Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Unless otherwise specified, answers may be left in terms of \( \pi \) or in radical form.

1. Express \( \frac{1}{\sqrt{3} + 1} \) as an equivalent fraction with a rational denominator.

2. Find the value of \((2x)^5 + 8x^{\frac{2}{3}}\) if \(x = 8\).

3. If \(s\) varies directly as the square of \(t\) and if \(s = 45\) when \(t = 3\), find the constant of variation.

4. Find the slope of the graph whose equation is \(2x + 3y = 6\).

5. The sum of the roots of the equation \(ax^2 + 6x - 8 = 0\) is 12. Find the value of \(a\).

6. Solve for \(x\): \(\sqrt{x^2 - 4} = x - 2\)

7. Solve for \(x\): \(\frac{1}{x} - \frac{1}{a} = 1\)

8. If the numbers 3, \(x\), \(y\) and \(-24\) form a geometric progression in this order, find the value of \(x\).

9. In triangle \(ABC\), \(a = 6\), \(b = 3\) and \(A = 150^\circ\). Find the value of \(\sin B\).

10. Find the number whose logarithm is 9.8783 - 10.
11 Find \( \cot 32^\circ 54' \).

12 Find the numerical value of \( \cos \frac{2x}{3} \).

13 If \( x \) is an obtuse angle, express \( \cos x \) in terms of \( \sin x \).

14 If \( \theta \) is an acute angle and \( \cos \theta = m \), express \( \cos \frac{\theta}{2} \) in terms of \( m \).

15 Each of the equal sides of an isosceles triangle is \( a \) and a base angle is \( 15^\circ \). Express the area of the triangle in terms of \( a \).

16 Find the number of degrees in the positive acute angle \( x \)
if \( \sin^2 x - \cos^2 x = 0 \).

17 If \( \theta \) is an angle in quadrant IV and \( \theta = \arccos \frac{1}{2} \), find the value of \( \sin \theta \).

18 Express \( \tan \left( \frac{\pi}{4} + x \right) \) in terms of \( \tan x \).

19 In triangle \( ABC \), \( a = 3 \), \( b = 3 \) and \( c = 2 \). Find the value of \( \cos C \).

Directions (20–27): Write on the line at the right of each of the following the number preceding the expression that best completes the statement.

20 The value of \( \frac{1}{\frac{1}{2} + \frac{1}{2}} \) is

(1) 1
(2) 2
(3) 3
(4) 4

21 If \( a \), \( b \) and \( c \) are positive unequal numbers, the graph of \( ax^2 + by^2 = c \) is

(1) a circle
(2) a parabola
(3) an ellipse
(4) a hyperbola

22 If the discriminant of a quadratic equation with integral coefficients is 1, the roots are

(1) rational and equal
(2) rational and unequal
(3) irrational and unequal
(4) imaginary

[2]
23 \, \text{Log} \, \frac{x^3}{100} \, \text{equals} \, \\
(1) \, 3 \log x - 2 \quad \text{(3) \, 3x - 100} \\
(2) \, \frac{3 \log x}{2} \quad \text{(4) \, \frac{3x}{100}} \, \underline{23} \ldots ..

24 \, \text{The tens digit of a two-digit number is 3 times the units digit. If } x \text{ represents the units digit, then the number can be represented by} \\
(1) \, 4x \quad \text{(3) \, 13x} \\
(2) \, 11x \quad \text{(4) \, 31x} \, \underline{24} \ldots ..

25 \, \text{In triangle } ABC, \, a = 12, \, b = 9 \text{ and } A = 42^\circ. \, \text{These data determine for } B \\
(1) \, \text{two values, one less than } 90^\circ \text{ and one greater than } 90^\circ \\
(2) \, \text{two values, both less than } 90^\circ \\
(3) \, \text{exactly one value, that value being less than } 90^\circ \\
(4) \, \text{exactly one value, that value being greater than } 90^\circ \, \underline{25} \ldots ..

26 \, \text{An equivalent expression for } \sqrt{-8} \text{ is} \\
(1) \, 8i \quad \text{(3) \, 2i \sqrt{2}} \\
(2) \, -\sqrt{8} \quad \text{(4) \, -2 \sqrt{2}} \, \underline{26} \ldots ..

27 \, \text{The minimum value of } \frac{1}{2} \sin 2x \text{ is} \\
(1) \, -1 \quad \text{(3) \, -\frac{1}{2}} \\
(2) \, -2 \quad \text{(4) \, \frac{1}{2}} \, \underline{27} \ldots ..

\textit{Directions (28–30): If the blank space in each statement below is replaced by the word always, sometimes (but not always) or never, the resulting statement will be true. Select the word that will correctly complete each statement and write this word on the line at the right.}

28 \, \text{The expression } \cos (A - B) \text{ is ... equal to } \cos (B - A). \, \underline{28} \ldots ..

29 \, \text{If } A + B = 90^\circ, \sin^2 A + \sin^2 B \text{ is ... equal to 1.} \, \underline{29} \ldots ..

30 \, \text{If } a, \, b \text{ and } c \text{ represent positive unequal numbers,} \\
\frac{a + b}{c + b} \text{ is ... equal to } \frac{a}{c}. \, \underline{30} \ldots ..
31 \ a \ Solve \ for \ x, \ expressing \ the \ answer \ in \ \textit{inverse \ trigonometric \ form}: \ \ [8] \\
\sin^2x - 2 \sin x + \frac{1}{4} = 0 \\
b \ Using \ the \ result \ obtained \ in \ part \ a, \ determine \ the \ quadrant(s) \ in \ which \ angle \ x \ lies. \ \ [2] \\

32 \ a \ On \ the \ same \ set \ of \ axes, \ draw \ the \ graphs \ of \ y = \frac{1}{2}x^2 \ and \ y = -x + 3. \ \ [5, 3] \\
b \ From \ your \ graph, \ estimate \ to \ \textit{tenths} \ the \ roots \ of \ the \ equation \ \frac{1}{2}x^2 = -x + 3. \ \ [2] \\

33 \ a \ Starting \ with \ a \ formula \ for \ \sin (x + y), \ develop \ a \ formula \ for \ \sin (x - y). \ \ [3] \\
b \ Prove \ that \ the \ following \ equality \ is \ an \ identity: \ \ [7] \\
\sin (x + y) \sin (x - y) = \sin^2x - \sin^2y \\

34 \ The \ area \ of \ a \ regular \ pentagon \ inscribed \ in \ a \ circle \ of \ radius \ R \ is \ given \ by \ the \ formula \ \ [10] \\
A = \frac{5R^2}{2} \sin 72^\circ. \ By \ means \ of \ logarithms, \ find \ R \ to \ the \ \textit{nearest \ integer} \ if \ A = 340. \\

35 \ Three \ positive \ numbers \ a, \ b \ and \ c \ form \ an \ arithmetic \ progression \ in \ this \ order. \ \ [4] \\
a \ Express \ b \ in \ terms \ of \ a \ and \ c. \\
b \ If \ the \ ratio \ of \ a \ to \ b \ is \ \frac{1}{3}, \ find \ the \ ratio \ of \ a \ to \ c. \ \ [6] \\

36 \ a \ Two \ forces \ of \ 8 \ pounds \ and \ 6 \ pounds, \ respectively, \ act \ on \ a \ body \ so \ that \ the \ magnitude \ of \ the \ resultant \ is \ 9 \ pounds. \ Find \ to \ the \ \textit{nearest \ degree} \ the \ angle \ between \ the \ two \ forces. \ \ [8] \\
b \ If \ the \ angle \ between \ the \ 8-pound \ force \ and \ the \ 6-pound \ force \ changes, \ the \ magnitude \ of \ the \ resultant \ changes. \ When \ the \ angle \ between \ the \ two \ forces \ is \ 90^\circ, \ what \ is \ the \ value \ of \ the \ magnitude \ of \ the \ resultant? \ \ [2] \\

*37 \ In \ triangle \ ABC, \ angle \ C \ is \ 120^\circ \ and \ side \ a \ is \ twice \ side \ b. \ \ [5] \\
a \ Using \ the \ law \ of \ tangents, \ show \ that \ \tan \frac{1}{2} (A - B) = \frac{\sqrt{3}}{9}. \\
b \ Using \ the \ result \ in \ part \ a, \ find \ angle \ A \ to \ the \ \textit{nearest \ degree}. \ \ [5] \\

*This \ question \ is \ based \ upon \ an \ optional \ topic \ in \ the \ syllabus.
Use only red ink or pencil in rating Regents papers. Do not attempt to correct the pupil's work by making insertions or changes of any kind. Use checkmarks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. In problems involving logarithms, answers should be left correct to four significant digits unless directions say otherwise. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow 2 credits for each correct answer; allow no partial credit. For questions 20–27, allow credit if the pupil has written the correct answer instead of the number 1, 2, 3 or 4.

1. \( \frac{\sqrt{3} - 1}{2} \)  
2. 33  
3. 5  
4. \(-\frac{3}{5}\)  
5. \(-\frac{1}{2}\)  
6. 2  
7. \(\frac{a}{a + 1}\)  
8. \(-6\)  
9. \(\frac{1}{2}\)  
10. 0.7557  
11. 1.5458  
12. \(-\frac{1}{2}\)  
13. \(-\sqrt{1 - \sin^2 x}\)  
14. \(\sqrt{\frac{1 + m}{2}} \text{ or } \frac{1}{2}\sqrt{2 + 2m}\)  
15. \(\frac{1}{2}a^2\)  
16. 45  
17. \(-\frac{\sqrt{3}}{2}\)  
18. \(\frac{1 + \tan x}{1 - \tan x}\)  
19. \(\frac{1}{5}\)  
20. 1  
21. 3  
22. 2  
23. 1  
24. 4  
25. 3  
26. 3  
27. 3  
28. always  
29. always  
30. never
Please refer to the Department's pamphlet *Suggestions on the Rating of Regents Examination Papers in Mathematics*. Care should be exercised in making deductions as to whether the error is purely a mechanical one or due to a violation of some principle. A mechanical error generally should receive a deduction of 10 percent, while an error due to a violation of some cardinal principle should receive a deduction ranging from 30 percent to 50 percent, depending on the relative importance of the principle in the solution of the problem.

**Part II**

(31) \( a \, x = \arcsin \frac{2 - \sqrt{3}}{2} \)  
\[ \text{[8]} \]

\( b \) I, II  
\[ \text{[2]} \]

(32) \( b \) 1.6 and -3.6  
\[ \text{[2]} \]

(34) 12  
\[ \text{[10]} \]

(35) \( a \, \frac{a + c}{2} \)  
\[ \text{[4]} \]

\( b \) \( \frac{5}{19} \)  
\[ \text{[6]} \]

(36) \( a \) 101  
\[ \text{[8]} \]

\( b \) 10  
\[ \text{[2]} \]

(37) \( b \) 41  
\[ \text{[5]} \]