The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

ELEVENTH YEAR

MATHEMATICS

Thursday, August 14, 1980 — 8:30 to 11:30 a.m., only

The last page of the booklet is the answer sheet. Fold the last page along the perforations and, slowly and carefully, tear off the answer sheet. Then fill in the heading of your answer sheet.

The “Reference Tables for Mathematics” which you may need to answer some questions in this examination are stapled in the center of this booklet.

When you have completed the examination, you must sign the statement printed at the end of the answer paper, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer paper cannot be accepted if you fail to sign this declaration.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN
Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed.

Directions (1-17): Write in the space provided on the separate answer sheet the numeral preceding the expression that best completes each statement or answers each question.

1. If the number 0.0031 is written in the form \(3.1 \times 10^n\), then \(n\) is equal to
   (1) \(\frac{1}{3}\)  (3) 3
   (2) -2  (4) -3

2. What is the solution set for \(|x^2| = -1\)?
   (1) \{1\}  (3) \{0\}
   (2) \{1, -1\}  (4) \{

3. The graph of the relation \(x^2 - \frac{y^2}{16} = 1\) is
   (1) a circle  (3) an ellipse
   (2) a hyperbola  (4) a parabola

4. Which value of \(x\) will satisfy the equation \(\sin^2 x - 1 = 0\)?
   (1) 30°  (3) 60°
   (2) 45°  (4) 90°

5. What is the numerical value of \(6 \sin \frac{5\pi}{6}\)?
   (1) 1  (3) 3
   (2) \(3\sqrt{3}\)  (4) -3

6. Which is a member of the set of rational numbers?
   (1) \(i^2\)  (3) \(\sin 45°\)
   (2) \(2i\)  (4) \(\sin 60°\)

7. The expression \(\frac{\sqrt{2} + 1}{\sqrt{2} - 1}\) is equivalent to
   (1) \(\frac{\sqrt{2}}{3}\)  (3) \(3 + 2\sqrt{2}\)
   (2) \(2 - 3\sqrt{2}\)  (4) \(\frac{2 + 3\sqrt{2}}{3}\)

8. The graphs of \(x^2 + y^2 = 9\) and \(y = -3\) are drawn on the same set of axes. How many points do these graphs have in common?
   (1) 1  (3) 0
   (2) 2  (4) 4

9. What is the value of the expression \(\frac{2 - \frac{1}{2}}{5}\) when \(x = \frac{3}{2}\)?
   (1) \(\frac{1}{6}\)  (3) \(\frac{3}{8}\)
   (2) \(\frac{2}{3}\)  (4) \(\frac{1}{4}\)

10. Which value of \(B\) satisfies the following system of equations?
    \[
    \begin{align*}
    \sin A + \cos B &= 1 \\
    \sin A - \cos B &= 0
    \end{align*}
    \]
    (1) \(B = 30°\)  (3) \(B = 60°\)
    (2) \(B = 45°\)  (4) \(B = 90°\)

11. The expression \(\frac{\sin^2 x + \cos^2 x}{\cos x}\) is equivalent to
    (1) \(\sin x \cos x\)  (3) \(\csc x\)
    (2) \(\tan x \cos x\)  (4) \(\sec x\)

12. Which is the graph of the solution set of the inequality \(x^2 - 4x - 5 < 0\)?
    (1) \[\text{Graph A}\]
    (2) \[\text{Graph B}\]
    (3) \[\text{Graph C}\]
    (4) \[\text{Graph D}\]

13. One factor of the expression \((a - 2)^2 + 3(a - 2)\)
    (1) \(a - 1\)  (3) \(a + 1\)
    (2) \(a - 5\)  (4) \(a + 2\)
14 As an angle increases from $\frac{\pi}{2}$ to $\frac{3\pi}{2}$, its cosine will
(1) increase, only
(2) decrease, only
(3) increase, then decrease
(4) decrease, then increase

15 In triangle ABC, $a = \sqrt{3}$, $b = \sqrt{5}$, and $c = 2$. What is the value of $\cos C$?
(1) $\frac{3}{5}$
(2) $0$
(3) $\frac{-3}{10}$
(4) $\frac{-1}{4}$

16 The value of $\sin (-10^\circ)$ is equivalent to the value of
(1) $\sin 10^\circ$
(2) $-\sin 10^\circ$
(3) $-\cos 10^\circ$
(4) $\csc 10^\circ$

17 Which value of $x$ satisfies the equation
$\cos (3x - 10^\circ) = \sin (x + 20^\circ)$?
(1) 15°
(2) 20°
(3) 35°
(4) 50°

Directions (18–30): Write your answers in the spaces provided on the separate answer sheet. Unless otherwise specified, answers may be left in terms of $\pi$ or in radical form.

18 If $\sin A = \frac{3}{5}$ and $\sin B = \frac{4}{5}$, what is the value of $\sin (A + B)$?

19 If $y$ varies directly as the square of $x$, and $y = 32$ when $x = 4$, find the value of $y$ when $x = 5$.

20 The sum of the digits of a two-digit number is 9. If the number is divided by the sum of the digits, the quotient is 3. What is the number?

21 If $B = \text{Arc} \cos (0.7071)$, what is the measure of angle $B$?

22 If 5 is a root of the equation $ax^2 - 10x - 25 = 0$, find the value of $a$.

23 Find the numerical value of $c$ that will make the roots of $x^2 - 6x + c = 0$ real, rational, and equal.

24 Given the equation $2x^2 - y = a$. If $x = \frac{1}{4}$, express $y$ in terms of $a$.

25 If $\log_{10} x = 2$, what is the value of $x$?

26 Solve for $x$: $9^x = 27$

27 In triangle ABC, $a = 3$, $b = 5$, and $\sin B = \frac{1}{4}$. What is the value of $\sin A$?

28 If $f(x) = \frac{2x^2 - x}{9}$, what is the numerical value of $f(1)$?

29 Perform the indicated operations and express the result in simplest form: $(7i - 3) - 2(3i - 2)$

30 In a circle, a central angle of 1.5 radians intercepts an arc of 4.5 centimeters. Find the number of centimeters in the length of the radius of the circle.
Part II

Answers to the following questions are to be placed on paper provided by the school.

Answer four questions from this part. Show all work unless otherwise directed.

31. a Find to the nearest tenth the values of \( \tan x \) which satisfy the equation
   \[ \tan^2 x + 4 \tan x - 6 = 0. \] [8]
   b Using the answers obtained in part a, determine the number of values of \( x \) in the interval
   \( 0 \leq x < 2\pi \) which satisfy the equation
   \[ \tan^2 x + 4 \tan x - 6 = 0. \] [2]

32. a Draw the graph of the function \( f(x) = x^2 - 4x + 2 \)
   as \( x \) varies from \(-1\) to \(+5\) inclusive. [6]
   b Write an equation of the axis of symmetry. [2]
   c Find the minimum value of \( x^2 - 4x + 2. \) [2]

33. a In quadrilateral \( ABCD, \quad \angle DAB = 106, \)
   \( AB = 18, \quad AD = 12, \quad \angle ACB = 56, \)
   and \( \angle B = 90. \)
   Find \( AC \) to the nearest integer. [5]
   b Using the answer to part a, find the area of triangle \( ACD \) to the nearest integer. [5]

34. a On the same set of axes, sketch the graphs of
   \( y = \tan x \) and \( y = \cos 2x \) for values of \( x \) in the interval \( 0 \leq x \leq \pi. \) [4,4]
   b State the number of values of \( x \) in the interval
   \( 0 \leq x \leq \pi \) that satisfy the equation
   \( \tan x = \cos 2x. \) [2]

35. a Starting with the formula for \( \sin (x + y), \) derive the formula for \( \sin 2x. \) [4]
   b If \( \log_b x = 1.2346 \) and \( \log_b y = 2.4690, \) find the value of
     \[ (1) \log_b xy^2 \] [3]
     \[ (2) \log_b \frac{y}{\sqrt{x}} \] [3]

36. A woman bought a certain number of shares of stock for \$900. If she had bought these shares earlier when the price per share was \$3 less, she could have bought 10 more shares for the same investment. How many shares did she buy? [Only an algebraic solution will be accepted.] [5,5]

*37 Solve the following system of equations and check.
   \[ x + y + 2z = 6 \]
   \[ 3x - y + 4z = 3 \] [7,3]
   \[ 3x + 2y - 6z = 20 \]

* This question is based on an optional topic in the syllabus.
FOR TEACHERS ONLY

SCORING KEY

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Use only red ink or red pencil in rating Regents papers. Do not attempt to correct the pupil's work by making insertions or changes of any kind. Use checkmarks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. In problems involving logarithms, answers should be left correct to four significant digits unless directions say otherwise. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow 2 credits for each correct answer; allow no partial credit. For questions 1–17, allow credit if the pupil has written the correct answer instead of the numeral 1, 2, 3, or 4.

(1) 4

(2) 4

(3) 2

(4) 4

(5) 3

(6) 1

(7) 3

(8) 1

(9) 1

(10) 3

(11) 4

(12) 1

(13) 3

(14) 4

(15) 1

(16) 2

(17) 2

(18) 1

(19) 50

(20) 27

(21) \( \frac{\pi}{4} \) or 45°

(22) 3

(23) 9

(24) \( y = \frac{1}{2} - a \)

(25) 100

(26) \( \frac{1}{3} \)

(27) 0.3 or \( \frac{3}{10} \)

(28) \( \frac{1}{5} \)

(29) 1 + i or \( i + 1 \)

(30) 3

[OVER]
Part II

Please refer to the Department's pamphlet *Suggestions on the Rating of Regents Examination Papers in Mathematics*. Care should be exercised in making deductions as to whether the error is purely a mechanical one or due to a violation of some principle. A mechanical error generally should receive a deduction of 10 percent, while an error due to a violation of some cardinal principle should receive a deduction ranging from 30 percent to 50 percent, depending on the relative importance of the principle in the solution of the problem.

(31) $a$ 1.2, -5.2 [8]  
    $b$ 4 [2]

(32) $b$ $x = 2$ [2]  
    $c$ -2 [2]

(33) $a$ 22 [5]  
    $b$ 126 [5]

(34) $b$ 1 [2]

(35) $b$ (1) 6.1706 [3]  
    (2) 1.8507 [3]

(36) 50 [5,5]

(37) $x = 3$  
    $y = 4$  
    $z = -\frac{1}{2}$ [7,3]