for the height of the cone, and made an error by not dividing the volume of the sphere by 2. The student did not state if the water tower can be filled to 85%.

Question 35

35 The water tower in the picture below is modeled by the two-dimensional figure beside it. The water tower is composed of a hemisphere, a cylinder, and a cone. Let C be the center of the hemisphere and let D be the center of the base of the cone.



Source: <http://en.wikipedia.org>



Question 35 is continued on the next page.

Question 35 continued

If $AC = 8.5$ feet, $BF = 25$ feet, and $m\angle EFD = 47^\circ$, determine and state, to the nearest cubic foot, the volume of the water tower.

<u>Cone</u>	Cylinder
$V = \frac{1}{3}\pi r^2 h$	$\pi r^2 h$
$V = \frac{1}{3}\pi (8.5)^2 (8.5)$	$\pi (8.5)^2 (33.5)$
$V = 643.1$	7603.8

$V = 8247$

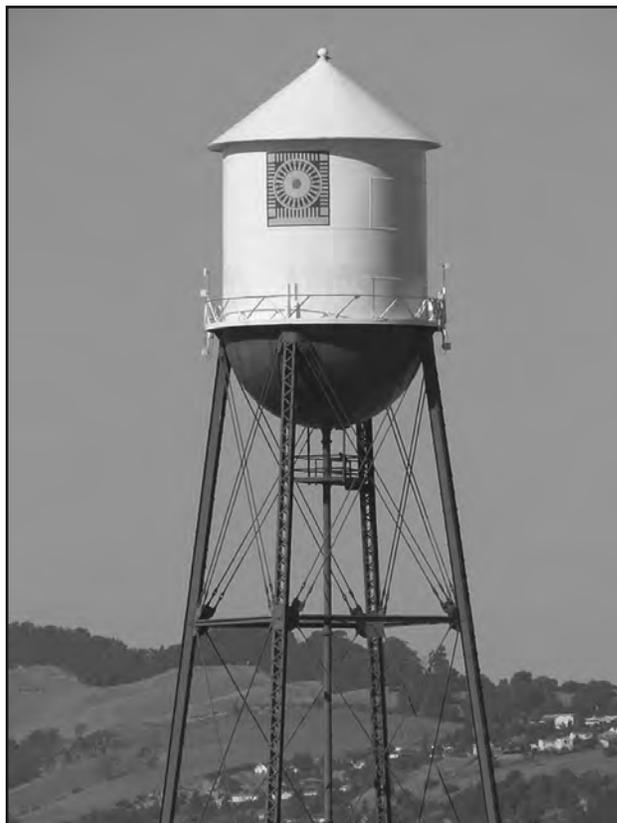
The water tower was constructed to hold a maximum of 400,000 pounds of water. If water weighs 62.4 pounds per cubic foot, can the water tower be filled to 85% of its volume and *not* exceed the weight limit? Justify your answer.

$8247 \times 62.4 = 514,612.8 \text{ lbs}$
NO

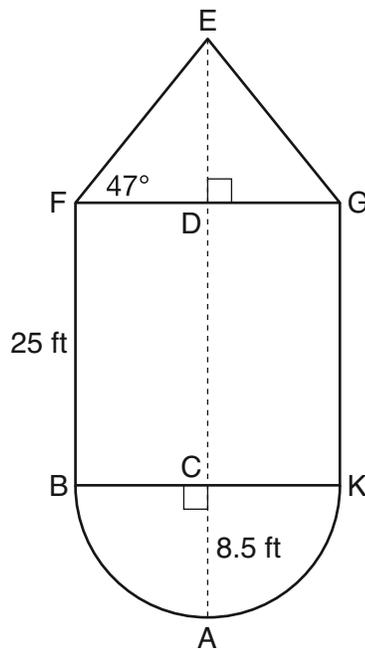
Score 1: The student made two conceptual errors in finding the volume of the water tower and one computational error by not multiplying by 85%.

Question 35

35 The water tower in the picture below is modeled by the two-dimensional figure beside it. The water tower is composed of a hemisphere, a cylinder, and a cone. Let C be the center of the hemisphere and let D be the center of the base of the cone.



Source: <http://en.wikipedia.org>



Question 35 is continued on the next page.

Question 35 continued

If $AC = 8.5$ feet, $BF = 25$ feet, and $m\angle EFD = 47^\circ$, determine and state, to the nearest cubic foot, the volume of the water tower.

$$\begin{array}{r} 17\pi \\ \hline | 8.5 \\ | 25 \\ | 8.5 \end{array} \left. \vphantom{\begin{array}{r} 17\pi \\ \hline | 8.5 \\ | 25 \\ | 8.5 \end{array}} \right\} (42)(17)^2\pi$$
$$12138\pi = 38,32.65$$

The water tower was constructed to hold a maximum of 400,000 pounds of water. If water weighs 62.4 pounds per cubic foot, can the water tower be filled to 85% of its volume and *not* exceed the weight limit? Justify your answer.

$$(400,000)(.85) = 340,000$$

Score 0: The student had a completely incorrect response.

Question 36

36 In the coordinate plane, the vertices of $\triangle RST$ are $R(6, -1)$, $S(1, -4)$, and $T(-5, 6)$.
Prove that $\triangle RST$ is a right triangle.

[The use of the set of axes on the next page is optional.]

$$m_{\overline{RS}} = \frac{3}{5}$$

$$m_{\overline{ST}} = \frac{-10}{6} = -\frac{5}{3}$$

Therefore the slopes of \overline{RS} and \overline{ST} are negative reciprocals and so $\overline{RS} \perp \overline{ST}$. Since the segments are \perp , $\triangle RST$ is a rt \triangle .

$\therefore \triangle RST$ is a rt \triangle because it has 1 rt \angle .

State the coordinates of point P such that quadrilateral $RSTP$ is a rectangle.

$(0, 9)$

Question 36 is continued on the next page.

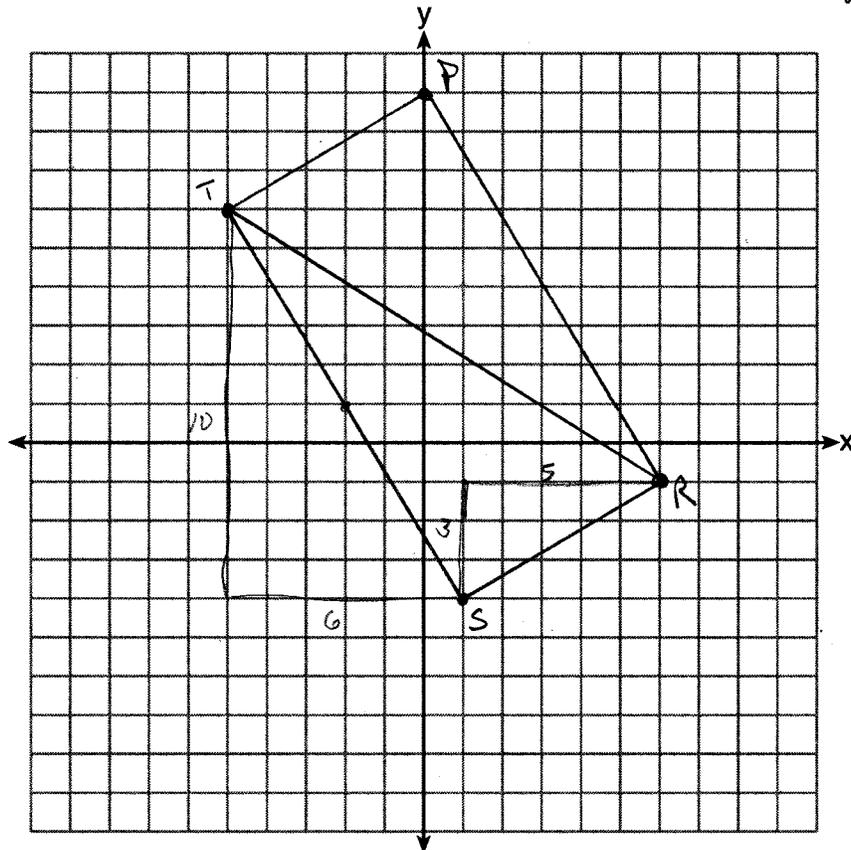
Question 36 continued

Prove that your quadrilateral $RSTP$ is a rectangle.
 [The use of the set of axes below is optional.]

$$\left. \begin{array}{l} m_{\overline{RS}} = \frac{3}{5} \\ m_{\overline{PT}} = \frac{3}{5} \end{array} \right\} \therefore \overline{RS} \parallel \overline{PT}$$

$$\left. \begin{array}{l} m_{\overline{ST}} = \frac{-10}{6} = -\frac{5}{3} \\ m_{\overline{RP}} = \frac{-10}{6} = -\frac{5}{3} \end{array} \right\} \therefore \overline{ST} \parallel \overline{RP}$$

Since $RSTP$ is a quadrilateral with both pairs of opposite sides \parallel and one \perp at S , it must be a rectangle.



Score 6: The student has a complete and correct response.

Question 36

36 In the coordinate plane, the vertices of $\triangle RST$ are $R(6, -1)$, $S(1, -4)$, and $T(-5, 6)$.

Prove that $\triangle RST$ is a right triangle.

[The use of the set of axes on the next page is optional.]

$$RS = \sqrt{3^2 + 5^2} = \sqrt{34}$$

$$ST = \sqrt{6^2 + 10^2} = \sqrt{136}$$

$$RT = \sqrt{7^2 + 11^2} = \sqrt{170}$$

$$RS^2 + ST^2 = RT^2$$

$$\sqrt{34}^2 + \sqrt{136}^2 = \sqrt{170}^2$$

$$34 + 136 = 170 \checkmark$$

$\triangle RST$ is a rt \triangle

b/c its side lengths
satisfy the pyth.

theorem

State the coordinates of point P such that quadrilateral $RSTP$ is a rectangle.

$$P(0, 9)$$

Question 36 is continued on the next page.

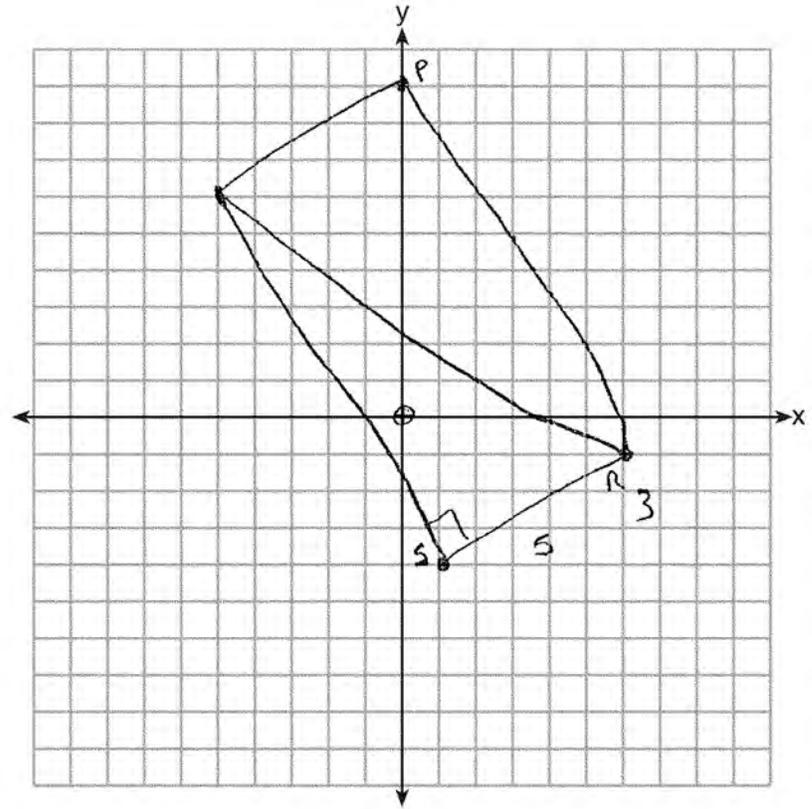
Question 36 continued

Prove that your quadrilateral $RSTP$ is a rectangle.
 [The use of the set of axes below is optional.]

$$\left. \begin{array}{l} m \overline{PT} = \frac{3}{5} \\ m \overline{RS} = \frac{3}{5} \end{array} \right\} \parallel$$

$$\left. \begin{array}{l} m \overline{ST} = -\frac{10}{6} \\ m \overline{RP} = -\frac{10}{6} \end{array} \right\} \parallel$$

$RSTP$ is a \square
 b/c both sets
 of opposite sides
 are \parallel .
 A \square w/ 1 $\text{Rt} \angle$,
 $\angle S$, is a rectangle
 $\therefore RSTP$ is a
 rectangle



Score 6: The student has a complete and correct response.

Question 36

36 In the coordinate plane, the vertices of $\triangle RST$ are $R(6, -1)$, $S(1, -4)$, and $T(-5, 6)$.
Prove that $\triangle RST$ is a right triangle.

[The use of the set of axes on the next page is optional.]

Slopes

$$\overline{TS} = -\frac{10}{6} = -\frac{5}{3}$$

$$\overline{SR} = \frac{3}{5}$$

$\overline{TS} \perp \overline{SR}$ because their slopes are negative reciprocals of each other. $\triangle RST$ is a right \triangle because \perp lines form rt. \angle s.
 $\triangle RST$ is a right \triangle because it has 1 right \angle .

State the coordinates of point P such that quadrilateral $RSTP$ is a rectangle.

$$P(0, 9)$$

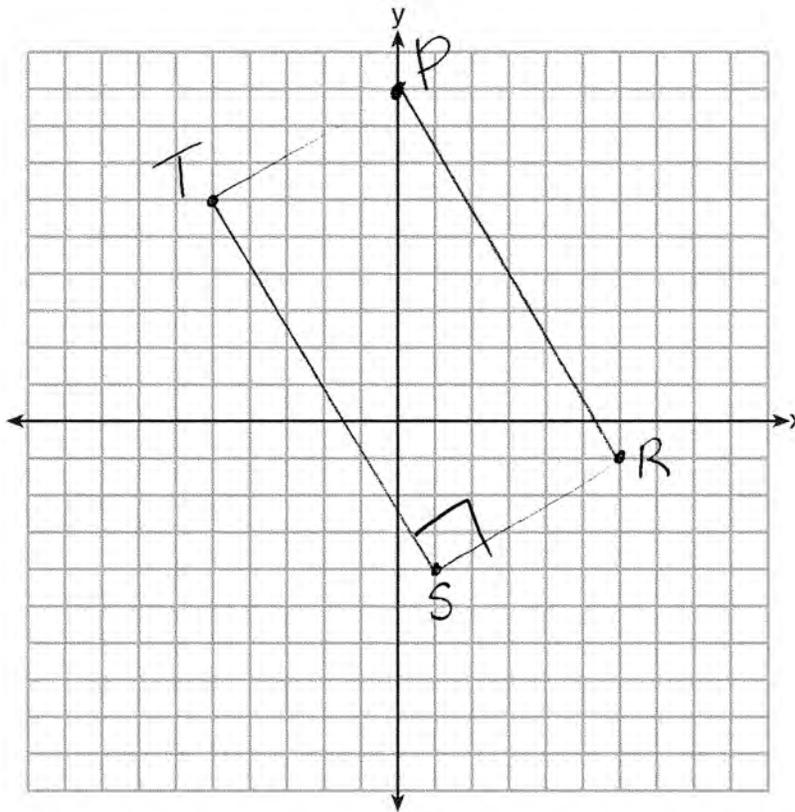
Question 36 is continued on the next page.

Question 36 continued

Prove that your quadrilateral $RSTP$ is a rectangle.
[The use of the set of axes below is optional.]

$m_{TP} = 3/5$
 $m_{SR} = 3/5$
 $m_{TS} = -5/3$
 $m_{PR} = -5/3$

Opposite sides are parallel
because they have the same
slope. $RSTP$ is a parallelogram
because opposite sides are
parallel.



Score 5: The student proved $RSTP$ is a parallelogram, but did not have a concluding statement proving $RSTP$ is a rectangle.

Question 36

36 In the coordinate plane, the vertices of $\triangle RST$ are $R(6, -1)$, $S(1, -4)$, and $T(-5, 6)$.

Prove that $\triangle RST$ is a right triangle.

[The use of the set of axes on the next page is optional.]

$$\text{slope } \overline{RS} = \frac{3}{5} \quad \text{slope } \overline{TS} = \frac{-10}{6} = -\frac{5}{3}$$

$\overline{RS} \perp \overline{TS}$ since they have negative reciprocal slopes.

Therefore $\angle S$ is a right \angle .

Since $\triangle RST$ contains a right \angle , it is a right \triangle .

State the coordinates of point P such that quadrilateral $RSTP$ is a rectangle.

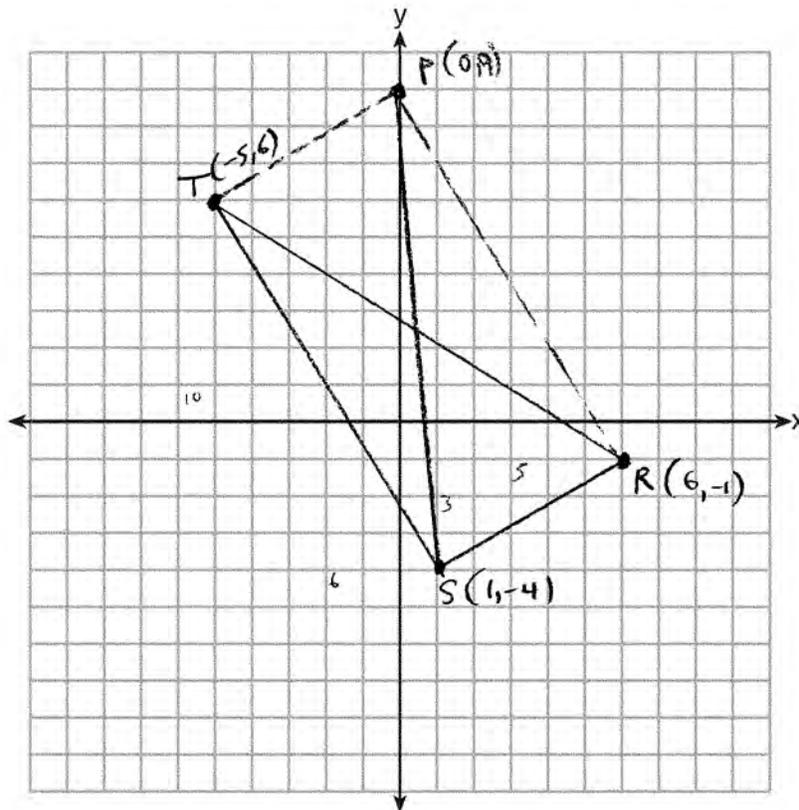
$$P(0, 9)$$

Question 36 is continued on the next page.

Question 36 continued

Prove that your quadrilateral $RSTP$ is a rectangle.
 [The use of the set of axes below is optional.]

Length \overline{RT} $\sqrt{7^2 + 11^2}$ $\sqrt{49 + 121}$ $RT = \sqrt{170}$	Length \overline{PS} $\sqrt{13^2 + 1^2}$ $\sqrt{169 + 1}$ $PS = \sqrt{170}$	Since the diagonals of $RSTP$ are \cong , then it is a rectangle.
---	--	--



Score 4: The student made one conceptual error when proving the rectangle, because no work is shown to prove that $RSTP$ is a parallelogram.

Question 36

- 36** In the coordinate plane, the vertices of $\triangle RST$ are $R(6, -1)$, $S(1, -4)$, and $T(-5, 6)$.
Prove that $\triangle RST$ is a right triangle.
[The use of the set of axes on the next page is optional.]

State the coordinates of point P such that quadrilateral $RSTP$ is a rectangle.

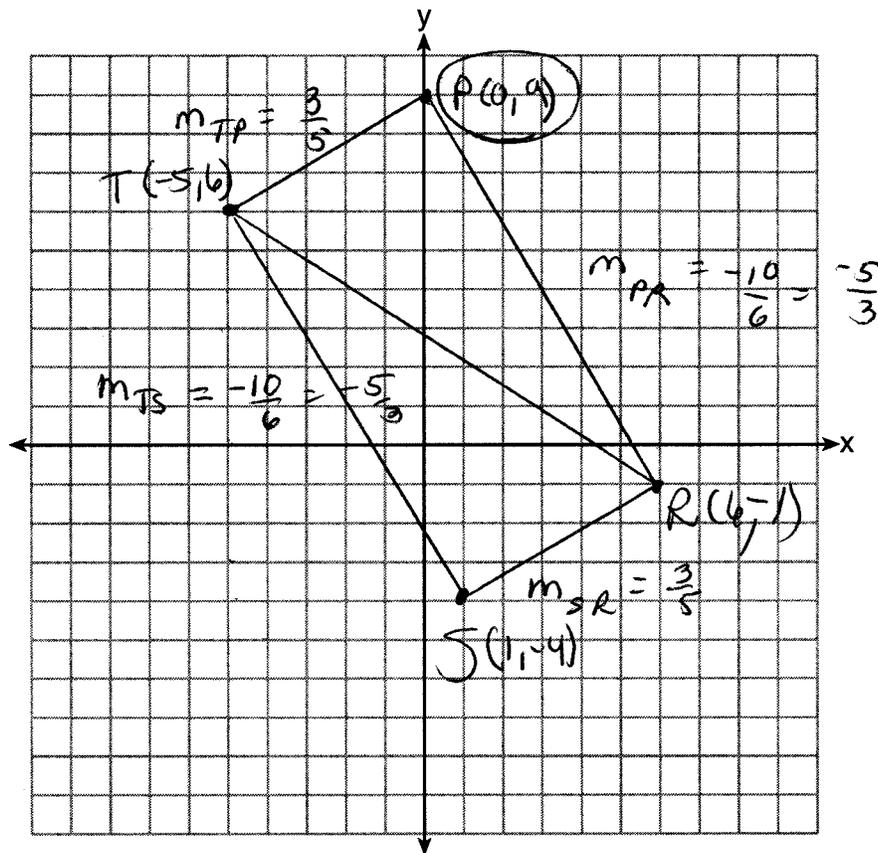
Question 36 is continued on the next page.

Question 36 continued

Prove that your quadrilateral $RSTP$ is a rectangle.

[The use of the set of axes below is optional.]

$m_{TP} = \frac{3}{5}$ $\overline{TP} \perp \overline{RP}$ b/c their slopes are neg. reciprocals
 $m_{RP} = -\frac{5}{3}$ $\angle P$ is a rt \angle b/c \perp lines form right \angle s
 $m_{SR} = \frac{3}{5}$ $\overline{TP} \parallel \overline{SR}$ b/c their slopes are equal
 $m_{TS} = -\frac{5}{3}$ $\overline{RP} \parallel \overline{TS}$
 $RSTP$ is a \square b/c it \perp a pair of \parallel sides
 $RSTP$ is a ~~square~~ ^{rectangle} b/c a \square with a right \angle
 is a rectangle



Score 4: The student did not prove $\triangle RST$ is a right triangle. The student found point P and stated its coordinates correctly. The student's proof for rectangle $RSTP$ is correct.

Question 36

- 36 In the coordinate plane, the vertices of $\triangle RST$ are $R(6, -1)$, $S(1, -4)$, and $T(-5, 6)$.
Prove that $\triangle RST$ is a right triangle.
[The use of the set of axes on the next page is optional.]

sides are \perp
because their
slopes are
negative
reciprocals

$$\begin{cases} m(TS) = -\frac{10}{6} = -\frac{5}{3} \\ m(SR) = \frac{3}{5} \end{cases}$$

\perp lines form right angles

$\triangle RST$ is a right \triangle
because it has a
right \angle .

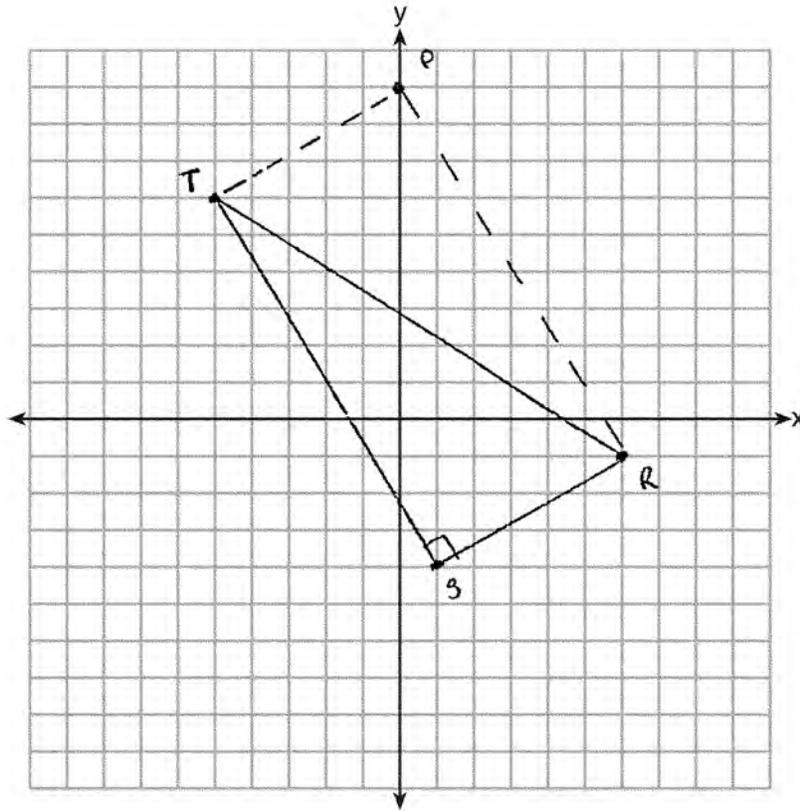
State the coordinates of point P such that quadrilateral $RSTP$ is a rectangle.

$P(0, 4)$

Question 36 is continued on the next page.

Question 36 continued

Prove that your quadrilateral $RSTP$ is a rectangle.
[The use of the set of axes below is optional.]



Score 3: The student correctly proved the right triangle and stated the coordinates of P , but no further correct work was shown.

Question 36

36 In the coordinate plane, the vertices of $\triangle RST$ are $R(6, -1)$, $S(1, -4)$, and $T(-5, 6)$.

Prove that $\triangle RST$ is a right triangle.

[The use of the set of axes on the next page is optional.]

$$R(6, -1)$$

$$S(1, -4)$$

$$T(-5, 6)$$

$$d_{RS} = \sqrt{(6-1)^2 + (-1+4)^2} = \sqrt{25+9} = \sqrt{34}$$

$$d_{ST} = \sqrt{(1+5)^2 + (-4-6)^2} = \sqrt{36+100} = \sqrt{136}$$

$$d_{RT} = \sqrt{(6+5)^2 + (-1-6)^2} = \sqrt{121+49} = \sqrt{170}$$

$$(RS)^2 + (ST)^2 \stackrel{?}{=} (RT)^2$$

$$\frac{(\sqrt{34})^2 + (\sqrt{136})^2}{34 + 136} \quad \left| \quad (\sqrt{170})^2 \right.$$

$$34 + 136$$

$$170 = 170$$

State the coordinates of point P such that quadrilateral $RSTP$ is a rectangle.

$$(0, 9)$$

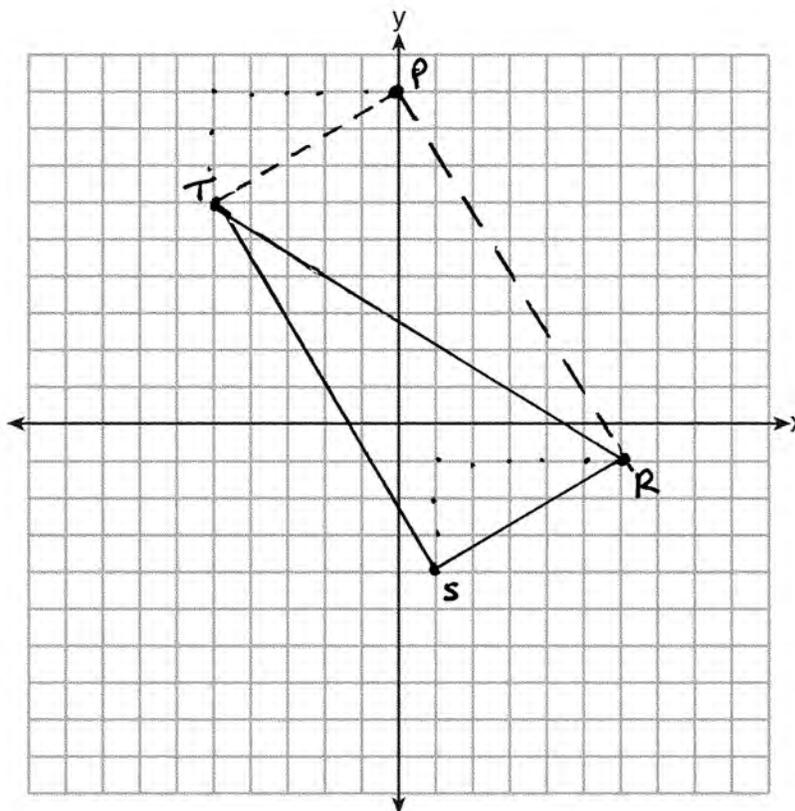
Question 36 is continued on the next page.

Question 36 continued

Prove that your quadrilateral $RSTP$ is a rectangle.
[The use of the set of axes below is optional.]

$\angle P$ is $Rt \angle$

$RSTP$ is Rectangle because opposite
 \angle 's are Right \angle 's



Score 2: The student was missing a concluding statement when proving the right triangle, and the coordinates of P were correctly stated, but no further correct work is shown.

Question 36

36 In the coordinate plane, the vertices of $\triangle RST$ are $R(6, -1)$, $S(1, -4)$, and $T(-5, 6)$.

Prove that $\triangle RST$ is a right triangle.

[The use of the set of axes on the next page is optional.]

$\triangle RST$ is a right triangle.

Slopes are negative
reciprocals

$$m_{SR} = 5/3$$

$$m_{ST} = -4/10 = -3/5$$

State the coordinates of point P such that quadrilateral $RSTP$ is a rectangle.

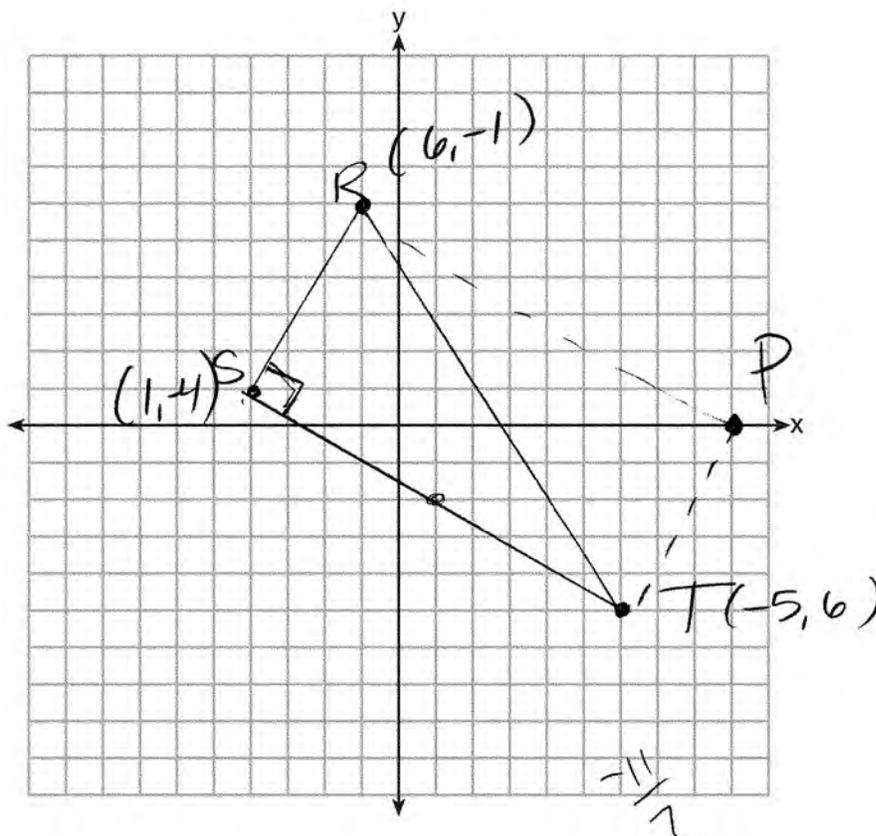
$$P(0, 9)$$

Question 36 is continued on the next page.

Question 36 continued

Prove that your quadrilateral $RSTP$ is a rectangle.
[The use of the set of axes below is optional.]

RSTP is a rectangle.



Score 2: The student had an incomplete triangle proof. When graphing the triangle, the student mixed up the x - and y -coordinates, which is one graphing error. The student stated appropriate coordinates for P based on this error. No further correct work was shown.

Question 36

- 36** In the coordinate plane, the vertices of $\triangle RST$ are $R(6, -1)$, $S(1, -4)$, and $T(-5, 6)$.
Prove that $\triangle RST$ is a right triangle.
[The use of the set of axes on the next page is optional.]

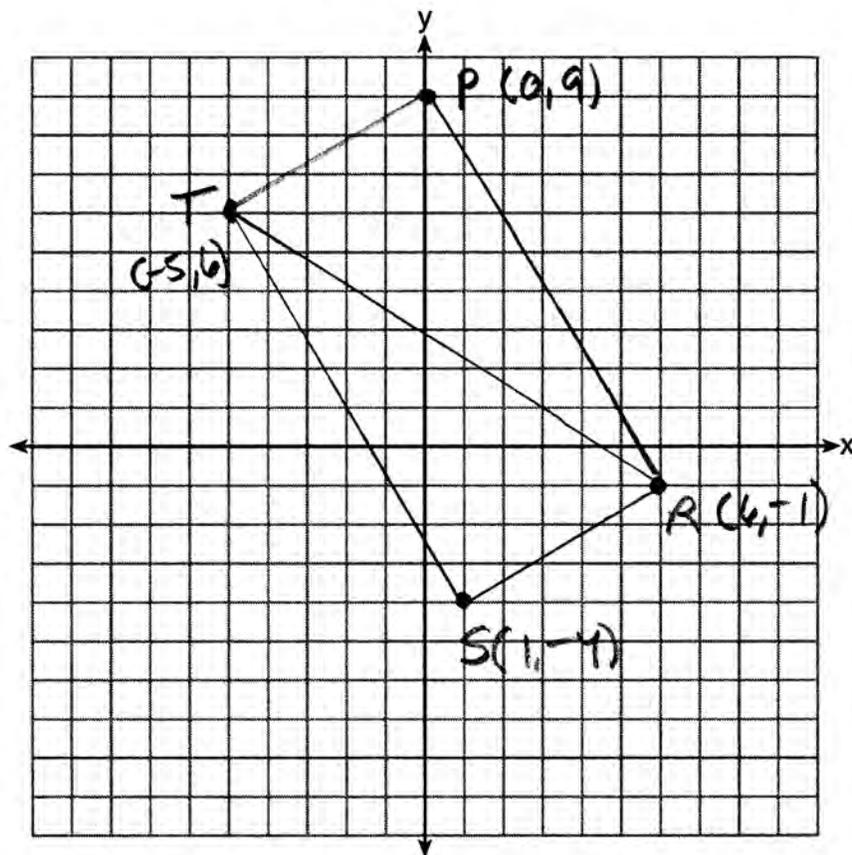
State the coordinates of point P such that quadrilateral $RSTP$ is a rectangle.

$(0, 9)$

Question 36 is continued on the next page.

Question 36 continued

Prove that your quadrilateral $RSTP$ is a rectangle.
[The use of the set of axes below is optional.]



Score 1: The student graphed point P correctly and stated its coordinates. No further work was shown.

Question 36

- 36** In the coordinate plane, the vertices of $\triangle RST$ are $R(6, -1)$, $S(1, -4)$, and $T(-5, 6)$.
Prove that $\triangle RST$ is a right triangle.
[The use of the set of axes on the next page is optional.]

$\triangle RST$ is a right \triangle because $\angle S$ is a right angle.

State the coordinates of point P such that quadrilateral $RSTP$ is a rectangle.

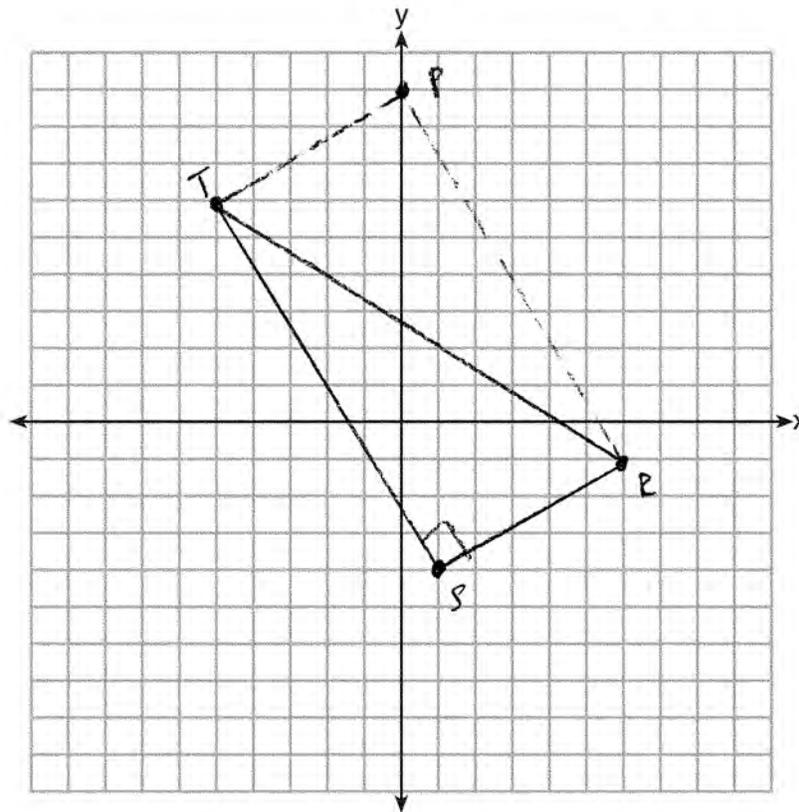
0, 9

Question 36 is continued on the next page.

Question 36 continued

Prove that your quadrilateral $RSTP$ is a rectangle.
[The use of the set of axes below is optional.]

RSTP is a rectangle because it has a right \angle .



Score 0: The student had no work to justify the statements, and the parentheses are missing on the coordinates of P .

Regents Examination in Geometry (Common Core) – June 2015

Chart for Converting Total Test Raw Scores to Final Exam Scores (Scale Scores)

(Use for the June 2015 exam only.)

Raw Score	Scale Score	Performance Level	Raw Score	Scale Score	Performance Level	Raw Score	Scale Score	Performance Level
86	100	5	57	79	3	28	60	2
85	99	5	56	79	3	27	59	2
84	98	5	55	78	3	26	58	2
83	97	5	54	78	3	25	56	2
82	96	5	53	77	3	24	55	2
81	95	5	52	77	3	23	54	1
80	94	5	51	77	3	22	52	1
79	93	5	50	76	3	21	51	1
78	92	5	49	76	3	20	49	1
77	91	5	48	75	3	19	47	1
76	90	5	47	75	3	18	46	1
75	90	5	46	74	3	17	44	1
74	89	5	45	73	3	16	42	1
73	88	5	44	73	3	15	40	1
72	88	5	43	72	3	14	38	1
71	87	5	42	72	3	13	36	1
70	86	5	41	71	3	12	34	1
69	86	5	40	70	3	11	32	1
68	85	5	39	70	3	10	29	1
67	84	4	38	69	3	9	27	1
66	84	4	37	68	3	8	24	1
65	83	4	36	68	3	7	22	1
64	83	4	35	67	3	6	19	1
63	82	4	34	66	3	5	16	1
62	82	4	33	65	3	4	13	1
61	81	4	32	64	2	3	10	1
60	81	4	31	63	2	2	7	1
59	80	4	30	62	2	1	3	1
58	80	4	29	61	2	0	0	1

To determine the student’s final examination score (scale score), find the student’s total test raw score in the column labeled “Raw Score” and then locate the scale score that corresponds to that raw score. The scale score is the student’s final examination score. Enter this score in the space labeled “Scale Score” on the student’s answer sheet.

Schools are not permitted to rescore any of the open-ended questions on this exam after each question has been rated once, regardless of the final exam score. Schools are required to ensure that the raw scores have been added correctly and that the resulting scale score has been determined accurately.

Because scale scores corresponding to raw scores in the conversion chart change from one administration to another, it is crucial that for each administration the conversion chart provided for that administration be used to determine the student’s final score. The chart above is usable only for this administration of the Regents Examination in Geometry (Common Core).



New York State Testing Program

Regents Examination in Geometry (Common Core)

Selected Questions with Annotations

June 2015

engage^{ny}

Our Students. Their Moment.

DRAFT



**New York State Testing Program
Regents Examination in Geometry (Common Core)
Selected Questions with Annotations**

With the adoption of the New York P-12 Common Core Learning Standards (CCLS) in ELA/Literacy and Mathematics, the Board of Regents signaled a shift in both instruction and assessment. In Spring 2014, New York State administered the first set of Regents Exams designed to assess student performance in accordance with the instructional shifts and the rigor demanded by the Common Core State Standards (CCSS). To aid in the transition to new tests, New York State released a number of resources including sample questions, test blueprints and specifications, and criteria for writing test questions. These resources can be found at <http://www.engageny.org/resource/regents-exams>.

New York State administered the first Geometry (Common Core) Regents Exam in June 2015 and is now annotating a portion of the questions from this tests available for review and use. These annotated questions will help students, families, educators, and the public better understand how the test has changed to assess the instructional shifts demanded by the Common Core and to assess the rigor required to ensure that all students are on track to college and career readiness.

Annotated Questions Are Teaching Tools

The annotated questions are intended to help students, families, educators, and the public understand how the Common Core is different. The annotated questions will demonstrate the way the Common Core should drive instruction and how tests have changed to better assess student performance in accordance with the instructional shifts demanded by the Common Core. They are also intended to help educators identify how the rigor of the Regents Examinations can inform classroom instruction and local assessment. The annotations will indicate common student misunderstandings related to content clusters; educators should use these to help inform unit and lesson planning. In some cases, the annotations may offer insight into particular instructional elements (conceptual thinking, mathematical modeling) that align to the Common Core that may be used in curricular design. It should not be assumed, however, that a particular cluster will be measured with identical items in future assessments.

The annotated questions include both multiple-choice and constructed-response questions. With each multiple-choice question annotated, a commentary will be available to demonstrate why the question measures the intended cluster. The rationales describe why the wrong answer choices are plausible but incorrect and are based in common misconceptions or common procedural errors and why the correct answer is correct. While these rationales speak to a possible and likely reason for the selection of the incorrect option by the student, these rationales do not contain definitive statements as to why the student chose the incorrect option or what we can infer about knowledge and skills of the student based on the students selection of an incorrect response. These multiple-choice questions are designed to assess student proficiency, not to diagnose specific misconceptions/errors with each and every incorrect option.

For each constructed-response question, there will be a commentary describing how the question measures the intended cluster, and sample student responses representing possible student errors or misconceptions at each possible score point.

The annotated questions do not represent the full spectrum of standards assessed on the State test, nor do they represent the full spectrum of how the Common Core should be taught and assessed in the classroom. Specific criteria for writing test questions as well as test information are available at <http://www.engageny.org/resource/regents-exams>.

Understanding Math Annotated Questions

All questions on the Regents Exam in Geometry (Common Core) are designed to measure the Common Core Learning Standards identified by the PARCC Model Content Framework for Geometry. More information about the relationship between the New York State Testing Program and PARCC can be found here: <http://www.p12.nysed.gov/assessment/math/ccmath/parccmcf.pdf>.

Multiple Choice

Multiple-choice questions will primarily be used to assess procedural fluency and conceptual understanding. Multiple-choice questions measure the Standards for Mathematical Content and may incorporate Standards for Mathematical Practices and real-world applications. Some multiple-choice questions require students to complete multiple steps. Likewise, questions may measure more than one cluster, drawing on the simultaneous application of multiple skills and concepts. Within answer choices, distractors will all be based on plausible missteps.

Constructed Response

Constructed-response questions will require students to show a deep understanding of mathematical procedures, concepts, and applications, as well as demonstrating geometric concepts through constructions. The Regents Examination in Geometry (Common Core) contains 2-, 4-, and 6-credit constructed-response questions.

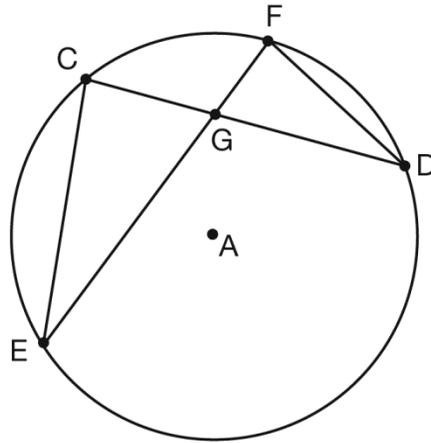
2-credit constructed-response questions require students to complete a task and show their work. Like multiple-choice questions, 2-credit constructed-response questions may involve multiple steps, the application of multiple mathematics skills, and real-world applications. These questions may ask students to explain or justify their solutions and/or show their process of problem solving.

Constructed-response questions that are worth 4 credits require students to show their work in completing more extensive problems which may involve multiple tasks and concepts. Students will need to reason abstractly by constructing viable arguments to explain, justify, and/or prove geometric relationships in order to demonstrate conceptual understanding. Students will also need to reason quantitatively when solving real-world modeling problems.

There are two 6-credit constructed-response questions on the Regents Examination in Geometry (Common Core). One 6-credit question requires students to develop multi-step, extended logical arguments and proofs involving major content, and one 6-credit question requires students to use modeling to solve real-world problems.

#8

8 In the diagram of circle A shown below, chords \overline{CD} and \overline{EF} intersect at G , and chords \overline{CE} and \overline{FD} are drawn.



Which statement is *not* always true?

(1) $\overline{CG} \cong \overline{FG}$

(3) $\frac{CE}{EG} = \frac{FD}{DG}$

(2) $\angle CEG \cong \angle FDG$

(4) $\triangle CEG \sim \triangle FDG$

Measured CCLS Cluster: G-SRT.B

Key: (1)

Commentary: This question measures the knowledge and skills described by the standards within G-SRT.B because it requires the student to apply similarity criteria to reason about geometric relationships. The student must conclude that the two triangles are similar because they have two pairs of congruent angles and therefore the corresponding sides of the similar triangles are proportional. The question is also an example of the instructional shift of coherence, as the student must draw on understandings from another domain, Circles (G-C), which includes angles inscribed in circles.

Rationale: Choices (2), (3), and (4) are plausible but incorrect. They represent common student errors made when recognizing geometric relationships between triangles, angles, and segments in a circle and using triangle similarity to reason about those relationships. Choosing the correct solution requires students to know how to analyze a geometric diagram and apply the AA criterion for triangle similarity. Compare with questions 11, 15, and 31, which also assess G-SRT.B.

Answer Choice: (1) $\overline{CG} \cong \overline{FG}$. This response is correct because it is a statement that is not always true; line segment CG is not always congruent to line segment FG . These segments are only congruent when

$\triangle CEG \cong \triangle FDG$. A student that selects this response understands that the diagram implies that $\triangle CEG$ and $\triangle FDG$ are similar, but not necessarily congruent.

Answer Choice: (2) $\angle CEG \cong \angle FDG$. This response is incorrect because it is always true that $\angle CEG \cong \angle FDG$ because $\angle CEG$ and $\angle FDG$ are inscribed angles that intercept the same arc, CF . The student may not have recognized the angles as angles inscribed in the circle.

Answer Choice: (3) $\frac{CE}{EG} = \frac{FD}{DG}$. This response is incorrect because it is always true that $\frac{CE}{EG} = \frac{FD}{DG}$ because $\triangle CEG \sim \triangle FDG$ and corresponding sides of similar triangles are proportional. The student may have confused the concepts of congruence and similarity or made an error in defining the correspondence between $\triangle CEG$ and $\triangle FDG$.

Answer Choice: (4) $\triangle CEG \sim \triangle FDG$. This response is incorrect because it is always true that $\triangle CEG \sim \triangle FDG$; the triangles can be shown to satisfy the AA similarity criterion. The pairs of angles that can be used for the AA similarity criteria are the inscribed angles CEF and FDC because they intercept the same arc CF , inscribed angles ECD and DFE because they intercept the same arc, ED , and the vertical angles CGE and FGD because vertical angles are always congruent. The student may not have understood that the AA similarity criterion would apply to this situation or made an error in defining the correspondence between $\triangle CEG$ and $\triangle FDG$.

Answer Choice: (3) 14.3. This response is correct and is the length of \overline{AC} . This length is determined by recognizing that $\triangle ABE$ is similar to $\triangle ACD$. The student reasons that triangle ABE and triangle ACD are similar using the fact that parallel lines form corresponding congruent angles and/or using the reflexive angle A . The student uses the similar triangles to write an equation for the length of \overline{AC} .

$$\frac{9}{9.2} = \frac{9 + 5}{x}$$

$$9x = 128.8$$

$$x = 14.3111\dots$$

$$x \approx 14.3$$

Answer Choice: (4) 14.4. This response is incorrect and does not represent the length of \overline{AC} . The student may have attempted to find the length of \overline{BC} instead of \overline{AC} by assuming the difference between \overline{AB} and \overline{BC} had to be the same as the difference between \overline{AE} and \overline{AD} . The student then added it to the length of \overline{AB} to find the length of \overline{AC} .

#13

13 Quadrilateral $ABCD$ has diagonals \overline{AC} and \overline{BD} . Which information is *not* sufficient to prove $ABCD$ is a parallelogram?

- (1) \overline{AC} and \overline{BD} bisect each other.
- (2) $\overline{AB} \cong \overline{CD}$ and $\overline{BC} \cong \overline{AD}$
- (3) $\overline{AB} \cong \overline{CD}$ and $\overline{AB} \parallel \overline{CD}$
- (4) $\overline{AB} \cong \overline{CD}$ and $\overline{BC} \parallel \overline{AD}$

Measured CCLS Cluster: G-CO.C

Key: (4)

Commentary: This question measures the knowledge and skills described by the standards within G-CO.C because it requires the student to reason using the theorems involving the diagonals and the sides of a quadrilateral that would prove it a parallelogram.

Rationale: Choices (1), (2), and (3) are plausible but incorrect. They represent common student errors made when working with parallelograms and indicate a limited understanding of how to reason about theorems of parallelograms. Choosing the correct solution requires students to know how to reason using theorems involving parallelograms. Compare with questions 17, 26, 32, and 33, which also assess G-CO.C.

Answer Choice: (1) \overline{AC} and \overline{BD} bisect each other. This response is incorrect because this information *is* sufficient to prove that $ABCD$ is a parallelogram; if the diagonals of a quadrilateral bisect each other, then it is a parallelogram. The student may have assumed that this theorem applied only to a larger subset of quadrilaterals than the parallelograms, such as trapezoids.

Answer Choice: (2) $\overline{AB} \cong \overline{CD}$ and $\overline{BC} \cong \overline{AD}$. This response is incorrect because this information *is* sufficient to prove that $ABCD$ is a parallelogram; if both pairs of opposite sides are congruent, then the quadrilateral is a parallelogram. The student may have concluded that a quadrilateral whose opposite sides are congruent is a rectangle, without also reasoning that a rectangle is a parallelogram.

Answer Choice: (3) $\overline{AB} \cong \overline{CD}$ and $\overline{AB} \parallel \overline{CD}$. This response is incorrect because this information *is* sufficient to prove that $ABCD$ is a parallelogram; if a quadrilateral has one pair of opposite sides that are both congruent and parallel, then it is a parallelogram. The student may have assumed that information about only one pair of sides would not be sufficient to determine a parallelogram.

Answer Choice: (4) $\overline{AB} \cong \overline{CD}$ and $\overline{BC} \parallel \overline{AD}$. This response is correct because this information is *not* sufficient to prove that $ABCD$ is a parallelogram. A quadrilateral that has one pair of opposite sides congruent and the other pair of opposite sides parallel may not be a parallelogram. A student who selects this response understands how to reason about parallelograms using given information.

#14

14 The equation of a circle is $x^2 + y^2 + 6y = 7$. What are the coordinates of the center and the length of the radius of the circle?

- (1) center (0,3) and radius 4
- (2) center (0,−3) and radius 4
- (3) center (0,3) and radius 16
- (4) center (0,−3) and radius 16

Measured CCLS Cluster: G-GPE.A

Key: (2)

Commentary: This question measures the knowledge and skills described by the standards within G-GPE.A because it requires the student to complete the square to rewrite the given equation representing a circle and identify the coordinates of its center and the length of its radius. The question also requires the student to employ Mathematical Practice 7 (Look for and make use of structure) because the student must notice and use the structure of the equation to rewrite it and determine properties of the circle.

Rationale: Choices (1), (3), and (4) are plausible but incorrect. They represent common student errors made when completing the square and identifying the coordinates of the center and length of the radius from its equation. Choosing the correct solution requires students to know how to complete the square and identify the radius and the coordinates of the center.

Answer Choice: (1) center (0,3) and radius 4. This response is incorrect and does not show the center for the circle represented by the equation $x^2 + y^2 + 6y = 7$. The student may have completed the square to rewrite the equation of the circle and identified the radius by taking the square root of 16, but incorrectly interpreted the coordinates of the center as (0,3).

Answer Choice: (2) center (0,−3) and radius 4. This response is correct and shows the center and radius for the circle represented by the equation $x^2 + y^2 + 6y = 7$. A student who selects this response understands how to complete the square and identify the coordinates of the center and length of the radius.

$$x^2 + y^2 + 6y = 7$$

$$x^2 + y^2 + 6y + 9 = 7 + 9$$

$$x^2 + (y + 3)^2 = 16$$

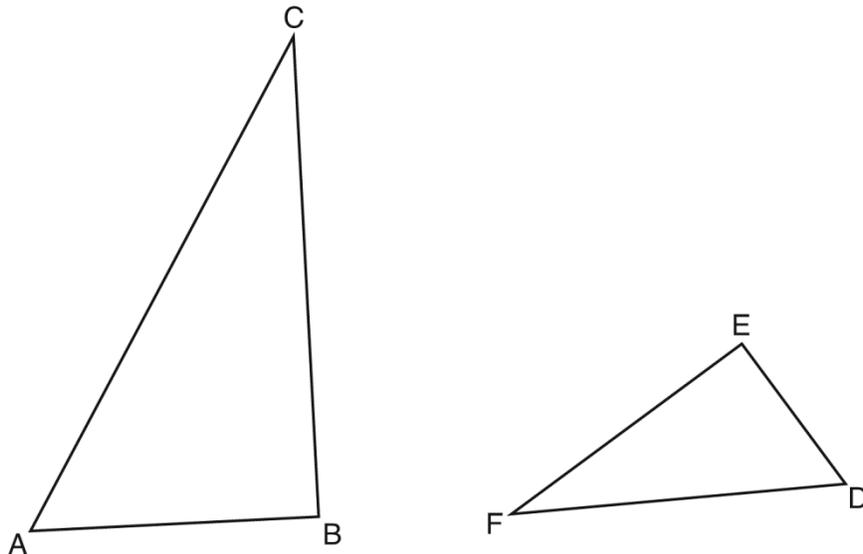
Center (0,−3); radius 4

Answer Choice: (3) center $(0,3)$ and radius 16. This response is incorrect and does not show the center and radius for the circle represented by the equation $x^2 + y^2 + 6y = 7$. The student may have completed the square to rewrite the equation of the circle, but made an error in interpreting the coordinates of the center, while also not taking the square root of the constant, 16, to find the length of the radius.

Answer Choice: (4) center $(0, -3)$ and radius 16. This response is incorrect and does not show the radius for the circle represented by the equation $x^2 + y^2 + 6y = 7$. The student may have completed the square and identified the coordinates of the center, but made an error by not taking the square root of 16 to find the length of the radius.

#15

15 Triangles ABC and DEF are drawn below.



If $AB = 9$, $BC = 15$, $DE = 6$, $EF = 10$, and $\angle B \cong \angle E$, which statement is true?

- (1) $\angle CAB \cong \angle DEF$ (3) $\triangle ABC \sim \triangle DEF$
(2) $\frac{AB}{CB} = \frac{FE}{DE}$ (4) $\frac{AB}{DE} = \frac{FE}{CB}$

Measured CCLS Cluster: G-SRT.B

Key: (3)

Commentary: This question measures the knowledge and skills described by the standards within G-SRT.B because it requires the student to apply triangle similarity criteria to reach a conclusion about geometric relationships. Specifically, the student must reason about the similarity of two triangles by applying the SAS similarity criterion.

Rationale: Choices (1), (2), and (4) are plausible but incorrect. They represent common student errors made when working with triangle similarity criteria and how to apply triangle similarity criteria.

Choosing the correct solution requires students to know how to analyze a diagram and apply similarity criteria to reach a conclusion. Compare with questions 8, 11, and 31, which also assess G-SRT.B.

Answer Choice: (1) $\angle CAB \cong \angle DEF$. This response is incorrect because the given information does not imply that $\angle CAB$ and $\angle DEF$ must be congruent. The student may have assumed that because the triangles are similar, any pair of angles would be congruent.

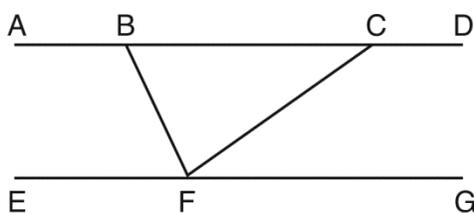
Answer Choice: (2) $\frac{AB}{CB} = \frac{FE}{DE}$. This response is incorrect because the given information does not imply the proportional relationship illustrated by this equation. The student may have concluded that the triangles are similar, and recognized a proportional relationship between sides, but incorrectly identified the relationship of the corresponding sides in the proportion.

Answer Choice: (3) $\triangle ABC \sim \triangle DEF$. This response is correct and shows a correct conclusion from the given information. The student understands that because the two pairs of corresponding sides are proportional and the included angles are congruent, the triangles are similar by the SAS similarity criterion. A student who selects this response understands how to apply triangle similarity criteria.

Answer Choice: (4) $\frac{AB}{DE} = \frac{FE}{CB}$. This response is incorrect because the given information does not imply the proportional relationship illustrated by this equation. The student may have concluded that the triangles are similar, and recognized a proportional relationship between sides, but incorrectly identified the relationship of the corresponding sides in the proportion.

#17

17 Steve drew line segments $ABCD$, EFG , BF , and CF as shown in the diagram below. Scalene $\triangle BFC$ is formed.



Which statement will allow Steve to prove $\overline{ABCD} \parallel \overline{EFG}$?

- (1) $\angle CFG \cong \angle FCB$ (3) $\angle EFB \cong \angle CFB$
(2) $\angle ABF \cong \angle BFC$ (4) $\angle CBF \cong \angle GFC$

Measured CCLS Cluster: G-CO.C

Key: (1)

Commentary: This question measures the knowledge and skills described by the standards within G-CO.C because it requires the student to reason about lines and angles by identifying congruent alternate interior angles to prove that lines are parallel. Additionally, the item requires the student to employ Mathematical Practice 3 (Construct viable arguments and critique the reasoning of others) because the student must identify evidence that will support the claim that two lines are parallel.

Rationale: Choices (2), (3), and (4) are plausible but incorrect. They represent common student errors made when working with lines and angles and indicate a limited understanding of how to reason about lines and angles. Choosing the correct solution requires students to identify the correct angle pairs needed for the parallel lines to be proven. Compare with questions 13, 26, 32, and 33, which also assess G-CO.C.

Answer Choice: (1) $\angle CFG \cong \angle FCB$. This response is correct and is valid evidence that will support the claim that the lines are parallel. The student identified that transversal \overline{CF} intersects \overline{ABCD} and \overline{EFG} to form the alternate interior angles CFG and FCB , and reasoned that that when segments are intersected by a transversal such that the alternate interior angles are congruent, the segments are parallel. A student who selects this response understands how to reason about lines and angles.

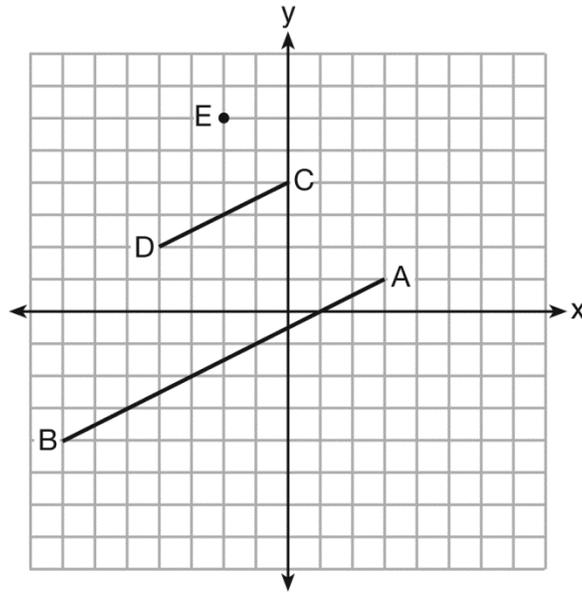
Answer Choice: (2) $\angle ABF \cong \angle BFC$. This response is incorrect and is not evidence that will support the claim that the lines are parallel. The student may have mistakenly identified these angles as alternate interior angles, reasoning then that because they are congruent, the lines would be parallel.

Answer Choice: (3) $\angle EFB \cong \angle CFB$. This response is incorrect and is not evidence that will support the claim that the lines are parallel. The student may have mistakenly identified these angles as alternate interior angles, reasoning then that because they are congruent, the lines would be parallel.

Answer Choice: (4) $\angle CBF \cong \angle GFC$. This response is incorrect and is not evidence that will support the claim that the lines are parallel. The student may have mistakenly identified these angles as corresponding angles, reasoning then that because they are congruent, the lines would be parallel.

#18

18 In the diagram below, \overline{CD} is the image of \overline{AB} after a dilation of scale factor k with center E .



Which ratio is equal to the scale factor k of the dilation?

(1) $\frac{EC}{EA}$

(3) $\frac{EA}{BA}$

(2) $\frac{BA}{EA}$

(4) $\frac{EA}{EC}$

Measured CCLS Cluster: G-SRT.A

Key: (1)

Commentary: This question measures the knowledge and skills described by the standards within G-SRT.A because it requires the student to use similarity transformations to reason about the effect of a dilation on a line segment. Specifically, the student must understand that the dilation of a line segment is longer or shorter in the ratio given by the scale factor. The student may also understand that the line segments joining the center of dilation and the corresponding endpoints of the given line segment and its image forms two similar triangles.

Rationale: Choices (2), (3), and (4) are plausible but incorrect. They represent common student errors made when working with dilations in the plane. Students who select these responses may not understand the effect of a dilation on the length of a segment. Choosing the correct solution requires students to know that the dilation of a line segment is longer or shorter in the ratio given by the scale factor. Compare with question 22, which also assesses G-SRT.A.

Answer Choice: (1) $\frac{EC}{EA}$. This response is correct and is the scale factor of the dilation. The student correctly chose the scale factor by identifying a ratio between a dimension of the image and the corresponding dimension of its pre-image. A student who selects this response understands the effect of a dilation on the length of a segment.

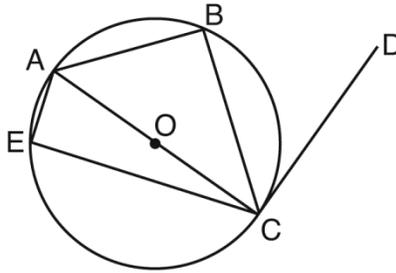
Answer Choice: (2) $\frac{BA}{EA}$. This response is incorrect and is not the scale factor of the dilation. The student may have incorrectly assumed that the ratio of any two distances in a diagram would be equal to the scale factor of the dilation, or lacked a general understanding of how figures are dilated.

Answer Choice: (3) $\frac{EA}{BA}$. This response is incorrect and is not the scale factor of the dilation. The student may have incorrectly assumed that the ratio of any two distances in a diagram would be equal to the scale factor of the dilation, or lacked a general understanding of how figures are dilated.

Answer Choice: (4) $\frac{EA}{EC}$. This response is incorrect and is not the scale factor of the dilation. The student may have incorrectly assumed that \overline{AB} was the image of \overline{CD} , or lacked a general understanding of how figures are dilated.

#20

20 In circle O shown below, diameter \overline{AC} is perpendicular to \overline{CD} at point C , and chords \overline{AB} , \overline{BC} , \overline{AE} , and \overline{CE} are drawn.



Which statement is *not* always true?

- (1) $\angle ACB \cong \angle BCD$ (3) $\angle BAC \cong \angle DCB$
(2) $\angle ABC \cong \angle ACD$ (4) $\angle CBA \cong \angle AEC$

Measured CCLS Cluster: G-C.A

Key: (1)

Commentary: This question measures the knowledge and skills described by the standards within G-C.A because it requires the student to apply theorems to circles. The student must reason using theorems about inscribed angles, angles formed by a chord and a tangent, and other circle relationships to determine the statement that is not always true.

Rationale: Choices (2), (3), and (4) are plausible but incorrect. They represent common student errors made when working with relationships in circles and applying theorems to circles. Choosing the correct solution requires that students be able to reason using relationships between angles and segments in circles.

Answer Choice: (1) $\angle ACB \cong \angle BCD$. This response is correct because it is *not* always true. The angles ACB and BCD are not always congruent since the arc intercepted by the inscribed angle ACB and the arc intercepted by the angle BCD , formed by the intersection of the chord BC and the tangent CD , are not always congruent. A student who selects this response understands how to apply theorems to circles.

Answer Choice: (2) $\angle ABC \cong \angle ACD$. This response is incorrect because it *is* always true. The inscribed $\angle ABC$ intercepts a semicircle, therefore $\angle ABC$ is a right angle. It is given that \overline{AC} and \overline{CD} are perpendicular, therefore $\angle ACD$ is a right angle. The student may not have used these relationships in the circle to reason that $\angle ABC$ is a right angle.

Answer Choice: (3) $\angle BAC \cong \angle DCB$. This response is incorrect because it *is* always true. The inscribed $\angle BAC$ and the angle formed by the intersection of chord \overline{BC} and tangent \overline{CD} intercept the same arc BC , therefore these angles are congruent because both their measures are half the measure of the intercepted arc. The student may not have used these relationships in the circle to reason that these angles are congruent.

Answer Choice: (4) $\angle CBA \cong \angle AEC$. This response is incorrect because it *is* always true. The inscribed angles CBA and AEC both intercept a semicircle, and are congruent because the measures of inscribed angles are half the measure of the intercepted arc and the arc, measure of all semicircles are equal. The student may not have used these relationships in the circle to reason that these angles are congruent.

#22

22 The line $3y = -2x + 8$ is transformed by a dilation centered at the origin. Which linear equation could be its image?

(1) $2x + 3y = 5$

(3) $3x + 2y = 5$

(2) $2x - 3y = 5$

(4) $3x - 2y = 5$

Measured CCLS Cluster: G-SRT.A

Key: (1)

Commentary: This question measures the knowledge and skills described by the standards within G-SRT.A because it requires the student to use similarity transformations to reason about lines. The student must reason that the image of a dilated line is always parallel to its pre-image when the line does not pass through the center of the dilation. The question is also an example of the instructional shift of coherence, since the student must draw on understandings from another domain, Expressing Geometric Properties with Equations (G-GPE), including work with coordinates and the slope criteria for parallel lines.

Rationale: Choices (2), (3), and (4) are plausible but incorrect. They represent common student errors made when working with dilated lines represented by equations. Choosing the correct solution requires that students be able to reason about how a dilation affects the equation of a line. Compare with question 18, which also assesses G-SRT.A.

Answer Choice: (1) $2x + 3y = 5$. This response is correct because it represents a line parallel to the given line. Knowing that the image of a dilated line is always parallel to its pre-image when the line does not pass through the center of dilation, the student found the slope of the line and chose the line with the same slope as the given line. When the lines $3y = -2x + 8$ and $2x + 3y = 5$ are rewritten in the form $y = mx + b$, the slopes are equal. Since both lines have the same slope but different y -intercepts, they are parallel. A student who selects this response understands that the image of a dilated line is always parallel to its pre-image when the line does not pass through the center of the dilation.

$$3y = -2x + 8$$

$$y = -\frac{2}{3}x + \frac{8}{3}$$

$$\text{slope} = -\frac{2}{3}$$

$$2x + 3y = 5$$

$$3y = -2x + 5$$

$$y = -\frac{2}{3}x + \frac{5}{3}$$

$$\text{slope} = -\frac{2}{3}$$

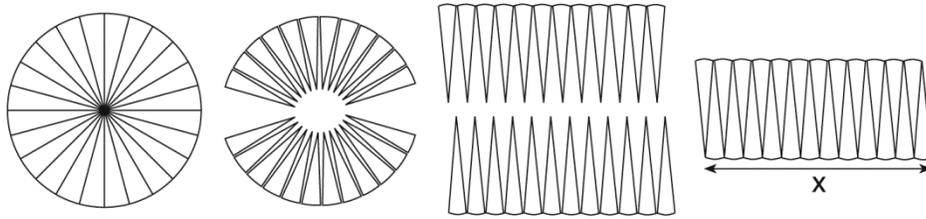
Answer Choice: (2) $2x - 3y = 5$. This response is incorrect because it represents a line that is not parallel to the given line. The line given by this equation has a slope of $\frac{2}{3}$, which is the opposite of the slope of the given line. The student may have incorrectly concluded that parallel lines have opposite slopes or the student may have incorrectly rewritten the equation to have a slope that is equal to the slope of the given line.

Answer Choice: (3) $3x + 2y = 5$. This response is incorrect because it represents a line that is not parallel to the given line. The line given by this equation has a slope of $-\frac{3}{2}$, which is the reciprocal of the slope of the given line. The student may have incorrectly concluded that parallel lines have reciprocal slopes or the student may have incorrectly rewritten the equation to have a slope that is equal to the slope of the given line.

Answer Choice: (4) $3x - 2y = 5$. This response is incorrect because it represents a line that is not parallel to the given line. The line given by this equation has a slope that is the negative reciprocal of the slope of the given line. The student may have incorrectly concluded that the image of a dilated line is perpendicular to its pre-image or the student may have incorrectly rewritten the equation to have a slope that is equal to the slope of the given line.

#23

23 A circle with a radius of 5 was divided into 24 congruent sectors. The sectors were then rearranged, as shown in the diagram below.



To the *nearest integer*, the value of x is

- | | |
|--------|--------|
| (1) 31 | (3) 12 |
| (2) 16 | (4) 10 |

Measured CCLS Cluster: G-GMD.A

Key: (2)

Commentary: This question measures the knowledge and skills described by the standards within G-GMD.A because it requires the student to analyze an informal argument for the area of a circle. The diagram shows a circle decomposed into congruent sectors, which are reassembled to form a figure that approximates a parallelogram with a base equal to approximately half the circumference of the circle and a height equal to the radius of the circle. The student must reason using the informal argument and knowledge of the circle area formula to determine an approximate length of the base of the parallelogram-like figure.

Rationale: Choices (1), (3), and (4) are plausible but incorrect. They represent common student errors made when working with the area of a circle and indicate a limited understanding of an informal argument for the area of a circle. Choosing the correct solution requires that students be able to reason using an informal argument for the area of a circle.

Answer Choice: (1) 31. This response is incorrect and does not represent an approximate length of the base of the figure. The student may have assumed that the value of x would be equivalent to the circumference of the circle.

Answer Choice: (2) 16. This response is correct and represents an approximate length of the base of the figure. The area of the circle can be determined using the formula $A = \pi r^2$; once this is found, the student must recognize that the area of the parallelogram-like figure is the same as area of the circle, $5x = 25\pi$. A student who selects this response understands an informal argument for the area of a circle.

$$A = \pi r^2$$

$$5x = 25\pi$$

$$x = 5\pi$$

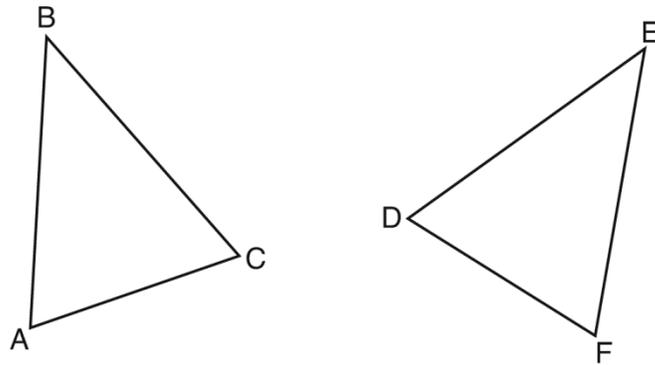
$$x \approx 16$$

Answer Choice: (3) 12. This response is incorrect and does not represent an approximate length of the base of the figure. The student may have assumed that because the circle was divided into 24 congruent sectors and the base of the figure accounts for half of these sectors, that the base is 12.

Answer Choice: (4) 10. This response is incorrect and does not represent an approximate length of the base of the figure. The student may have assumed that the parallelogram-like figure would have a base length that is equal to the length of the diameter of the circle.

#24

24 Which statement is sufficient evidence that $\triangle DEF$ is congruent to $\triangle ABC$?



- (1) $AB = DE$ and $BC = EF$
- (2) $\angle D \cong \angle A$, $\angle B \cong \angle E$, $\angle C \cong \angle F$
- (3) There is a sequence of rigid motions that maps \overline{AB} onto \overline{DE} , \overline{BC} onto \overline{EF} , and \overline{AC} onto \overline{DF} .
- (4) There is a sequence of rigid motions that maps point A onto point D, \overline{AB} onto \overline{DE} , and $\angle B$ onto $\angle E$.

Measured CCLS Cluster: G-CO.B

Key: (3)

Commentary: This question measures the knowledge and skills described by the standards within G-CO.B because it requires the student to use rigid motions to reason about the congruence of triangles. The diagram shows two triangles; the student must determine which evidence, including information about corresponding angles and sides and also various rigid motions, is sufficient to prove the triangles are congruent.

Rationale: Choices (1), (2), and (4) are plausible but incorrect. They represent common student errors made when reasoning about the congruence of triangles. Choosing the correct solution requires that students be able to carefully reason using triangle congruence criteria and the definition of congruence in terms of rigid motions. Compare with question 30, which also assesses G-CO.B.

Answer Choice: (1) $AB = DE$ and $BC = EF$. This response is incorrect and is insufficient evidence to prove that the triangles are congruent. Two pairs of corresponding sides of equal measure do not meet the congruence criteria for congruent triangles. The student may have a misconception about the criteria needed to conclude the triangles are congruent.

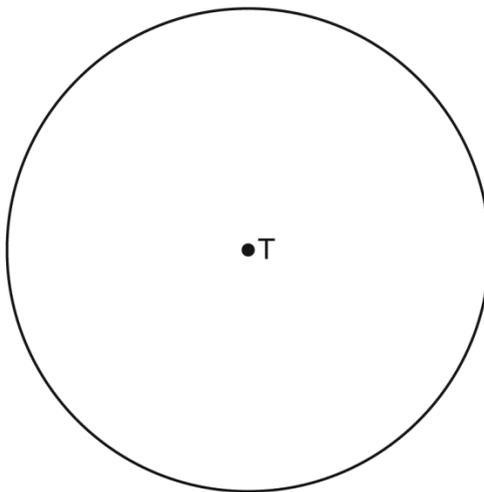
Answer Choice: (2) $\angle D \cong \angle A$, $\angle B \cong \angle E$, $\angle C \cong \angle F$. This response is incorrect and is insufficient evidence to prove that the triangles are congruent. Two or more pairs of congruent corresponding angles do not meet the congruence criteria for congruent triangles. The student may have confused the criteria for triangle congruence with the criteria for triangle similarity.

Answer Choice: (3) There is a sequence of rigid motions that maps \overline{AB} onto \overline{DE} , \overline{BC} onto \overline{EF} , and \overline{AC} onto \overline{DF} . This response is correct because it is sufficient evidence to prove the triangles congruent. If there exists a rigid motion that maps all three sides of one triangle onto three corresponding sides of another triangle, then the triangles are congruent. A student who selects this response understands how to reason about the congruence of triangles.

Answer Choice: (4) There is a sequence of rigid motions that maps point A onto point D , \overline{AB} onto \overline{DE} , and $\angle B$ onto $\angle E$. This response is incorrect and is insufficient evidence to prove that the triangles are congruent. There is one pair of corresponding congruent sides and one pair of corresponding congruent angles, but it is still possible that one triangle is not mapped onto the other. The student may have incorrectly assumed that mapping point A onto point D will result in $\angle A \cong \angle D$.

#25

25 Use a compass and straightedge to construct an inscribed square in circle T shown below.
[Leave all construction marks.]



Measured CCLS Cluster: G-CO.D

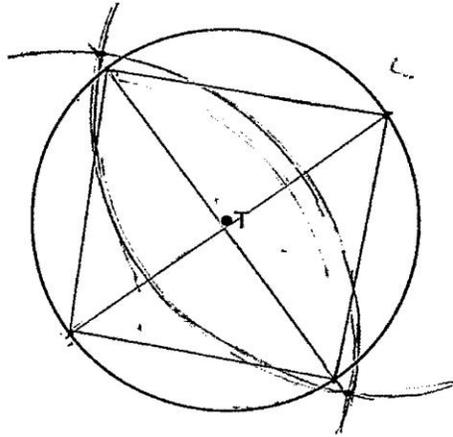
Commentary: The question measures the knowledge and skills described by the standards within G-CO.D because it requires the student to construct a square inscribed in a circle.

Rationale: This question requires students to construct a square inscribed in a circle. As indicated in the rubric, a correct response requires a correct construction showing all appropriate arcs.

Sample student responses and scores appear on the following pages.

Question 25

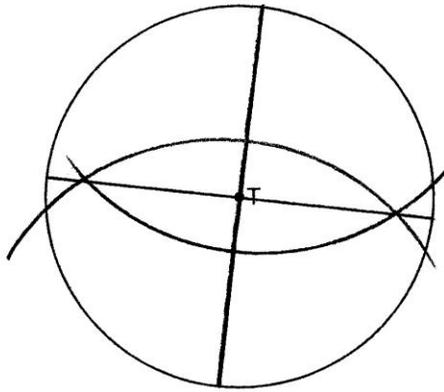
25 Use a compass and straightedge to construct an inscribed square in circle T shown below.
[Leave all construction marks.]



Score 2: The student drew a correct construction showing all appropriate construction marks and the square was drawn.

Question 25

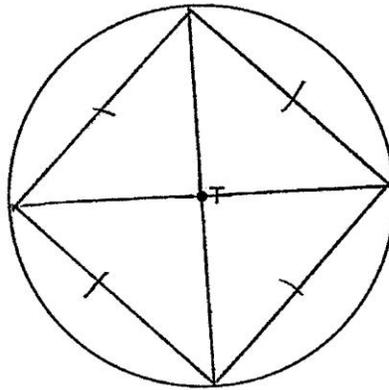
25 Use a compass and straightedge to construct an inscribed square in circle T shown below.
[Leave all construction marks.]



Score 1: The student drew a correct construction showing all appropriate construction marks, but the square was not drawn.

Question 25

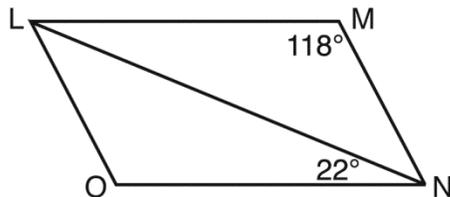
25 Use a compass and straightedge to construct an inscribed square in circle T shown below.
[Leave all construction marks.]



Score 0: The student made a drawing that is not a construction.

#26:

26 The diagram below shows parallelogram $LMNO$ with diagonal \overline{LN} , $m\angle M = 118^\circ$, and $m\angle LNO = 22^\circ$.



Explain why $m\angle NLO$ is 40 degrees.

Measured CCLS Cluster: G-CO.C

Commentary: The question measures the knowledge and skills described by the standards within G-CO.C because the student is required to reason using theorems about parallelograms and triangles to explain the measure of the noted angle. Theorems that students might use include: opposite angles of a parallelogram are congruent, consecutive angles of a parallelogram are supplementary, and/or angles of a triangle add up to 180 degrees. Additionally, the item requires the student to employ Mathematical Practice 3 because the student must identify and explain evidence that will support the claim that $\angle NLO$ measures 40 degrees.

Rationale: This question requires students to explain the measure of the noted angle in a diagram. One possible line of reasoning could be that since $m\angle M = 118^\circ$ and opposite angles of a parallelogram are congruent, $m\angle O = 118^\circ$. Then, the angles of triangle LNO add up to 180° so $m\angle NLO + m\angle LNO + m\angle O = 180^\circ$, therefore $m\angle NLO + 22^\circ + 118^\circ = 180^\circ$ and $m\angle NLO = 40^\circ$.

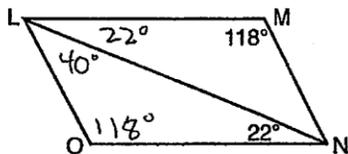
Another line of reasoning is that since opposite sides of a parallelogram are parallel, then the alternate interior angles LNO and NLM are congruent and therefore $m\angle NLM = 22^\circ$. Then, consecutive angles M and MLO of parallelogram $LMNO$ are supplementary and therefore $m\angle M + m\angle NLM + m\angle NLO = 180^\circ$. So, $118^\circ + 22^\circ + m\angle NLO = 180^\circ$ and $m\angle NLO = 40^\circ$.

Compare with items 13, 17, 32, and 33, which also assess G-CO.C.

Sample student responses and scores appear on the following pages.

Question 26

26 The diagram below shows parallelogram $LMNO$ with diagonal \overline{LN} , $m\angle M = 118^\circ$, and $m\angle LNO = 22^\circ$.



Explain why $m\angle NLO$ is 40 degrees.

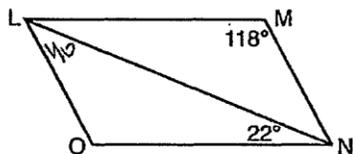
$\angle LON$ is 118° b/c opposite \angle 's of a \square are \cong .

A \triangle 's \angle measures add up to 180° .
 $118 + 22 = 140$ so $\angle NLO$ must
be 40° .

Score 2: The student has a complete and correct response.

Question 26

26 The diagram below shows parallelogram $LMNO$ with diagonal \overline{LN} , $m\angle M = 118^\circ$, and $m\angle LNO = 22^\circ$.



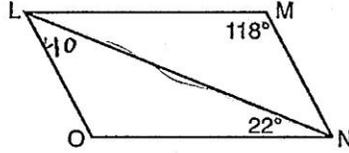
Explain why $m\angle NLO$ is 40 degrees.

because if you add 118 and 22
you get 140 and every triangle
equals 180 so you subtract
140 from 180 to get 40.

Score 1: The student gave an incomplete explanation, because a geometric relationship between 118° and 22° was not established.

Question 26

26 The diagram below shows parallelogram $LMNO$ with diagonal \overline{LN} , $m\angle M = 118^\circ$, and $m\angle LNO = 22^\circ$.



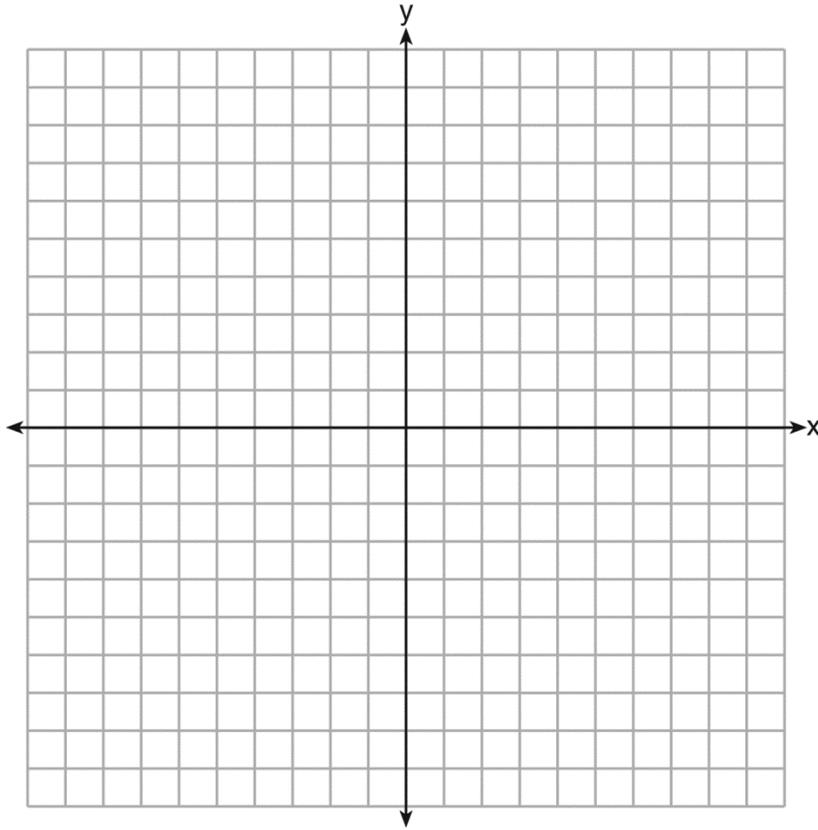
Explain why $m\angle NLO$ is 40 degrees.

because $\angle M$ & $\angle N$ are complementary angles
So when you add them up and equal it
to 180 you get 140 then subtract that from
180 and you get 40°

Score 0: The student gave a completely incorrect explanation.

#27:

- 27 The coordinates of the endpoints of \overline{AB} are $A(-6, -5)$ and $B(4, 0)$. Point P is on \overline{AB} . Determine and state the coordinates of point P , such that $AP:PB$ is 2:3.
[The use of the set of axes below is optional.]



Measured CCLS Cluster: G-GPE.B

Commentary: The question measures the knowledge and skills described by the standards within G-GPE.B because the student is required to use coordinates to apply understanding of geometric figures. The student uses coordinates to determine the location of a point dividing a directed line segment in the given ratio.

Rationale: This question requires students to find the coordinates of a point on a line segment that divides the line segment into a given ratio, optionally using the provided set of axes. The student who uses the set of axes must graph the line segment and divide it into five congruent parts. This can be done by dividing both the vertical change and the horizontal change between points A and B by 5, resulting in vertical increments of 1 and horizontal increments of 2; the segment can be divided into five equal parts by starting at point A and repeatedly moving right two units, then up one unit to mark the segment. The student will see that the line segment has been divided into five equal parts. The point that divides the line segment, such that $AP:PB$ is 2:3, is $(-2, -3)$.

A second method to solve the problem involves the same principle, but uses numerical coordinates. Two-fifths of the horizontal and vertical distance between A and B is added to the x - and y -coordinates of A , respectively:

$$x = -6 + \frac{2}{5}(4 - -6) \qquad y = -5 + \frac{2}{5}(0 - -5)$$

$$x = -6 + \frac{2}{5}(10) \qquad y = -5 + \frac{2}{5}(5)$$

$$x = -6 + 4 \qquad y = -5 + 2$$

$$x = -2 \qquad y = -3$$

$$(-2, -3)$$

Compare with question 36, which also assesses G-GPE.B.

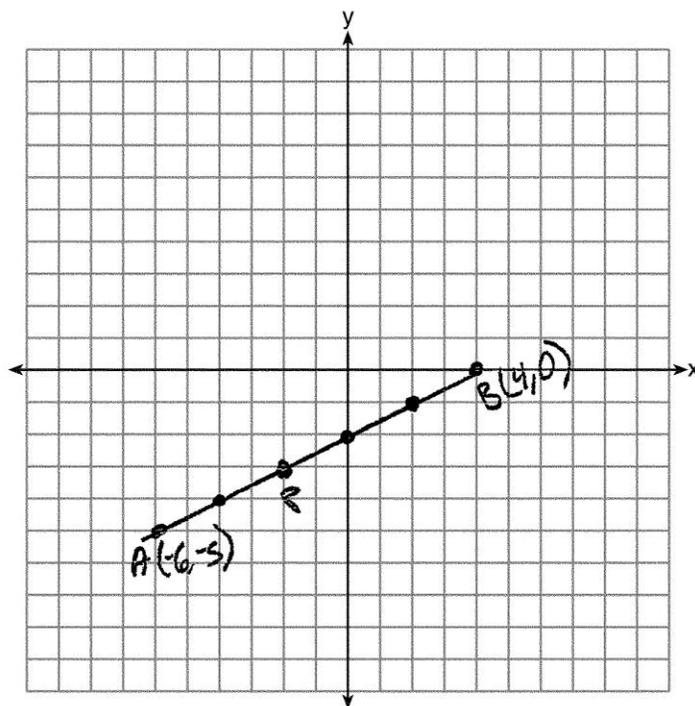
Sample student responses and scores appear on the following pages.

Question 27

- 27 The coordinates of the endpoints of \overline{AB} are $A(-6, -5)$ and $B(4, 0)$. Point P is on \overline{AB} . Determine and state the coordinates of point P , such that $AP:PB$ is 2:3.
[The use of the set of axes below is optional.]

$$\begin{aligned}d &= \sqrt{(x-x)^2 + (y-y)^2} \\ &= \sqrt{(-6-4)^2 + (-5-0)^2} \\ &= \sqrt{(-10)^2 + (-5)^2} \\ &= \sqrt{100+25} \\ &= \sqrt{125} \\ &= \sqrt{25 \cdot 5} \\ &= 5\sqrt{5}\end{aligned}$$

$P(-2, -3)$



Score 2: The student has a complete and correct response. The student showed correct work that was not necessary.

Question 27

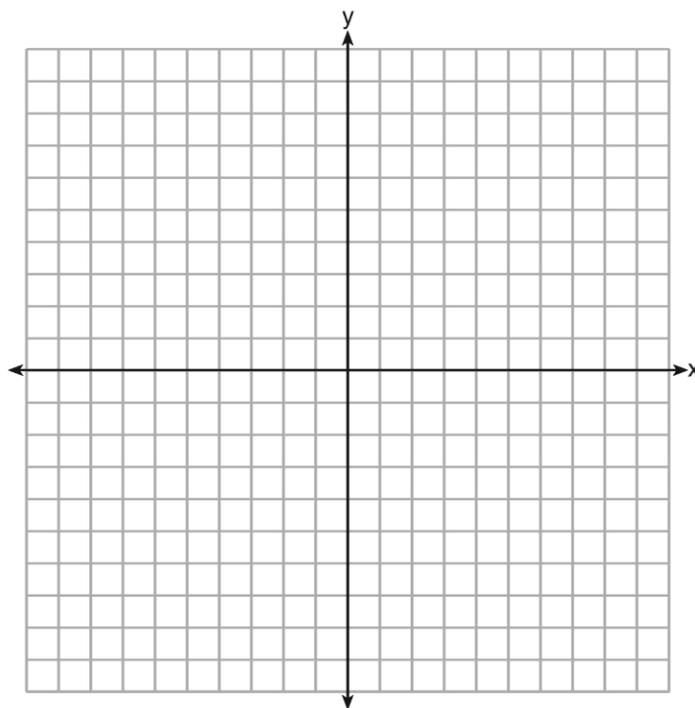
- 27 The coordinates of the endpoints of \overline{AB} are $A(-6, -5)$ and $B(4, 0)$. Point P is on \overline{AB} . Determine and state the coordinates of point P , such that $AP:PB$ is 2:3.
[The use of the set of axes below is optional.]

$$P \left(-6 + \frac{2}{3}(10), -5 + \frac{2}{3}(5) \right)$$

$$P \left(-6 + \frac{20}{3}, -5 + \frac{10}{3} \right)$$

$$P \left(-6 + 6\frac{2}{3}, -5 + 3\frac{1}{3} \right)$$

$$P \left(2\frac{2}{3}, -1\frac{2}{3} \right)$$

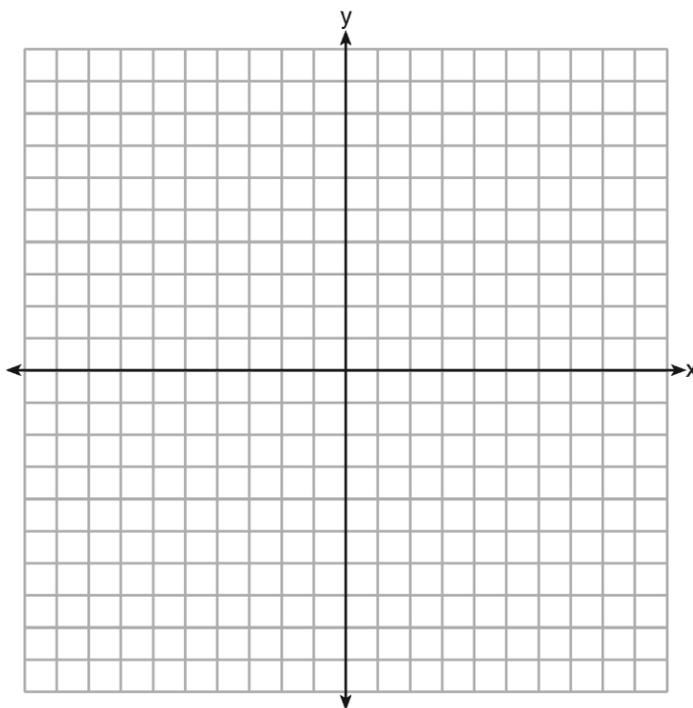


Score 1: The student made an error by multiplying by $\frac{2}{3}$ instead of $\frac{2}{5}$.

Question 27

- 27 The coordinates of the endpoints of \overline{AB} are $A(-6, -5)$ and $B(4, 0)$. Point P is on \overline{AB} . Determine and state the coordinates of point P , such that $AP:PB$ is 2:3.
[The use of the set of axes below is optional.]

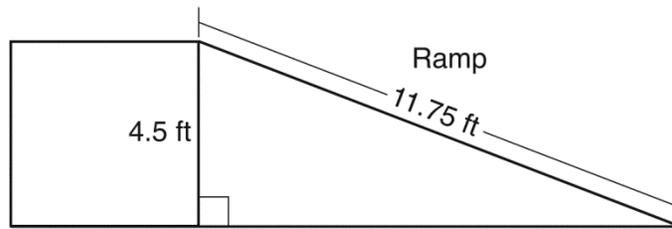
$$\begin{aligned} & \frac{-6+4}{2}, \frac{-5+0}{2} \\ & \frac{-2}{2}, \frac{-5}{2} \\ & (-1, -\frac{5}{2}) \end{aligned}$$



Score 0: The student's use of the midpoint formula was irrelevant to the question.

#28:

28 The diagram below shows a ramp connecting the ground to a loading platform 4.5 feet above the ground. The ramp measures 11.75 feet from the ground to the top of the loading platform.



Determine and state, to the *nearest degree*, the angle of elevation formed by the ramp and the ground.

Measured CCLS Cluster: G-SRT.C

Commentary: The question measures the knowledge and skills described by the standards within G-SRT.C because the student is required to apply understanding of relationships between angles and sides of right triangles using trigonometry. The question also requires the student to employ Mathematical Practice 4 (Model with mathematics), because the student must use right triangle trigonometry to solve a real-world problem.

Rationale: This question instructs the student to determine and state the angle of elevation a ramp makes with the ground given the length of the ramp and the height the ramp will reach. The student must determine which trigonometric ratio is appropriate for finding the angle of elevation. Although there are multiple methods of solving this problem, the most direct method is to use arcsine to find the angle of elevation, since the two given side lengths of the right triangle are the side opposite the angle of elevation and the hypotenuse of the right triangle.

Let x be the angle of elevation of the ramp.

$$\sin x = \frac{4.5}{11.75}$$

$$x = \arcsin\left(\frac{4.5}{11.75}\right)$$

$$x = 22.51831413$$

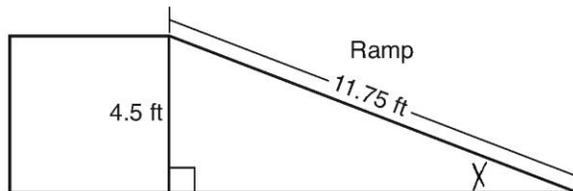
$$x = 23$$

Compare with question 34, which also assesses G-SRT.C.

Sample student responses and scores appear on the following pages.

Question 28

28 The diagram below shows a ramp connecting the ground to a loading platform 4.5 feet above the ground. The ramp measures 11.75 feet from the ground to the top of the loading platform.



Determine and state, to the *nearest degree*, the angle of elevation formed by the ramp and the ground.

$$X = \sin^{-1}\left(\frac{4.5}{11.75}\right)$$

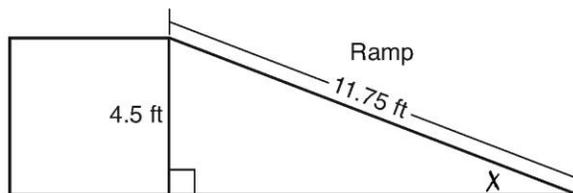
$$X = 22.518$$

$$X = 23^\circ$$

Score 2: The student has a complete and correct response.

Question 28

28 The diagram below shows a ramp connecting the ground to a loading platform 4.5 feet above the ground. The ramp measures 11.75 feet from the ground to the top of the loading platform.



Determine and state, to the *nearest degree*, the angle of elevation formed by the ramp and the ground.

$$\tan X = \frac{4.5}{11.75}$$

$$X = \tan^{-1} \frac{4.5}{11.75}$$

$$X = 20.9557767306$$

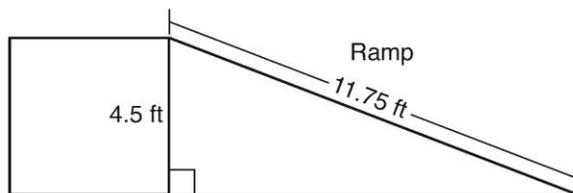
$$X = 21$$

21

Score 1: The student made an error by using the wrong trigonometric function, but found an appropriate angle of elevation.

Question 28

28 The diagram below shows a ramp connecting the ground to a loading platform 4.5 feet above the ground. The ramp measures 11.75 feet from the ground to the top of the loading platform.



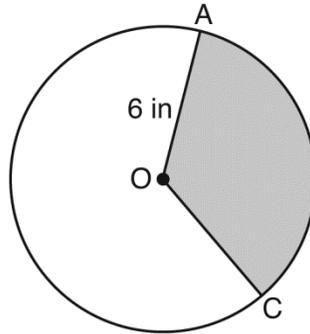
Determine and state, to the *nearest degree*, the angle of elevation formed by the ramp and the ground.

$$\begin{aligned} a^2 + b^2 &= c^2 \\ (4.5)^2 + b^2 &= (11.75)^2 \\ 20.25 + b^2 &= 138.0625 \\ -20.25 & \quad -20.25 \\ \hline \sqrt{b^2} &= \sqrt{117.8125} \\ b &= 10.854146673 \\ \boxed{11^\circ} \end{aligned}$$

Score 0: The student had a completely incorrect response.

#29

29 In the diagram below of circle O , the area of the shaded sector AOC is 12π in² and the length of \overline{OA} is 6 inches. Determine and state $m\angle AOC$.



Measured CCLS Cluster: G-C.B

Commentary: The question measures the knowledge and skills described by the standards within G-C.B because the student is required to apply their understanding of equations that relate radii, arc length, and/or areas of sectors and circles. Specifically, given the area of a sector and radius of the circle, the student must determine the central angle that defines the sector. The angle may be determined in degrees or radians, since it is not specified in the question stem.

Rationale:

$$A = \pi r^2$$

OR

$$A = \pi r^2$$

OR

$$A = \pi r^2$$

$$A = \pi 6^2$$

$$A = \pi 6^2$$

$$A = \pi 6^2$$

$$A = 36\pi$$

$$A = 36\pi$$

$$A = 36\pi$$

Let x represent $m\angle AOC$

Let x represent $m\angle AOC$

$$C = \pi d$$

$$\frac{12\pi}{36\pi} = \frac{x}{360}$$

$$\frac{12\pi}{36\pi} = \frac{x}{2\pi}$$

$$C = 12\pi$$

$$36\pi x = 4320\pi$$

$$36\pi x = 24(\pi)(\pi)$$

Let S represent $m\widehat{AC}$

$$x = 120^\circ$$

$$x = \frac{24(\pi)(\pi)}{36\pi}$$

$$\frac{12\pi}{36\pi} = \frac{S}{12\pi}$$

$$x = \frac{2\pi}{3} \text{ radians}$$

$$36\pi S = 144(\pi)(\pi)$$

$$S = 4\pi$$

Let θ represent $m\angle AOC$

$$S = \theta r$$

$$4\pi = \theta(6)$$

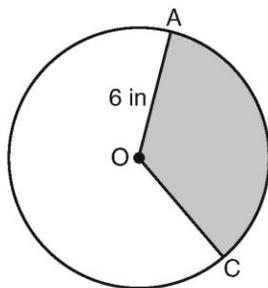
$$\theta = \frac{4\pi}{6}$$

$$\theta = \frac{2\pi}{3} \text{ radians}$$

Sample student responses and scores appear on the following pages.

Question 29

29 In the diagram below of circle O , the area of the shaded sector AOC is $12\pi \text{ in}^2$ and the length of OA is 6 inches. Determine and state $m\angle AOC$.



$$\begin{aligned} A &= \pi r^2 \\ &= 6^2 \cdot \pi \\ &= 36\pi \end{aligned}$$

$$\frac{12\pi}{36\pi} = \frac{1}{3}$$

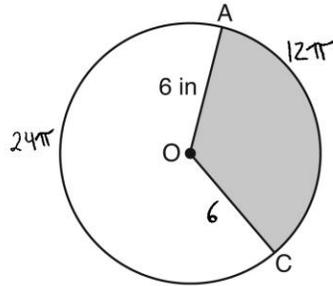
$$\frac{1}{3} \cdot 360$$

120°

Score 2: The student has a complete and correct response.

Question 29

29 In the diagram below of circle O , the area of the shaded sector AOC is 12π in² and the length of OA is 6 inches. Determine and state $m\angle AOC$.



$$A = \pi r^2$$

$$A = \pi 6^2$$

$$A = 36\pi$$

$$36\pi - 12\pi = 24\pi$$

$$\frac{24\pi}{36\pi} = \frac{x}{360}$$

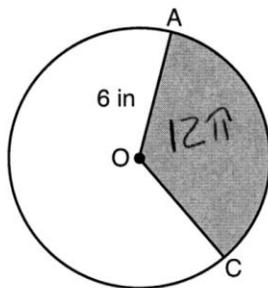
$$8640 = 36x$$

$$240^\circ = x$$

Score 1: The student made an error by finding the central angle for the unshaded sector.

Question 29

29 In the diagram below of circle O , the area of the shaded sector AOC is $12\pi \text{ in}^2$ and the length of OA is 6 inches. Determine and state $m\angle AOC$.



$$\text{Sector} = \theta r$$

$$\frac{12\pi}{6} = \frac{\theta(6)}{6}$$

$$2\pi = \theta$$

$$m\angle AOC = 2\pi$$

Score 0: The student had a completely incorrect response.

#30

30 After a reflection over a line, $\triangle A'B'C'$ is the image of $\triangle ABC$. Explain why triangle ABC is congruent to triangle $A'B'C'$.

Measured CCLS Cluster: G-CO.B

Commentary: The question measures the knowledge and skills described by the standards within G-CO.B because the student is required to use rigid motions to reason about the congruence of geometric figures. Additionally, the item requires the student to employ Mathematical Practice 3 because the student must identify and explain evidence that will support the claim that the triangles are congruent.

Rationale: This question asks students to explain why a triangle is congruent to its image after a reflection. Since a reflection is a rigid motion and all rigid motions will map one triangle onto the other and preserve side lengths, then the triangle and its image are congruent.

Compare with question 24, which also assesses G-CO.B.

Sample student responses and scores appear on the following pages.

Question 30

30 After a reflection over a line, $\triangle A'B'C'$ is the image of $\triangle ABC$. Explain why triangle ABC is congruent to triangle $A'B'C'$.

Reflections are rigid motions and Rigid
motions ~~do~~ keep distances the same.
So $\overline{AB} \cong \overline{A'B'}$ and $\overline{BC} \cong \overline{B'C'}$ and
 $\overline{AC} \cong \overline{A'C'}$, so $\triangle ABC \cong \triangle A'B'C'$ by SSS

Score 2: The student has a complete and correct response.

Question 30

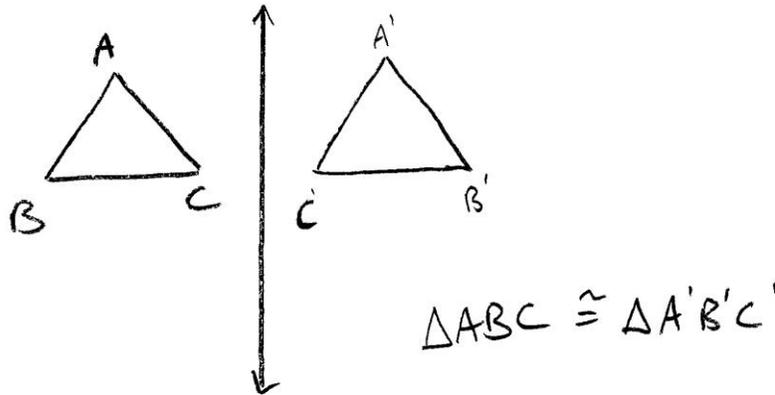
30 After a reflection over a line, $\triangle A'B'C'$ is the image of $\triangle ABC$. Explain why triangle ABC is congruent to triangle $A'B'C'$.

Because reflections are rigid motions.

Score 1: The student wrote an incomplete explanation.

Question 30

30 After a reflection over a line, $\triangle A'B'C'$ is the image of $\triangle ABC$. Explain why triangle ABC is congruent to triangle $A'B'C'$.



Score 0: The student did not provide an explanation.

#31

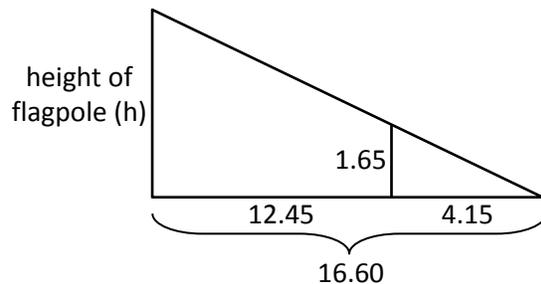
31 A flagpole casts a shadow 16.60 meters long. Tim stands at a distance of 12.45 meters from the base of the flagpole, such that the end of Tim’s shadow meets the end of the flagpole’s shadow. If Tim is 1.65 meters tall, determine and state the height of the flagpole to the *nearest tenth of a meter*.

Measured CCLS Cluster: G-SRT.B

Commentary: The question measures the knowledge and skills described by the standards within G-SRT.B because the student is required to apply similarity criteria to geometric figures to solve problems. The student must recognize that the situation can be modeled with two triangles that are similar by the AA similarity criteria, then apply that corresponding sides of similar triangles are proportional to solve the problem. The question is also an example of the instructional shift of coherence, as the student may draw on understandings from another cluster, G-SRT.C, in using right triangle trigonometry to find the height of the flagpole. The question also requires the student to employ Mathematical Practice 4, because the student must model with triangles to solve a real-world problem.

Rationale: This question asks students to find the height of a flagpole using its shadow length, the height of Tim, and his shadow length. This scenario can be modeled using two triangles which are similar by AA. Once similar triangles are determined, the height of the flagpole can be found using the corresponding sides of the similar triangles in a proportion.

$$\begin{aligned} \frac{16.60}{h} &= \frac{4.15}{1.65} \\ 4.15h &= 27.39 \\ h &= 6.6 \end{aligned}$$

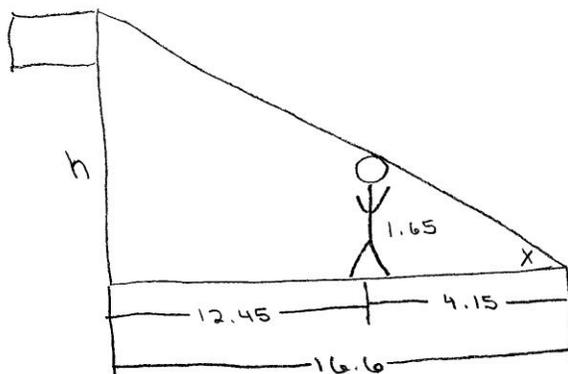


Compare with questions 8 and 11, which also assess G-SRT.B.

Sample student responses and scores appear on the following pages.

Question 31

- 31 A flagpole casts a shadow 16.60 meters long. Tim stands at a distance of 12.45 meters from the base of the flagpole, such that the end of Tim's shadow meets the end of the flagpole's shadow. If Tim is 1.65 meters tall, determine and state the height of the flagpole to the *nearest tenth of a meter*.



$$\tan x = \frac{1.65}{4.15}$$

$$x = 21.7$$

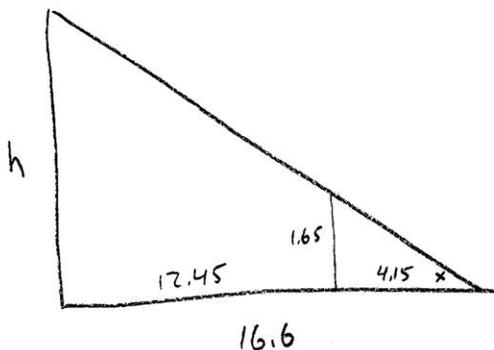
$$\tan 21.7 = \frac{h}{16.6}$$

$$h = 6.6$$

Score 2: The student has a complete and correct response.

Question 31

- 31 A flagpole casts a shadow 16.60 meters long. Tim stands at a distance of 12.45 meters from the base of the flagpole, such that the end of Tim's shadow meets the end of the flagpole's shadow. If Tim is 1.65 meters tall, determine and state the height of the flagpole to the *nearest tenth of a meter*.



$$\sin x = \frac{1.65}{4.15}$$

$$x = \sin^{-1}\left(\frac{1.65}{4.15}\right)$$

$$x = 23.427626509$$

$$16.6 \cdot \sin(23.427626509) = \frac{h}{16.6} \cdot 16.6$$

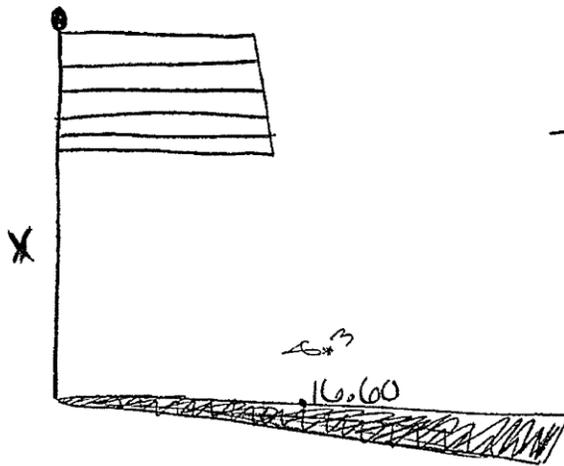
$$6.6 = h$$

6.6 meters

Score 1: The student made an error using the incorrect trigonometric function, and found an incorrect angle measure for x . The student made the same error in finding the height.

Question 31

- 31 A flagpole casts a shadow 16.60 meters long. Tim stands at a distance of 12.45 meters from the base of the flagpole, such that the end of Tim's shadow meets the end of the flagpole's shadow. If Tim is 1.65 meters tall, determine and state the height of the flagpole to the *nearest tenth of a meter*.



$$\frac{X}{1.65} = \frac{12.45}{16.60}$$

$$\frac{20.5425}{16.60} = \frac{16.60x}{16.60}$$

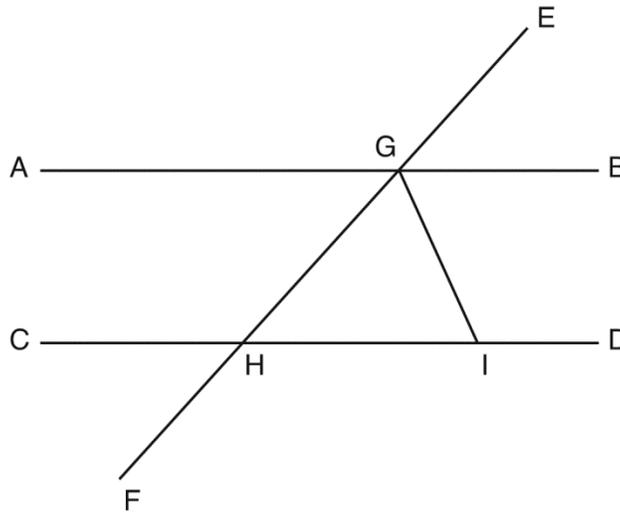
$$1.2375 = X$$

height of Flagpole = 1.2 meters long

Score 0: The student did not subtract 12.45 from 16.60. The student also wrote an incorrect proportion.

#32

32 In the diagram below, \overline{EF} intersects \overline{AB} and \overline{CD} at G and H , respectively, and \overline{GI} is drawn such that $\overline{GH} \cong \overline{IH}$.



If $m\angle EGB = 50^\circ$ and $m\angle DIG = 115^\circ$, explain why $\overline{AB} \parallel \overline{CD}$.

Measured CCLS Cluster: G-CO.C

Commentary: The question measures the knowledge and skills described by the standards within G-CO.C because the student is required to reason about lines and angles. The student must use theorems (e.g., linear pairs form supplementary angles, base angles of an isosceles triangle are equal, or if two lines are cut by a transversal and the corresponding angles are equal, the two lines are parallel) in order to explain why $\overline{AB} \parallel \overline{CD}$. Additionally, the item requires the student to employ Mathematical Practice 3, because the student must identify and explain evidence that will support the claim.

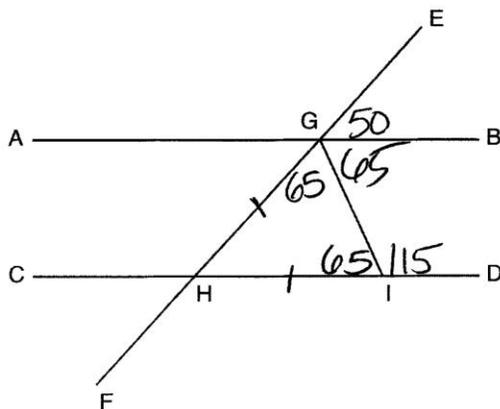
Rationale: This question asks students to explain why two lines are parallel given a diagram and two angle measures in the diagram. Since $\angle DIG$ and $\angle HIG$ are supplementary, then $m\angle HIG = 65^\circ$ because $m\angle DIG = 115^\circ$. Triangle GHI is an isosceles triangle because $\overline{GH} \cong \overline{IH}$. The base angles of the isosceles triangle are equal, so $m\angle HGI = 65^\circ$. The angles of triangle GHI add to 180° , so $m\angle GHI = 50^\circ$. Since the lines AB and CD are cut by a transversal and corresponding angles EGB and GHI are equal, then $\overline{AB} \parallel \overline{CD}$.

Compare with questions 13, 17, 26, and 33, which also assess G-CO.C.

Sample student responses and scores appear on the following pages.

Question 32

32 In the diagram below, \overline{EF} intersects \overline{AB} and \overline{CD} at G and H , respectively, and \overline{GI} is drawn such that $\overline{GH} \cong \overline{IH}$.



If $m\angle EGB = 50^\circ$ and $m\angle DIG = 115^\circ$, explain why $\overline{AB} \parallel \overline{CD}$.

$m\angle GIH = 65$ linear pairs are supplementary

$m\angle HGI = 65$ - Base angles of an isosceles triangle are equal

$$m\angle EGB + m\angle BGI + m\angle HGI = 180$$

$$50 + m\angle BGI + 65 = 180$$

$$115 + m\angle BGI = 180$$

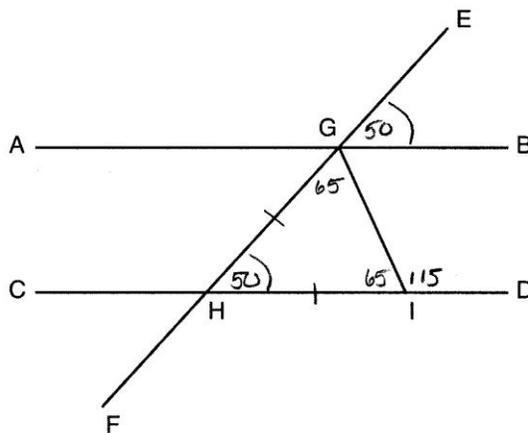
$$\begin{array}{r} 115 + m\angle BGI = 180 \\ -115 \qquad \qquad -115 \\ \hline m\angle BGI = 65 \end{array}$$

$\angle BGI$ and $\angle DIG$ are same-side interior \angle 's,
and since they are supplementary, $\overline{AB} \parallel \overline{CD}$.

Score 4: The student has a complete and correct response.

Question 32

32 In the diagram below, \overline{EF} intersects \overline{AB} and \overline{CD} at G and H , respectively, and \overline{GI} is drawn such that $\overline{GH} \cong \overline{HI}$.



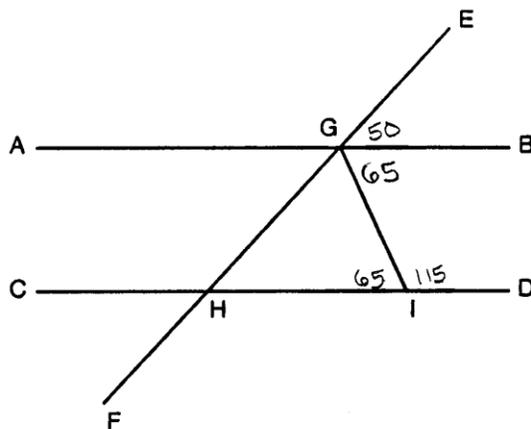
If $m\angle EGB = 50^\circ$ and $m\angle DIG = 115^\circ$, explain why $\overline{AB} \parallel \overline{CD}$.

$\angle DIG$ is supplementary to $\angle HIG$, so $m\angle HIG = 65$.
 $\angle HIG = \angle HGI$ because angles opposite equal sides are equal.
 The sum of angles of a triangle is 180° so $\angle GHI = 50$.
 So, $\overline{AB} \parallel \overline{CD}$.

Score 3: The student stated correct angle measures with explanations, but did not explain why $\overline{AB} \parallel \overline{CD}$.

Question 32

32 In the diagram below, \overline{EF} intersects \overline{AB} and \overline{CD} at G and H , respectively, and \overline{GI} is drawn such that $\overline{GH} \cong \overline{IH}$.



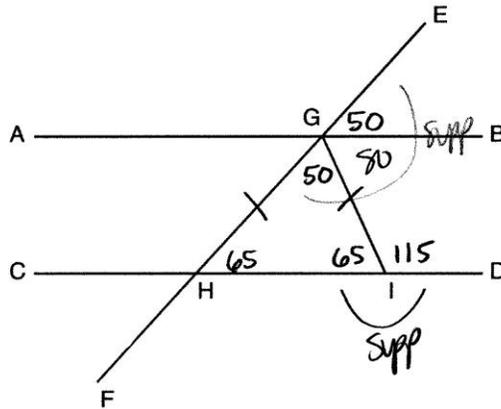
If $m\angle EGB = 50^\circ$ and $m\angle DIG = 115^\circ$, explain why $\overline{AB} \parallel \overline{CD}$.

$m\angle DIG + m\angle HIG = 180$ supplementary
 $m\angle HIG \cong m\angle BGI$ alternate interior
 $m\angle BGI + m\angle DIG = 180$ same side interior
 $65 + 115 = 180$
 $180 = 180$ lines parallel when
 same side interior
 angles add up to 180.

Score 2: The student made one conceptual error using alternate interior angles of parallel lines to prove the same lines parallel.

Question 32

32 In the diagram below, \overline{EF} intersects \overline{AB} and \overline{CD} at G and H , respectively, and \overline{GI} is drawn such that $\overline{GH} \cong \overline{IH}$.

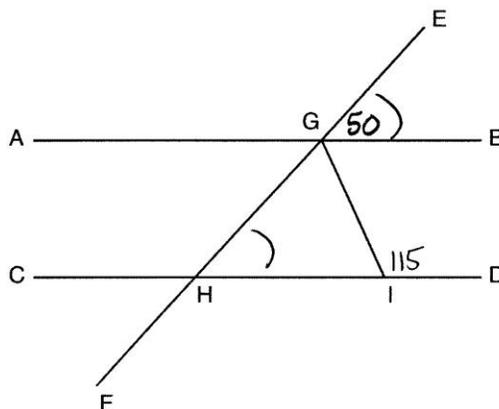


If $m\angle EGB = 50^\circ$ and $m\angle DIG = 115^\circ$, explain why $\overline{AB} \parallel \overline{CD}$.

Score 1: The student found appropriate angle measures based on a mislabeled diagram, and the explanation was missing.

Question 32

32 In the diagram below, \overline{EF} intersects \overline{AB} and \overline{CD} at G and H , respectively, and \overline{GI} is drawn such that $\overline{GH} \cong \overline{IH}$.



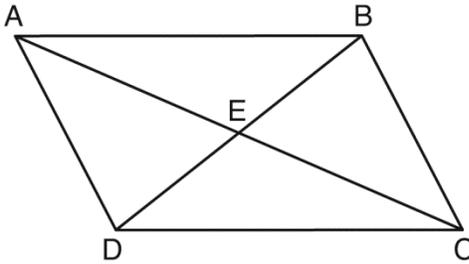
If $m\angle EGB = 50^\circ$ and $m\angle DIG = 115^\circ$, explain why $\overline{AB} \parallel \overline{CD}$.

Corresponding \angle s are \cong ,
so $AB \parallel CD$.

Score 0: The student did not show enough work on which to base the explanation.

#33

33 Given: Quadrilateral $ABCD$ is a parallelogram with diagonals \overline{AC} and \overline{BD} intersecting at E



Prove: $\triangle AED \cong \triangle CEB$

Describe a single rigid motion that maps $\triangle AED$ onto $\triangle CEB$.

Measured CCLS Cluster: G-CO.C

Commentary: The question measures the knowledge and skills described by the standards within G-CO.C because the student is required to reason about lines, angles, triangles and parallelograms. The student must construct a proof using theorems about these figures (e.g., the diagonals of a parallelogram bisect each other, parallel lines cut by a transversal form alternate interior angles, or vertical angles are congruent) to prove two triangles are congruent. The question is also an example of the instructional shift of coherence, as the student must draw on understandings from another cluster, G-CO.A, in describing the rigid motion that will map one triangle onto the other.

Rationale: This question asks students to prove triangles are congruent given a parallelogram with both diagonals drawn. The student must construct a proof using facts about parallelograms and parallel lines. An example is as follows:

$AD = BC$ because opposite sides of a parallelogram are equal in length. Additionally, $BE = DE$ and $CE = EA$ because the diagonals of a parallelogram bisect each other. Therefore, $\triangle AED \cong \triangle CEB$ by the SSS criterion.

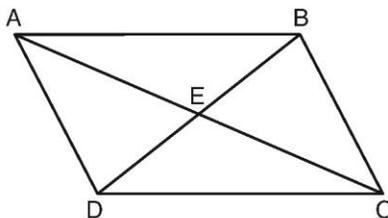
For the second part, the student must describe any valid single transformation that would map $\triangle AED$ onto $\triangle CEB$. An example of this is a rotation 180 degrees about point E .

Compare with questions 13, 17, 26, and 32, which also assess G-CO.C.

Sample student responses and scores appear on the following pages.

Question 33

33 Given: Quadrilateral $ABCD$ is a parallelogram with diagonals \overline{AC} and \overline{BD} intersecting at E



Prove: $\triangle AED \cong \triangle CEB$

Statement	Reason
1. Quad $ABCD$ is a parallelogram	1. given
2. $\overline{AD} \cong \overline{CB}$	2. opposite sides of parallelogram are congruent
3. \overline{AC} and \overline{DB} intersect at E	3. given
4. $\angle AED \cong \angle CEB$	4. vertical angles are congruent
5. $\overline{BC} \parallel \overline{DA}$	5. def. of \square
6. $\angle DBC \cong \angle BDA$	6. alt. interior angles are \cong
7. $\triangle AED \cong \triangle CEB$	7. AAS \cong AAS

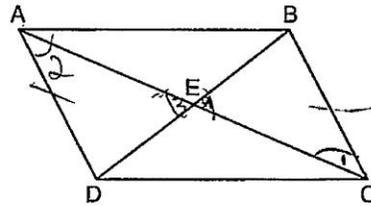
Describe a single rigid motion that maps $\triangle AED$ onto $\triangle CEB$.

Rotation of $\triangle AED$ around point E of 180°

Score 4: The student has a complete and correct proof, and a correct rigid motion is described.

Question 33

33 Given: Quadrilateral $ABCD$ is a parallelogram with diagonals \overline{AC} and \overline{BD} intersecting at E



Prove: $\triangle AED \cong \triangle CEB$

Statements	Reasons
1. Quadrilateral $ABCD$ is a \square with diagonals \overline{AC} + \overline{DB} intersecting at E	1. GN
2. $AD \cong BC$	2. Opposite sides of a \square are \cong
3. $AD \parallel BC$	3. Opposite sides of a \square are \parallel
4. $\angle 1 \cong \angle 2$	4. If 2 \parallel lines are cut by a transversal, the alternate interior \angle 's are \cong .
5. $\angle 3 \cong \angle 4$	5. Vertical \angle 's are \cong
6. $\triangle AED \cong \triangle BEC$	6. AAS \cong AAS

AAS

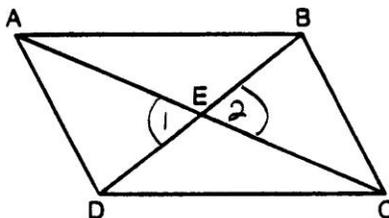
Describe a single rigid motion that maps $\triangle AED$ onto $\triangle CEB$.

Reflection

Score 3: The student wrote an incomplete description of the rigid motion.

Question 33

33 Given: Quadrilateral $ABCD$ is a parallelogram with diagonals \overline{AC} and \overline{BD} intersecting at E



Prove: $\triangle AED \cong \triangle CEB$

In a parallelogram, the diagonals bisect each other, so $\overline{AE} \cong \overline{CE}$ and $\overline{BE} \cong \overline{DE}$. $\angle 1 \cong \angle 2$. So $\triangle AED \cong \triangle CEB$ by SAS.

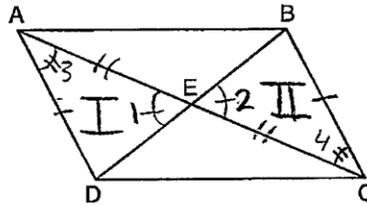
Describe a single rigid motion that maps $\triangle AED$ onto $\triangle CEB$.

180° rotation

Score 2: The student was missing the reason $\angle 1 \cong \angle 2$ and wrote an incomplete description of the rigid motion.

Question 33

33 Given: Quadrilateral $ABCD$ is a parallelogram with diagonals \overline{AC} and \overline{BD} intersecting at E



Prove: $\triangle AED \cong \triangle CEB$

Statement	Reason
1. $ABCD$ is a Parallelogram	1. Given
\overline{AC} + \overline{BD} intersect at E	2. verticle \angle s are \cong
2. $\angle 2 \cong \angle 1$	3. Opposite inverse are \cong
3. $\angle 3 \cong \angle 4$	4. Diagonals of a parallelogram are \cong
4. $\overline{AE} \cong \overline{EC}$	5. ASA
5. $\triangle I \cong \triangle II$	

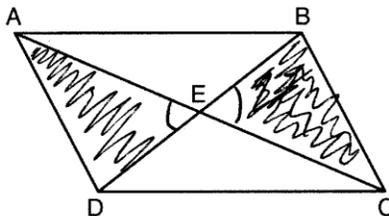
Describe a single rigid motion that maps $\triangle AED$ onto $\triangle CEB$.

Rotation 180°

Score 1: The student had some correct statements about the proof. The description of the rigid motion was incomplete.

Question 33

33 Given: Quadrilateral $ABCD$ is a parallelogram with diagonals \overline{AC} and \overline{BD} intersecting at E



Prove: $\triangle AED \cong \triangle CEB$

1) parallelogram $ABCD$ 1) given
diagonals \overline{AC} and \overline{BD}
intersecting at E

Prove: $\triangle AED \cong \triangle CEB$

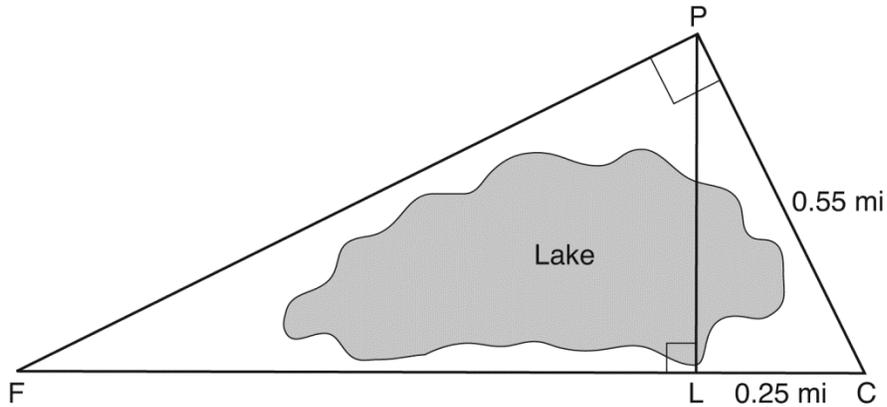
Describe a single rigid motion that maps $\triangle AED$ onto $\triangle CEB$.

Rotate 180°

Score 0: The student wrote only the “given” information, and an incomplete description of the rigid motion.

#34

34 In the diagram below, the line of sight from the park ranger station, P , to the lifeguard chair, L , on the beach of a lake is perpendicular to the path joining the campground, C , and the first aid station, F . The campground is 0.25 mile from the lifeguard chair. The straight paths from both the campground and first aid station to the park ranger station are perpendicular.



If the path from the park ranger station to the campground is 0.55 mile, determine and state, to the *nearest hundredth of a mile*, the distance between the park ranger station and the lifeguard chair.

Gerald believes the distance from the first aid station to the campground is at least 1.5 miles. Is Gerald correct? Justify your answer.

Measured CCLS Cluster: G-SRT.C

Commentary: The question measures the knowledge and skills described by the standards within G-SRT.C because the student is required to apply understanding of relationships between angles and sides in right triangles. Specifically, the student must use the Pythagorean Theorem to determine the distance between the park ranger station and the lifeguard chair. The question is also an example of the instructional shift of coherence, as the student must draw on understandings from another cluster, G-SRT.B, in using similarity to respond to Gerald’s claim that the distance from the first aid station to the campground is greater than 1.5 miles. In doing so, the question also requires the student to employ Mathematical Practice 4, because the student must model with triangles to solve a real-world problem.

Rationale:

The distance between the park ranger station and the lifeguard chair can be determined using the Pythagorean Theorem:

$$(0.25)^2 + (PL)^2 = (0.55)^2$$

$$(PL)^2 = 0.3025 - 0.0625$$

$$(PL)^2 = 0.24$$

$$PL \approx 0.49$$

To determine the distance from the first aid station to the campground, the student employed the understanding of similar right triangles by solving an appropriate proportion of corresponding sides.

$$\frac{FC}{0.55} = \frac{0.55}{0.25}$$

$$0.25(FC) = 0.3025$$

$$FC = 1.21$$

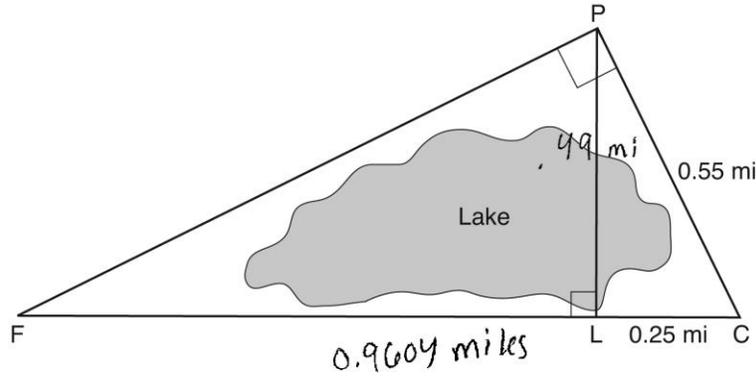
The total distance of 1.21 is less than 1.5, so Gerald is incorrect.

Compare with question 28, which also assesses G-SRT.C.

Sample student responses and scores appear on the following pages.

Question 34

34 In the diagram below, the line of sight from the park ranger station, P , to the lifeguard chair, L , on the beach of a lake is perpendicular to the path joining the campground, C , and the first aid station, F . The campground is 0.25 mile from the lifeguard chair. The straight paths from both the campground and first aid station to the park ranger station are perpendicular.



If the path from the park ranger station to the campground is 0.55 mile, determine and state, to the nearest hundredth of a mile, the distance between the park ranger station and the lifeguard chair.

$$a^2 + b^2 = c^2$$

$$a^2 + 0.0625 = 0.3025$$

$$a^2 = 0.24$$

$$a = 0.489897...$$

The distance is 0.49 miles

Gerald believes the distance from the first aid station to the campground is at least 1.5 miles. Is Gerald correct? Justify your answer.

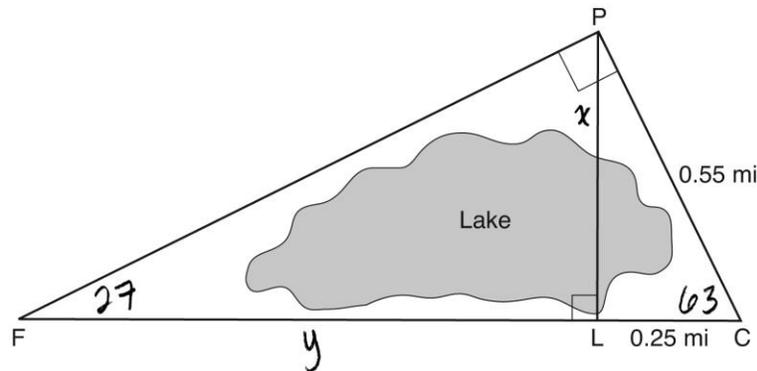
Altitude = $\frac{x}{h} = \frac{h}{y}$ $0.25y = 0.2401$ NO, the distance from F to L is 0.9604 miles. When added to the distance from L to C, it's only around 1.2 miles, not 1.5 miles.

$$\frac{0.25}{0.49} = \frac{0.49}{y}$$

Score 4: The student has a complete and correct response.

Question 34

- 34 In the diagram below, the line of sight from the park ranger station, P , to the lifeguard chair, L , on the beach of a lake is perpendicular to the path joining the campground, C , and the first aid station, F . The campground is 0.25 mile from the lifeguard chair. The straight paths from both the campground and first aid station to the park ranger station are perpendicular.



If the path from the park ranger station to the campground is 0.55 mile, determine and state, to the *nearest hundredth of a mile*, the distance between the park ranger station and the lifeguard chair.

$$\cos C = \frac{.25}{.55}$$

$$\tan 63 = \frac{x}{.25}$$

$$LC = 63^\circ$$

$$x = .4906$$

$$x = .49$$

Gerald believes the distance from the first aid station to the campground is at least 1.5 miles. Is Gerald correct? Justify your answer.

$$180 - 90 - 63 = 27$$

$$y = .96$$

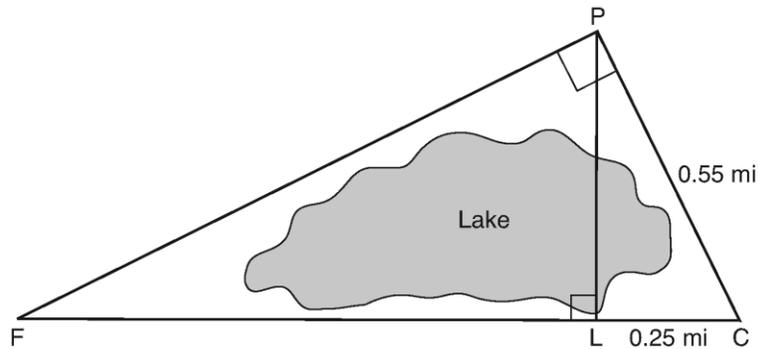
$$\tan 27 = \frac{.49}{y}$$

$$\frac{.49}{.96} + .25 = 1.21$$

Score 3: The student did not state if Gerald is correct.

Question 34

34 In the diagram below, the line of sight from the park ranger station, P , to the lifeguard chair, L , on the beach of a lake is perpendicular to the path joining the campground, C , and the first aid station, F . The campground is 0.25 mile from the lifeguard chair. The straight paths from both the campground and first aid station to the park ranger station are perpendicular.



If the path from the park ranger station to the campground is 0.55 mile, determine and state, to the nearest hundredth of a mile, the distance between the park ranger station and the lifeguard chair.

$$\frac{0.55}{x} = \frac{0.25}{0.55}$$

$$\frac{x}{0.96} = \frac{0.25}{x}$$

$$0.25 FC = 0.3025$$

$$FC = 1.21$$

$$x^2 = 0.24$$

$$x = 0.55$$

Distance between P and L = 0.5 mi.

Gerald believes the distance from the first aid station to the campground is at least 1.5 miles. Is Gerald correct? Justify your answer.

let x be FC

No because it is 1.21 miles. $\frac{0.55}{x} = \frac{0.25}{0.55}$

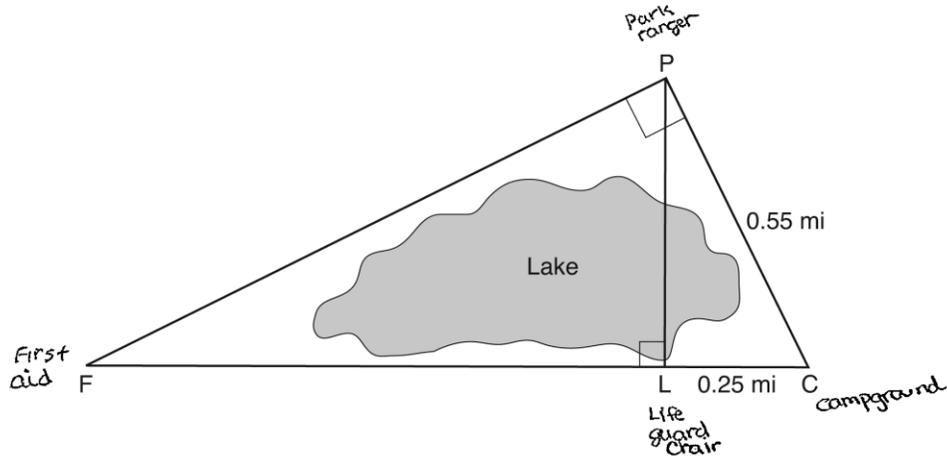
$$0.25 FC = 0.3025$$

$$FC = 1.21$$

Score 2: The student made one computational error and one rounding error in finding the distance between the park ranger station and the lifeguard chair.

Question 34

34 In the diagram below, the line of sight from the park ranger station, P , to the lifeguard chair, L , on the beach of a lake is perpendicular to the path joining the campground, C , and the first aid station, F . The campground is 0.25 mile from the lifeguard chair. The straight paths from both the campground and first aid station to the park ranger station are perpendicular.



If the path from the park ranger station to the campground is 0.55 mile, determine and state, to the *nearest hundredth of a mile*, the distance between the park ranger station and the lifeguard chair.

$$\begin{aligned}
 (0.25)^2 + b^2 &= (0.55)^2 \\
 0.0625 + b^2 &= .3025 \\
 \underline{- .0625} \quad \quad \underline{- .0625} \\
 b^2 &= \sqrt{.24} \\
 b &= 0.4898979486
 \end{aligned}$$

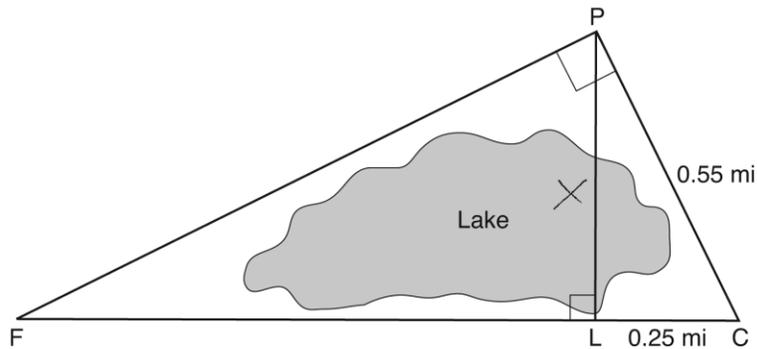
Distance \approx 0.5 miles

Gerald believes the distance from the first aid station to the campground is at least 1.5 miles. Is Gerald correct? Justify your answer.

Score 1: The student made one rounding error, and no further correct work was shown.

Question 34

- 34 In the diagram below, the line of sight from the park ranger station, P , to the lifeguard chair, L , on the beach of a lake is perpendicular to the path joining the campground, C , and the first aid station, F . The campground is 0.25 mile from the lifeguard chair. The straight paths from both the campground and first aid station to the park ranger station are perpendicular.



If the path from the park ranger station to the campground is 0.55 mile, determine and state, to the *nearest hundredth of a mile*, the distance between the park ranger station and the lifeguard chair.

$$\begin{aligned} .25^2 + .55^2 &= x^2 \\ .0625 + .3025 &= x^2 \\ .365 &= x^2 \\ \boxed{.6} &= x \end{aligned}$$

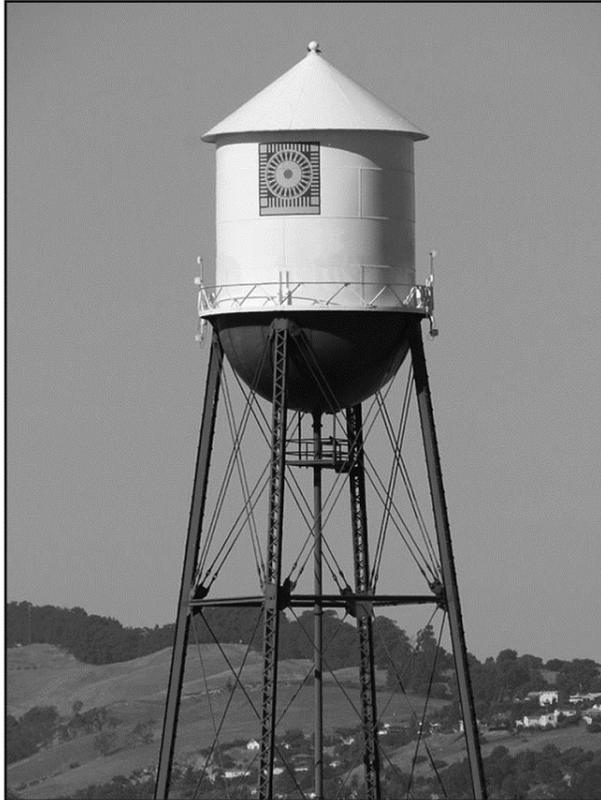
Gerald believes the distance from the first aid station to the campground is at least 1.5 miles. Is Gerald correct? Justify your answer.

Yes - its far away.

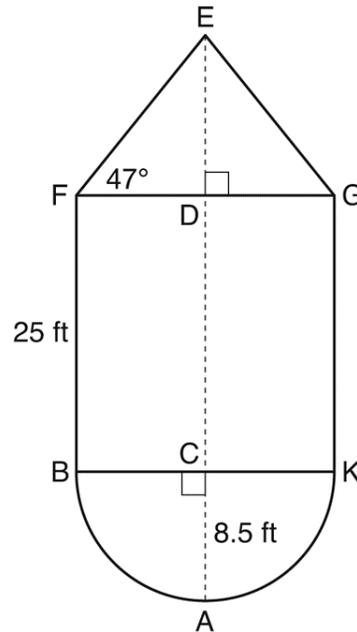
Score 0: The student had a completely incorrect response.

#35

35 The water tower in the picture below is modeled by the two-dimensional figure beside it. The water tower is composed of a hemisphere, a cylinder, and a cone. Let C be the center of the hemisphere and let D be the center of the base of the cone.



Source: <http://en.wikipedia.org>



If $AC = 8.5$ feet, $BF = 25$ feet, and $m\angle EFD = 47^\circ$, determine and state, to the *nearest cubic foot*, the volume of the water tower.

The water tower was constructed to hold a maximum of 400,000 pounds of water. If water weighs 62.4 pounds per cubic foot, can the water tower be filled to 85% of its volume and *not* exceed the weight limit? Justify your answer.

Measured CCLS Cluster: G-MG.A

Commentary: The question measures the knowledge and skills described by the standards within G-MG.A because the student is required to use volume and density to solve a real-world problem. The student must find the volume of a compound figure composed of a cone, a cylinder, and a hemisphere. Then the student must apply the concept of density to the volume to find the solution to the problem. In doing so, the question also requires the student to employ Mathematical Practice 4, because the student must model with geometric figures to solve a real-world problem.

Rationale: This question asks students to find the volume of the water tower and determine if the water tower can be filled to 85% capacity without exceeding the weight limit. The sum volumes of a cone ($V = \frac{1}{3}\pi r^2 h$), a cylinder ($V = \pi r^2 h$), and a hemisphere ($V = \frac{1}{2} \cdot \frac{4}{3}\pi r^3$) will be used to find the total volume of the water tower. A key piece is using right triangle trigonometry to find the height of the cone. The angle of incline of the cone and its radius can be used with the tangent to find the height of the cone.

<u>Height of the cone is h</u>	<u>Volume of cone</u>	<u>Volume of cylinder</u>	<u>Volume of hemisphere</u>
$\tan 47^\circ = \frac{h}{8.5}$	$V = \frac{1}{3}\pi r^2 h$	$V = \pi r^2 h$	$V = \frac{1}{2} \cdot \frac{4}{3}\pi r^3$
$h = 8.5(\tan 47^\circ)$	$V = \frac{1}{3}\pi 8.5^2 \cdot 9.11513$	$V = \pi 8.5^2 \cdot 25$	$V = \frac{1}{2} \cdot \frac{4}{3}\pi 8.5^3$
$h = 9.11513$	$V = 689.6509461$	$V = 5674.501731$	$V = 1286.220392$

Total volume of the water tower is 7650 cubic feet.

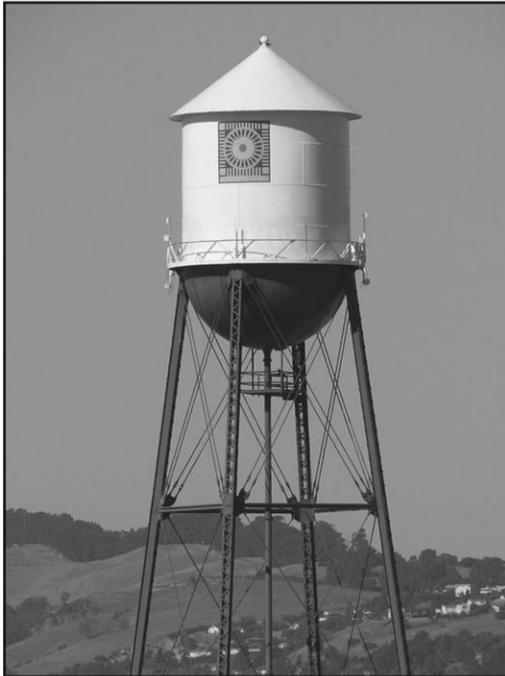
The water tower can hold a maximum of 400,000 pounds of water. If the tower were filled to 85% capacity and water weighs 62.4 pounds per cubic foot then, $62.4(7650)(0.85) = 405,756$ pounds of water. So, the water tower cannot be filled to 85% capacity because it would exceed the weight limit by 5,756 pounds.

Compare with question 7, which also assesses G-MG.A.

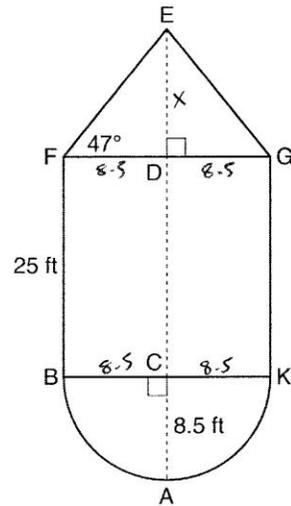
Sample student responses and scores appear on the following pages.

Question 35

35 The water tower in the picture below is modeled by the two-dimensional figure beside it. The water tower is composed of a hemisphere, a cylinder, and a cone. Let C be the center of the hemisphere and let D be the center of the base of the cone.



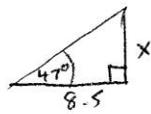
Source: <http://en.wikipedia.org>



Question 35 is continued on the next page.

Question 35 continued

If $AC = 8.5$ feet, $BF = 25$ feet, and $m\angle EFD = 47^\circ$, determine and state, to the nearest cubic foot, the volume of the water tower.



	Volume cone	Volume cylinder	Volume Hemisphere
$\tan 47^\circ = \frac{x}{8.5}$	$V = \frac{1}{3}\pi r^2 h$	$V = \pi r^2 h$	$V = \frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$
$x = 8.5 \tan 47^\circ$	$= \frac{1}{3}\pi (8.5)^2 (9.11513)$	$V = \pi (8.5)^2 (25)$	$= \frac{2}{3}\pi (8.5)^3$
$x = 9.11513$	$V = 689.65125$	$V = 5674.50173$	$= 1286.22039$

$$V = 689.65125 + 5674.50173 + 1286.22039 = 7650.37337$$

$$= \boxed{7650 \text{ ft}^3}$$

The water tower was constructed to hold a maximum of 400,000 pounds of water. If water weighs 62.4 pounds per cubic foot, can the water tower be filled to 85% of its volume and *not* exceed the weight limit? Justify your answer.

$$7650 \times 62.4 = 477,360 \text{ lbs}$$

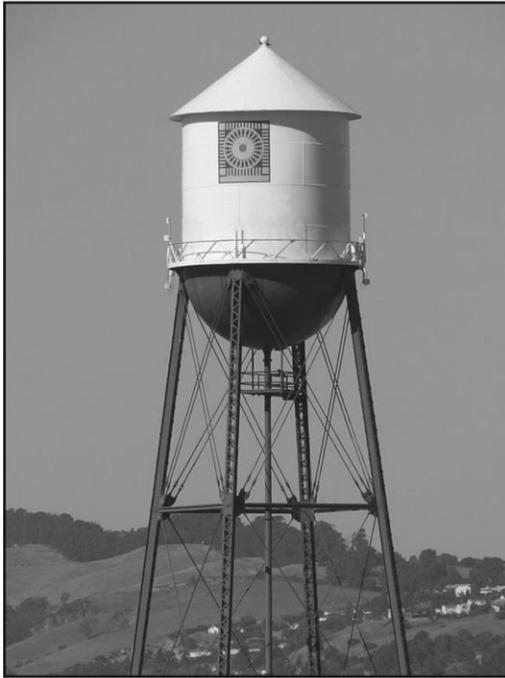
$$.85 \times 477,360 = \boxed{405,756 \text{ lbs}}$$

No - the weight would exceed 400,000 lbs

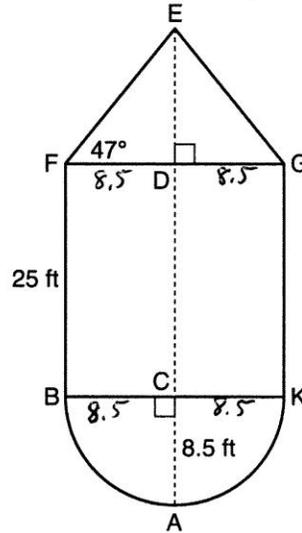
Score 6: The student had a complete and correct response.

Question 35

35 The water tower in the picture below is modeled by the two-dimensional figure beside it. The water tower is composed of a hemisphere, a cylinder, and a cone. Let C be the center of the hemisphere and let D be the center of the base of the cone.



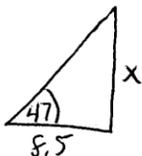
Source: <http://en.wikipedia.org>



Question 35 is continued on the next page.

Question 35 continued

If $AC = 8.5$ feet, $BF = 25$ feet, and $m\angle EFD = 47^\circ$, determine and state, to the nearest cubic foot, the volume of the water tower.



$$\tan 47 = \frac{x}{8.5}$$

$$x = 9.115$$

$$V = \frac{1}{3}\pi r^2 h + \pi r^2 h + \frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$$

$$V = \frac{1}{3}(3.14)(8.5)^2(9.115) + 3.14(8.5)^2(25) + \frac{1}{2} \cdot \frac{4}{3}(3.14)(8.5)^3$$

$$= 689.2914917 + 5671.625 + 1285.568333$$

$$= 7646.484825$$

$$V = 7646$$

The water tower was constructed to hold a maximum of 400,000 pounds of water. If water weighs 62.4 pounds per cubic foot, can the water tower be filled to 85% of its volume and *not* exceed the weight limit? Justify your answer.

$$7646(62.4) = 477,110.4 \text{ pounds}$$

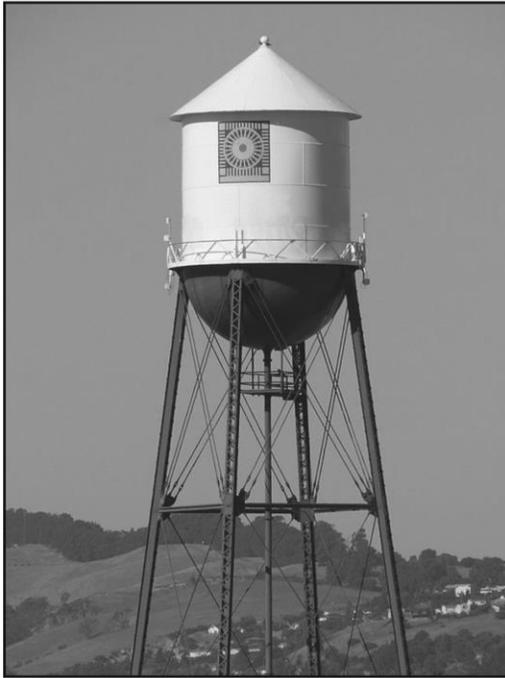
$$477,110.4(.85) = 405,543.84 \text{ pounds}$$

No because it would exceed 400,000 pounds

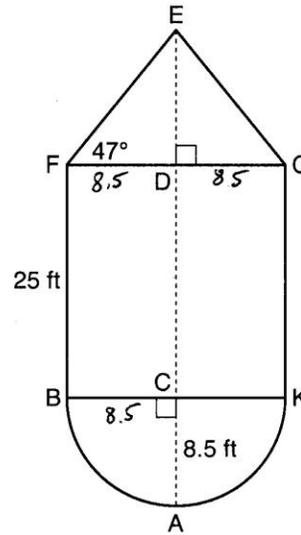
Score 5: The student used 3.14 instead of π to calculate the volume.

Question 35

35 The water tower in the picture below is modeled by the two-dimensional figure beside it. The water tower is composed of a hemisphere, a cylinder, and a cone. Let C be the center of the hemisphere and let D be the center of the base of the cone.



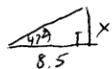
Source: <http://en.wikipedia.org>



Question 35 is continued on the next page.

Question 35 continued

If $AC = 8.5$ feet, $BF = 25$ feet, and $m\angle EFD = 47^\circ$, determine and state, to the nearest cubic foot, the volume of the water tower.



$$\tan 47 = \frac{x}{8.5}$$

$$x = 8.5 \tan 47$$

$$x = 9.1151$$

$$\boxed{x = 9.1}$$

Volume Hemisphere

$$V = \frac{1}{2} \left(\frac{4}{3} \pi r^3 \right)$$

$$= \frac{2}{3} \pi (8.5)^3$$

$$V = 1286.22039$$

Volume Cylinder

$$V = \pi r^2 h$$

$$V = \pi (8.5^2) (25)$$

$$V = 5674.5017$$

Volume Cone

$$V = \frac{1}{3} \pi r^2 h$$

$$V = \frac{1}{3} \pi (8.5)^2 (9.1)$$

$$V = 688.5062$$

$$V = 1286.2704 + 5674.5017 + 688.5062$$

$$= 7649.2183$$

$$V = \boxed{7649 \text{ ft}^3}$$

The water tower was constructed to hold a maximum of 400,000 pounds of water. If water weighs 62.4 pounds per cubic foot, can the water tower be filled to 85% of its volume and *not* exceed the weight limit? Justify your answer.

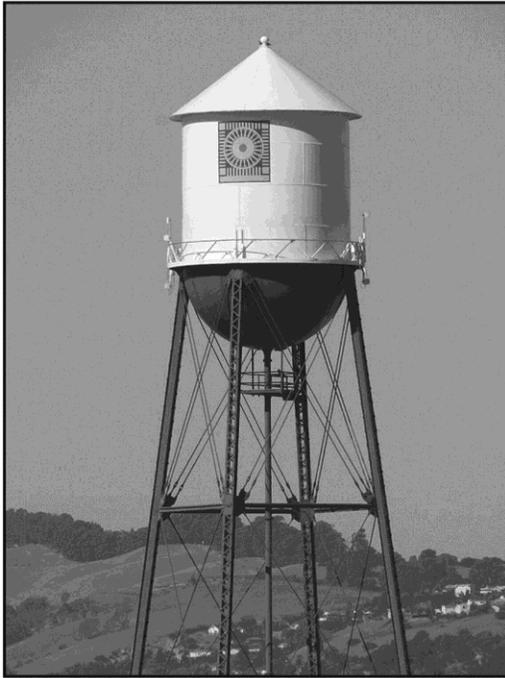
$$7649 \times 62.4 = 477,297.6 \text{ lbs.}$$

$$477,297.6 \times .85 = \boxed{405,702.96} \text{ lbs.}$$

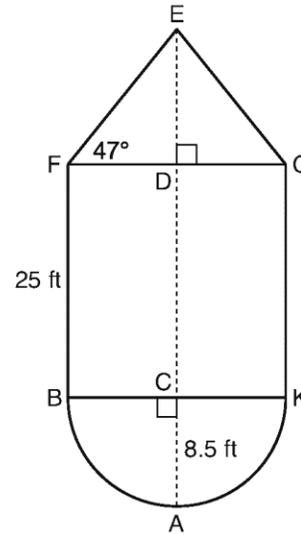
Score 4: The student rounded early with $x = 9.1$, and did not state if the water tower can be filled to 85%.

Question 35

35 The water tower in the picture below is modeled by the two-dimensional figure beside it. The water tower is composed of a hemisphere, a cylinder, and a cone. Let C be the center of the hemisphere and let D be the center of the base of the cone.



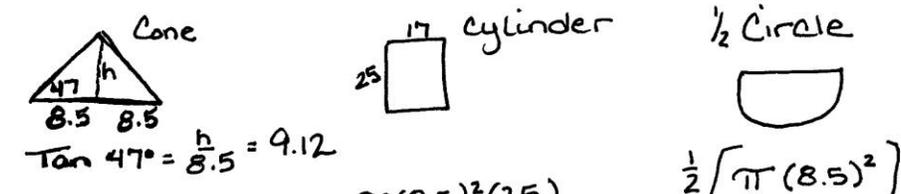
Source: <http://en.wikipedia.org>



Question 35 is continued on the next page.

Question 35 continued

If $AC = 8.5$ feet, $BF = 25$ feet, and $m\angle EFD = 47^\circ$, determine and state, to the nearest cubic foot, the volume of the water tower.



$\tan 47^\circ = \frac{h}{8.5} = 9.12$
 $\frac{1}{3}\pi(8.5)^2(9.12) = 219.64\pi$

$\pi(8.5)^2(25) = 1806.25\pi$

$\frac{1}{2}[\pi(8.5)^2] = 36.125\pi$

$219.64\pi + 1806.25\pi + 36.125\pi = 2062.015\pi$
 6478.01
 $\boxed{6478}$

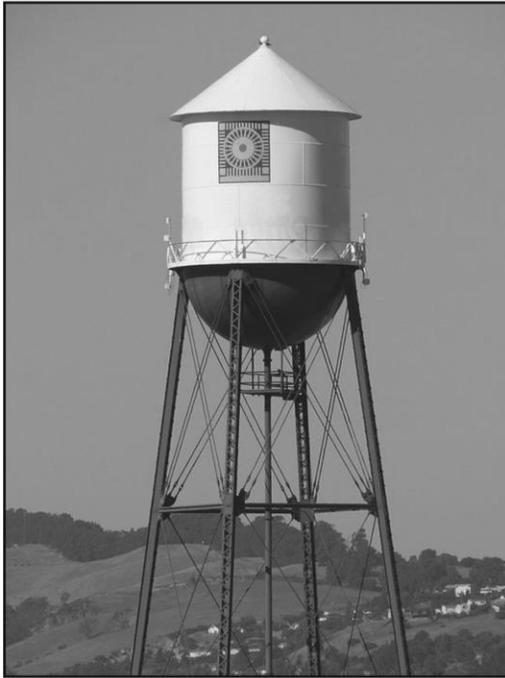
The water tower was constructed to hold a maximum of 400,000 pounds of water. If water weighs 62.4 pounds per cubic foot, can the water tower be filled to 85% of its volume and *not* exceed the weight limit? Justify your answer.

$(.85)(6478) = 5506.3$
 $(5506.3)(62.4) = 343,593.12$
 yes because less than 400,000

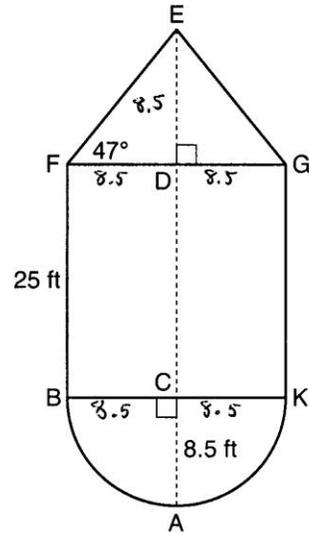
Score 3: The student made one conceptual error by finding the area of half of a circle instead of the volume of a hemisphere. The height of the cone was rounded incorrectly. The student used the answer from the first part to answer the second part appropriately.

Question 35

35 The water tower in the picture below is modeled by the two-dimensional figure beside it. The water tower is composed of a hemisphere, a cylinder, and a cone. Let C be the center of the hemisphere and let D be the center of the base of the cone.



Source: <http://en.wikipedia.org>



Question 35 is continued on the next page.

Question 35 continued

If $AC = 8.5$ feet, $BF = 25$ feet, and $m\angle EFD = 47^\circ$, determine and state, to the nearest cubic foot, the volume of the water tower.

$$V = \frac{1}{3} \pi r^2 h + \pi r^2 h + \frac{4}{3} \pi r^3$$

$$V = \frac{1}{3} \pi (8.5)^2 (8.5) + \pi (8.5)^2 (25) + \frac{4}{3} \pi (8.5)^3$$

$$V = \frac{1}{3} \pi (614.125) + \pi (1806.25) + \frac{4}{3} \pi (72.25)$$

$$V = 643.1101961 + 5674.501731 + 302.6\overset{4}{00923}$$

$$V = 6620.252019$$

6620

The water tower was constructed to hold a maximum of 400,000 pounds of water. If water weighs 62.4 pounds per cubic foot, can the water tower be filled to 85% of its volume and *not* exceed the weight limit? Justify your answer.

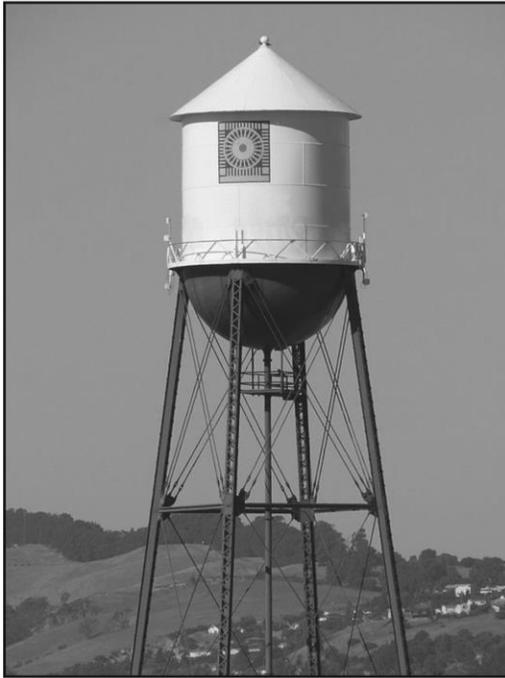
$$6620 (.85) = 5627$$

$$5627 (62.4) = 351,124.8$$

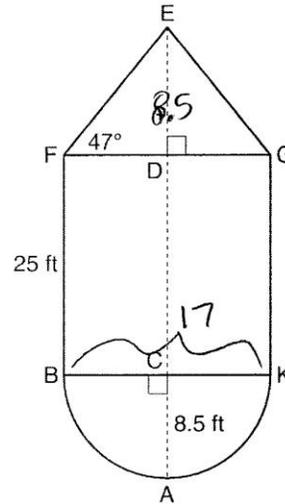
Score 2: The student made one conceptual error by using 8.5 for the height of the cone, and made an error by not dividing the volume of the sphere by 2. The student did not state if the water tower can be filled to 85%.

Question 35

35 The water tower in the picture below is modeled by the two-dimensional figure beside it. The water tower is composed of a hemisphere, a cylinder, and a cone. Let C be the center of the hemisphere and let D be the center of the base of the cone.



Source: <http://en.wikipedia.org>



Question 35 is continued on the next page.

Question 35 continued

If $AC = 8.5$ feet, $BF = 25$ feet, and $m\angle EFD = 47^\circ$, determine and state, to the nearest cubic foot, the volume of the water tower.

Cone	Cylinder
$V = \frac{1}{3}\pi r^2 h$	$\pi r^2 h$
$V = \frac{1}{3}\pi (8.5)^2 (8.5)$	$\pi (8.5)^2 (33.5)$
$V = 643.1$	7603.8

$$V = 8247$$

The water tower was constructed to hold a maximum of 400,000 pounds of water. If water weighs 62.4 pounds per cubic foot, can the water tower be filled to 85% of its volume and *not* exceed the weight limit? Justify your answer.

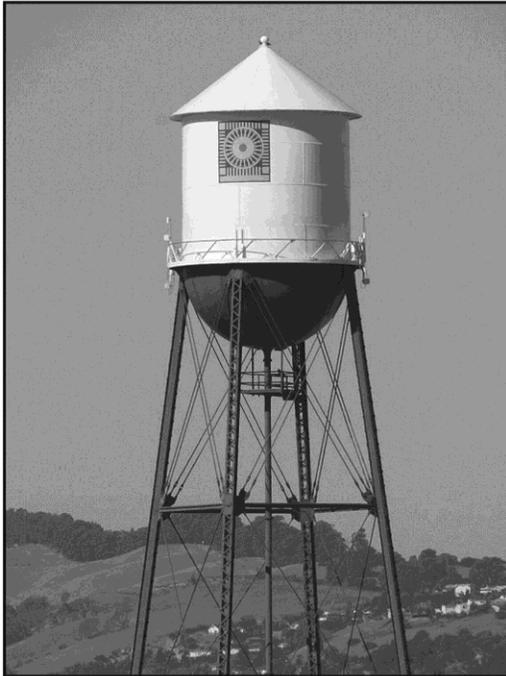
$$8247 \times 62.4 = 514,612.8 \text{ lbs}$$

NO

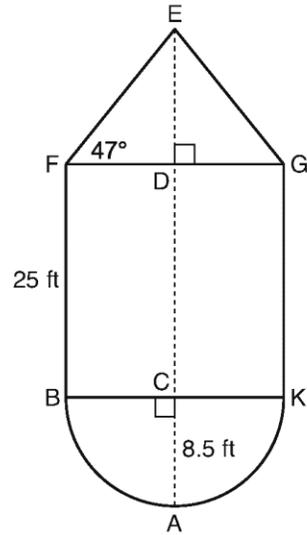
Score 1: The student made two conceptual errors in finding the volume of the water tower and one computational error by not multiplying by 85%.

Question 35

35 The water tower in the picture below is modeled by the two-dimensional figure beside it. The water tower is composed of a hemisphere, a cylinder, and a cone. Let C be the center of the hemisphere and let D be the center of the base of the cone.



Source: <http://en.wikipedia.org>



Question 35 is continued on the next page.

Question 35 continued

If $AC = 8.5$ feet, $BF = 25$ feet, and $m\angle EFD = 47^\circ$, determine and state, to the *nearest cubic foot*, the volume of the water tower.

$$\begin{array}{r} 17\pi \\ \hline | 8.5 \\ | 25 \\ | 8.5 \end{array} \left. \vphantom{\begin{array}{r} 17\pi \\ \hline | 8.5 \\ | 25 \\ | 8.5 \end{array}} \right\} (42)(17)^2\pi$$
$$12138\pi = 38,32.65$$

The water tower was constructed to hold a maximum of 400,000 pounds of water. If water weighs 62.4 pounds per cubic foot, can the water tower be filled to 85% of its volume and *not* exceed the weight limit? Justify your answer.

$$(400,000)(.85) = 340,000$$

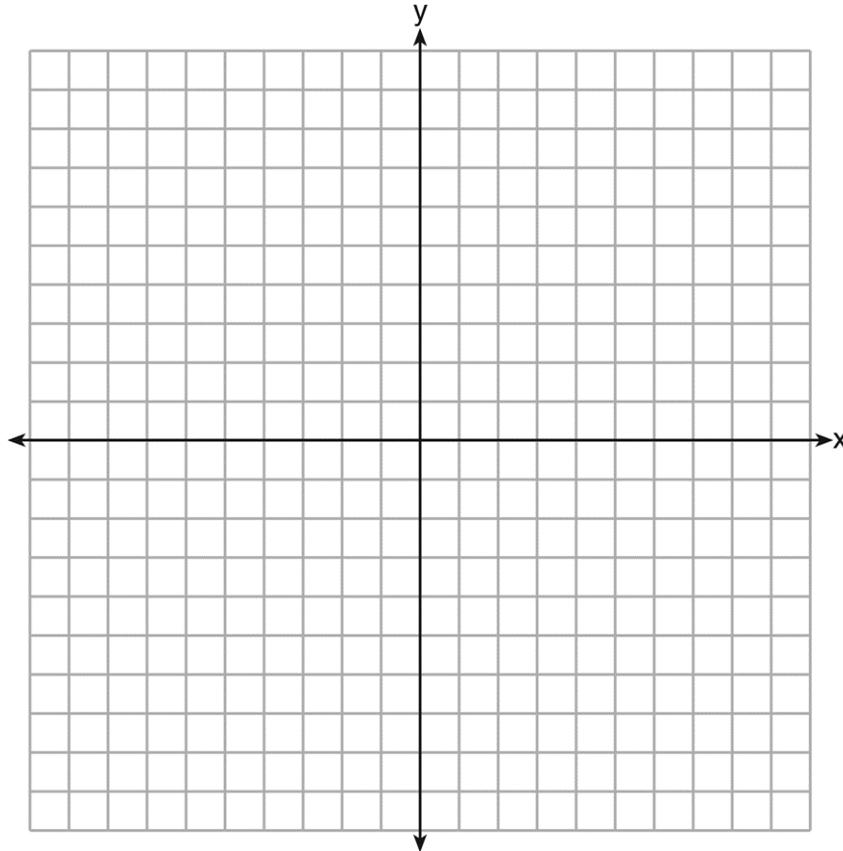
Score 0: The student had a completely incorrect response.

#36

36 In the coordinate plane, the vertices of $\triangle RST$ are $R(6, -1)$, $S(1, -4)$, and $T(-5, 6)$.
Prove that $\triangle RST$ is a right triangle.
[The use of the set of axes on the next page is optional.]

State the coordinates of point P such that quadrilateral $RSTP$ is a rectangle.

Prove that your quadrilateral $RSTP$ is a rectangle.
[The use of the set of axes below is optional.]



Measured CCLS Cluster: G-GPE.B

Commentary: The question measures the knowledge and skills described by the standards within G-GPE.B because the student is required to use coordinates to apply understanding of geometric figures. The student must use coordinates to prove a triangle is a right triangle and then determine the coordinates of a fourth point such that the three vertices of the right triangle and the fourth point are the four points of a rectangle. The student must prove this quadrilateral is a rectangle. In doing so, the question also requires the student to employ Mathematical Practice 3, because the student must construct a complete line of reasoning to prove an assertion.

Rationale: This question asks students to prove that the given triangle is a right triangle. The student must also determine the coordinates of a fourth point such that the three vertices of the right triangle and the fourth point are the four points of a rectangle and prove it is a rectangle. These parts can be accomplished several different ways such as using slopes, distances, and/or the midpoints of the diagonals.

The slope of \overline{SR} is $\frac{3}{5}$.

The slope of \overline{ST} is $-\frac{10}{6} = -\frac{5}{3}$.

The slopes of sides \overline{SR} and \overline{ST} are negative reciprocals, therefore $\overline{SR} \perp \overline{ST}$. Since perpendicular lines form right angles, then $\angle S$ is a right angle. Since $\triangle RST$ has a right angle, $\triangle RST$ is a right triangle.

The coordinates of point P that make $RSTP$ a rectangle are $(0,9)$.

The slope of \overline{TP} is $\frac{3}{5}$.

The slope of \overline{RP} is $-\frac{10}{6} = -\frac{5}{3}$.

Using $P(0,9)$, both pairs of opposite sides of $RSTP$ have the same slope, so $\overline{TP} \parallel \overline{RS}$ and $\overline{RP} \parallel \overline{ST}$. Since both pairs of opposite sides of $RSTP$ are parallel, $RSTP$ is a parallelogram. Since $RSTP$ is a parallelogram and has a right angle at vertex S , then $RSTP$ must be a rectangle.

Compare with question 27, which also assesses G-GPE.B.

Sample student responses and scores appear on the following pages.

Question 36

36 In the coordinate plane, the vertices of $\triangle RST$ are $R(6,-1)$, $S(1,-4)$, and $T(-5,6)$.

Prove that $\triangle RST$ is a right triangle.

[The use of the set of axes on the next page is optional.]

$$m_{\overline{RS}} = \frac{3}{5}$$

$$m_{\overline{ST}} = \frac{-10}{6} = -\frac{5}{3}$$

Therefore the slopes of \overline{RS} and \overline{ST} are negative reciprocals and so $\overline{RS} \perp \overline{ST}$. Since the segments are \perp , $\triangle RST$ is a rt \triangle .

$\therefore \triangle RST$ is a rt \triangle because it has 1 rt \angle .

State the coordinates of point P such that quadrilateral $RSTP$ is a rectangle.

$(0,9)$

Question 36 is continued on the next page.

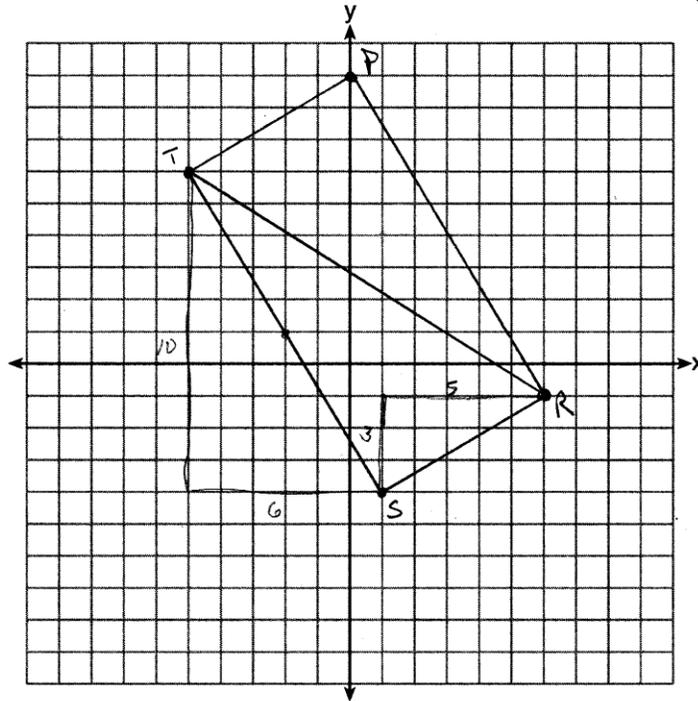
Question 36 continued

Prove that your quadrilateral $RSTP$ is a rectangle.
 [The use of the set of axes below is optional.]

$$\left. \begin{array}{l} m_{\overline{RS}} = \frac{3}{5} \\ m_{\overline{PT}} = \frac{3}{5} \end{array} \right\} \therefore \overline{RS} \parallel \overline{PT}$$

$$\left. \begin{array}{l} m_{\overline{ST}} = \frac{-10}{6} = \frac{-5}{3} \\ m_{\overline{RP}} = \frac{-10}{6} = \frac{-5}{3} \end{array} \right\} \therefore \overline{ST} \parallel \overline{RP}$$

Since $RSTP$ is a quadrilateral with both pairs of opposite sides \parallel and one \angle at S , it must be a rectangle.



Score 6: The student has a complete and correct response.

Question 36

36 In the coordinate plane, the vertices of $\triangle RST$ are $R(6,-1)$, $S(1,-4)$, and $T(-5,6)$.

Prove that $\triangle RST$ is a right triangle.

[The use of the set of axes on the next page is optional.]

Slopes

$$\overline{TS} = -\frac{10}{6} = -\frac{5}{3}$$

$$\overline{SR} = \frac{3}{5}$$

$\overline{TS} \perp \overline{SR}$ because their slopes are negative reciprocals of each other. $\triangle RST$ is a right \triangle because \perp lines form rt. \triangle , $\triangle RST$ is a right \triangle because it has 1 right \angle .

State the coordinates of point P such that quadrilateral $RSTP$ is a rectangle.

$$P(0,9)$$

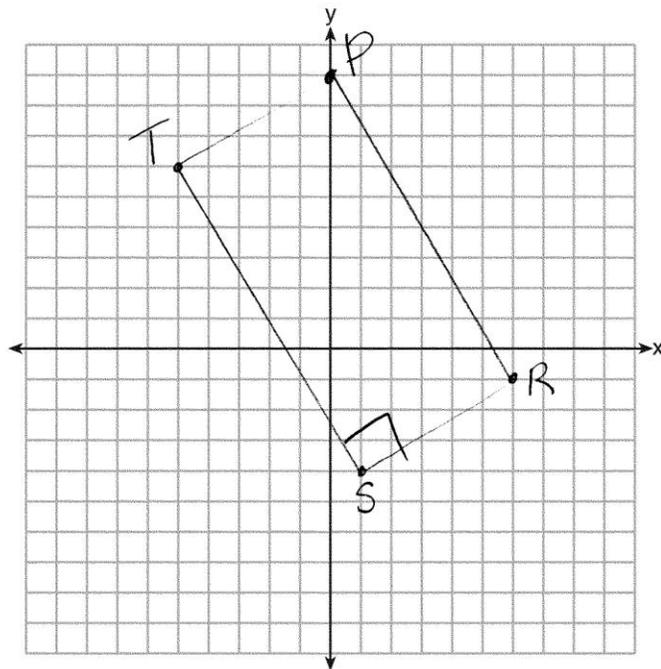
Question 36 is continued on the next page.

Question 36 continued

Prove that your quadrilateral $RSTP$ is a rectangle.
[The use of the set of axes below is optional.]

$m_{TP} = 3/5$
 $m_{SR} = 3/5$
 $m_{TS} = -5/3$
 $m_{PR} = -5/3$

Opposite sides are parallel
because they have the same
slope. $RSTP$ is a parallelogram
because opposite sides are
parallel.



Score 5: The student proved $RSTP$ is a parallelogram, but did not have a concluding statement proving $RSTP$ is a rectangle.

Question 36

36 In the coordinate plane, the vertices of $\triangle RST$ are $R(6,-1)$, $S(1,-4)$, and $T(-5,6)$.

Prove that $\triangle RST$ is a right triangle.

[The use of the set of axes on the next page is optional.]

$$\text{slope } \overline{RS} = \frac{3}{5} \quad \text{slope } \overline{TS} = \frac{-10}{6} = -\frac{5}{3}$$

$\overline{RS} \perp \overline{TS}$ since they have negative reciprocal slopes.

Therefore $\triangle RST$ is a right \triangle .

Since $\triangle RST$ contains a right \angle , it is a right \triangle .

State the coordinates of point P such that quadrilateral $RSTP$ is a rectangle.

$$P(0,9)$$

Question 36 is continued on the next page.

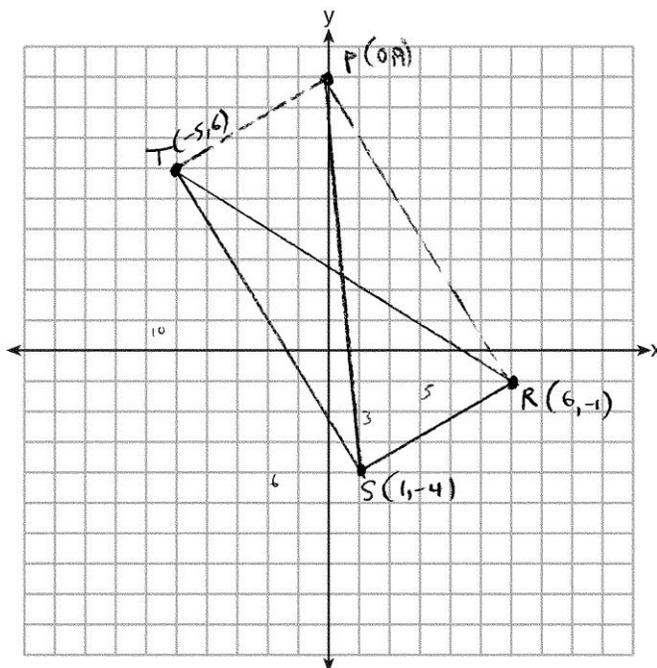
Question 36 continued

Prove that your quadrilateral $RSTP$ is a rectangle.
 [The use of the set of axes below is optional.]

$$\begin{aligned} \text{Length } \overline{RT} &= \sqrt{7^2 + 11^2} \\ &= \sqrt{49 + 121} \\ RT &= \sqrt{170} \end{aligned}$$

$$\begin{aligned} \text{Length } \overline{PS} &= \sqrt{13^2 + 1^2} \\ &= \sqrt{169 + 1} \\ PS &= \sqrt{170} \end{aligned}$$

Since the diagonals of $RSTP$ are \cong , then it is a rectangle.



Score 4: The student made one conceptual error when proving the rectangle, because no work is shown to prove that $RSTP$ is a parallelogram.

Question 36

36 In the coordinate plane, the vertices of $\triangle RST$ are $R(6, -1)$, $S(1, -4)$, and $T(-5, 6)$.
Prove that $\triangle RST$ is a right triangle.

[The use of the set of axes on the next page is optional.]

sides are \perp
because their
slopes are
negative
reciprocals

$$\begin{cases} m(TS) = -\frac{10}{6} = -\frac{5}{3} \\ m(SR) = \frac{3}{5} \end{cases}$$

\perp lines form right angles

$\triangle RST$ is a right \triangle
because it has a
right \angle .

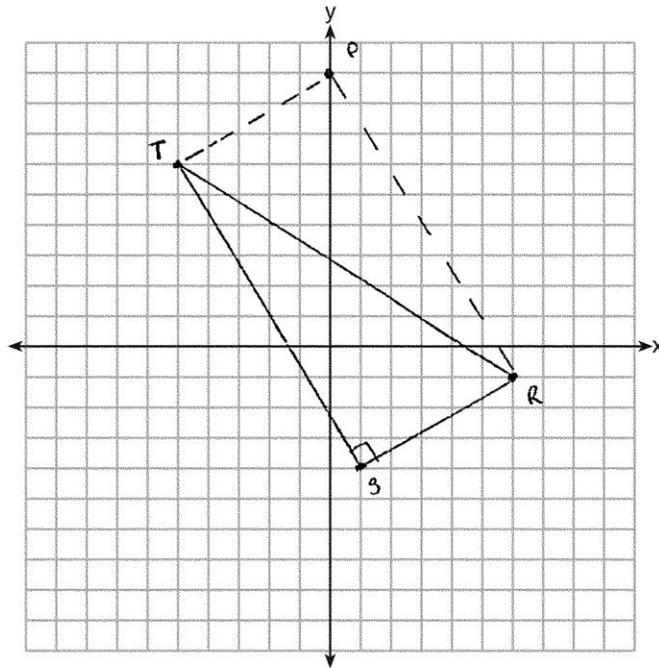
State the coordinates of point P such that quadrilateral $RSTP$ is a rectangle.

$P(0, 4)$

Question 36 is continued on the next page.

Question 36 continued

Prove that your quadrilateral $RSTP$ is a rectangle.
[The use of the set of axes below is optional.]



Score 3: The student correctly proved the right triangle and stated the coordinates of P , but no further correct work was shown.

Question 36

36 In the coordinate plane, the vertices of $\triangle RST$ are $R(6, -1)$, $S(1, -4)$, and $T(-5, 6)$.

Prove that $\triangle RST$ is a right triangle.

[The use of the set of axes on the next page is optional.]

$$R(6, -1)$$

$$S(1, -4)$$

$$T(-5, 6)$$

$$d_{RS} = \sqrt{(6-1)^2 + (-1+4)^2} = \sqrt{25+9} = \sqrt{34}$$

$$d_{ST} = \sqrt{(1+5)^2 + (-4-6)^2} = \sqrt{36+100} = \sqrt{136}$$

$$d_{RT} = \sqrt{(6+5)^2 + (-1-6)^2} = \sqrt{121+49} = \sqrt{170}$$

$$(RS)^2 + (ST)^2 \stackrel{?}{=} (RT)^2$$

$$\frac{(\sqrt{34})^2 + (\sqrt{136})^2}{34 + 136} \quad \left| \quad \frac{(\sqrt{170})^2}{170} \right.$$

$$34 + 136$$

$$170 = 170$$

State the coordinates of point P such that quadrilateral $RSTP$ is a rectangle.

$$(0, 9)$$

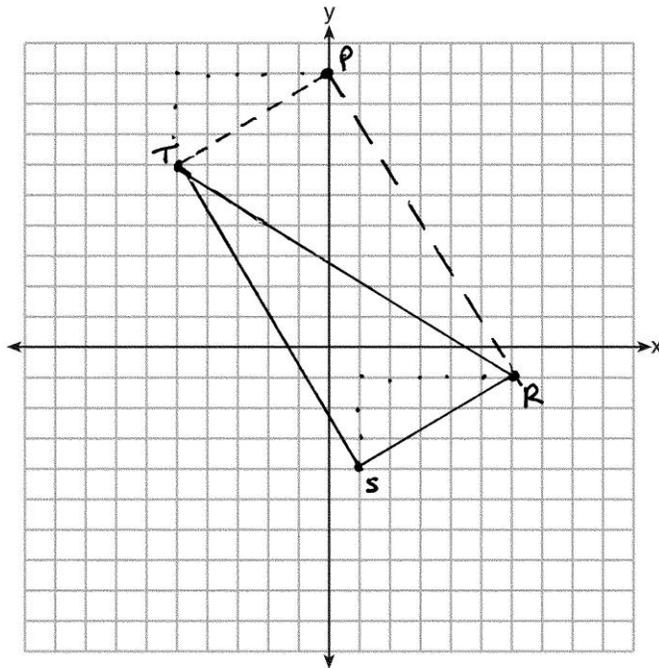
Question 36 is continued on the next page.

Question 36 continued

Prove that your quadrilateral $RSTP$ is a rectangle.
[The use of the set of axes below is optional.]

$\angle P$ is $Rt \angle$

$RSTP$ is Rectangle because opposite
 \angle 's are Right \angle 's



Score 2: The student was missing a concluding statement when proving the right triangle, and the coordinates of P were correctly stated, but no further correct work is shown.

Question 36

- 36** In the coordinate plane, the vertices of $\triangle RST$ are $R(6,-1)$, $S(1,-4)$, and $T(-5,6)$.
Prove that $\triangle RST$ is a right triangle.
[The use of the set of axes on the next page is optional.]

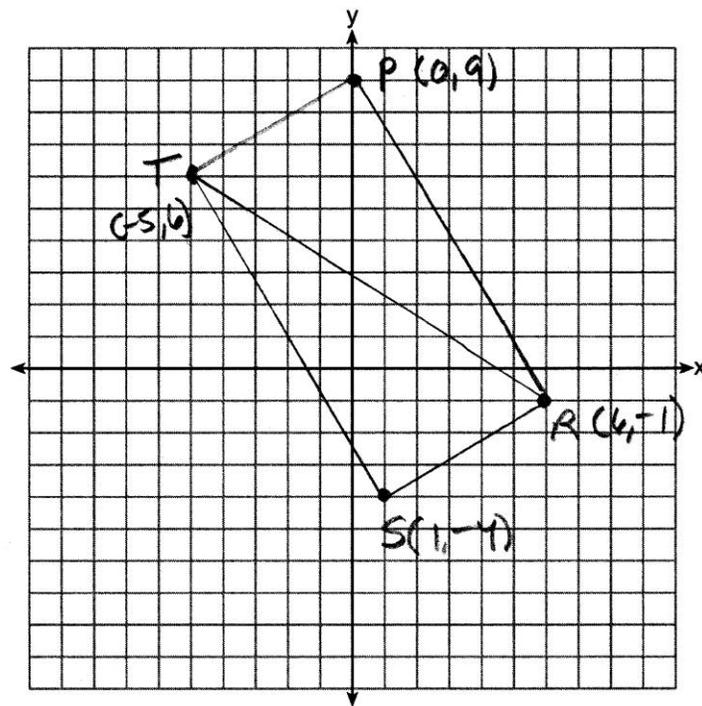
State the coordinates of point P such that quadrilateral $RSTP$ is a rectangle.

$(0,9)$

Question 36 is continued on the next page.

Question 36 continued

Prove that your quadrilateral $RSTP$ is a rectangle.
[The use of the set of axes below is optional.]



Score 1: The student graphed point P correctly and stated its coordinates. No further work was shown.

Question 36

36 In the coordinate plane, the vertices of $\triangle RST$ are $R(6,-1)$, $S(1,-4)$, and $T(-5,6)$.

Prove that $\triangle RST$ is a right triangle.

[The use of the set of axes on the next page is optional.]

$\triangle RST$ is a right \triangle because $\angle S$ is a right angle.

State the coordinates of point P such that quadrilateral $RSTP$ is a rectangle.

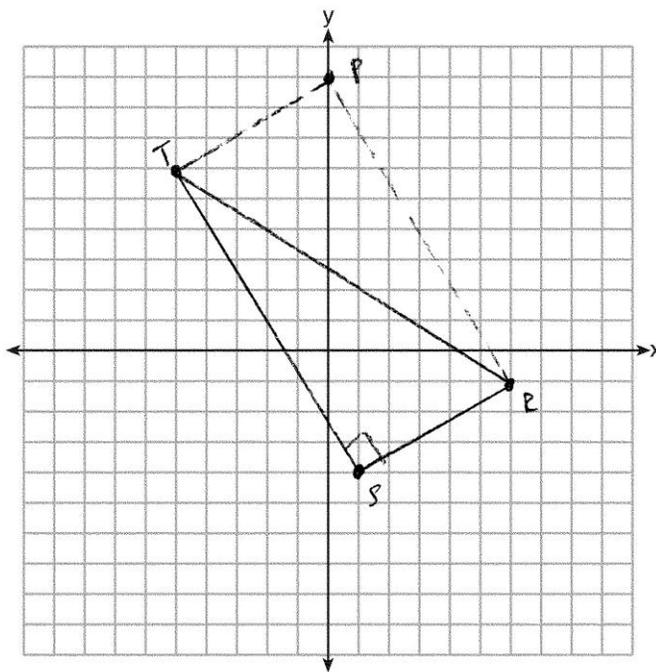
0,9

Question 36 is continued on the next page.

Question 36 continued

Prove that your quadrilateral $RSTP$ is a rectangle.
[The use of the set of axes below is optional.]

$RSTP$ is a rectangle because it has a right \angle .



Score 0: The student had no work to justify the statements, and the parentheses are missing on the coordinates of P .