The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

ELEVENTH YEAR

MATHEMATICS

Monday, June 19, 1972 — 1:15 to 4:15 p.m., only

The last page of the booklet is the answer sheet, which is perforated. Fold the last page along the perforation and then, slowly and carefully, tear off the answer sheet. Now fill in the heading of your answer sheet. When you have finished the heading, you may begin the examination immediately.
Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Unless otherwise specified, answers may be left in terms of π or in radical form. Write your answers in the spaces provided on the separate answer sheet.

1. Find the number of degrees in the positive acute angle $x$ that satisfies the equation $\tan^2 x - 1 = 0$.

2. Express an angle of 240° in radian measure.

3. What is the numerical value of the product $(\cos \frac{\pi}{6}) (\sin \frac{\pi}{2})$?

4. Solve for $d$: $t = a + (n - 1)d$

5. Find the value of $[(4^{\frac{1}{2}})]^{-1}$.

6. On a trip a man drove 30 miles per hour for 2 hours. He then changed his rate to 40 miles per hour and drove for 3 more hours. Find his average rate, in miles per hour, for the whole trip.

7. If $x + 2$ is a factor of $x^2 + kx - 8$, find the numerical value of $k$.

8. In acute triangle $ABC$, $a = 5$, $b = 6$, and $\sin B = \frac{5}{6}$. Find the number of degrees in angle $A$.

9. In a set of rectangles, the length of each varies inversely as the width. If the length of a rectangle in the set is 16 feet when its width is 3 feet, find the number of feet in the width of a rectangle whose length is 8 feet.

10. Find the value of $\cos (\arcsin \frac{1}{2} + \arccos \frac{1}{2})$.

11. Determine the value of $\tan 36°44'$, with accuracy to four decimal places.

12. Express $\sin (-42°)$ as a function of a positive acute angle.

13. Find the length of side $c$ in $\triangle ABC$ if $a = 3$, $b = 5$, and $\cos C = -\frac{1}{2}$.

Directions (14–30): Write in the space provided on the separate answer sheet the numeral preceding the expression that best completes each statement or answers each question.

14. A number whose square is less than zero is called
   (1) a whole number
   (2) an imaginary number
   (3) a negative real number
   (4) a negative rational number

15. Which value of $x$ satisfies the inequality $3 + x < 3x - 5$?
   (1) $x < 4$
   (2) $x \leq 4$
   (3) $x > 4$
   (4) $x \geq 4$

16. For all values of $x$ for which the expressions are defined, $\frac{1}{x - 1} + 1$ is equivalent to
   (1) $\frac{x}{x - 1}$
   (2) $\frac{x - 1}{x}$
   (3) $\frac{1}{x}$
   (4) $\frac{x}{1 - x}$

17. The expression $\sin (A + B) + \sin (A - B)$ is equivalent to
   (1) $2 \sin A \cos B$
   (2) $2 \sin B \cos A$
   (3) $2 \cos A \cos B$
   (4) $-2 \sin A \sin B$

18. If the straight line having the equation $y = \frac{1}{2}$ is drawn on the same set of axes as the graph of $y = \frac{1}{2} \sin x$, the number of intersections between 0 and $2\pi$ would be
   (1) 1
   (2) 2
   (3) 0
   (4) 4

19. The graph of $ax^2 + by^2 = c$, in which $a$, $b$, and $c$ are real numbers, is an ellipse if
   (1) $a = b$, $a > 0$, $b < 0$, $c > 0$
   (2) $a = b$, $a > 0$, $b > 0$, $c < 0$
   (3) $a \neq b$, $a > 0$, $b > 0$, $c > 0$
   (4) $a \neq b$, $a > 0$, $b < 0$, $c > 0$

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20 If the solution for $26 \times 13$ were obtained as follows:
$$26 \times 13 = 26 \times (10 + 3) = 26 \times 10 + 26 \times 3 = 260 + 78 = 338,$$ which law was demonstrated?
(1) the commutative law for addition
(2) the associative law for multiplication
(3) the associative law for addition
(4) the distributive law for multiplication over addition

21 What is the domain of the function defined by the equation $f(x) = \sqrt{x - 4}$?
(1) $\{x \mid x \geq 4\}$
(2) $\{x \mid x \leq 4\}$
(3) $\{x \mid x \geq 0\}$
(4) $\{x \mid x \leq 0\}$

22 If the discriminant of a quadratic equation with rational coefficients is equal to $-4$, the roots are
(1) imaginary
(2) real and irrational
(3) real, rational, and unequal
(4) real, rational, and equal

23 If $\cos x > 0$, which must always be true?
(1) $\sin x > 0$
(2) $\tan x > 0$
(3) $\sec x > 0$
(4) $\csc x > 0$

24 The additive inverse of $2 - \sqrt{3}$ is
(1) $1$
(2) $0$
(3) $2 + \sqrt{3}$
(4) $-2 + \sqrt{3}$

25 Which statement concerning the roots of the equation $\sqrt{x^2 - 3x + 3} = 1$ is true?
(1) The equation has 1 as its only root.
(2) The equation has 2 as its only root.
(3) The equation has both 1 and 2 as its roots.
(4) The equation has neither 1 nor 2 as its roots.

26 The reciprocal of $3 - \sqrt{5}$ is
(1) $\frac{3 - \sqrt{5}}{4}$
(2) $\frac{3 + \sqrt{5}}{14}$
(3) $\frac{3}{4}$
(4) $\frac{3 - \sqrt{5}}{14}$

27 Given the relation $\{(1,2),(2,3),(3,4)\}$. What is the inverse of the relation?
(1) $\{(1,\frac{1}{2}),(2,\frac{1}{3}),(3,\frac{1}{4})\}$
(2) $\{(-1,-2),(-2,-3),(-3,-4)\}$
(3) $\{(3,4),(2,3),(1,2)\}$
(4) $\{(2,1),(3,2),(4,3)\}$

28 For all values of $\theta$ for which the expression is defined, $\sin \theta \left( \frac{1}{\sin \theta} - \sin \theta \right)$ is equivalent to
(1) $-\cos^2 \theta$
(2) $\cos^2 \theta$
(3) $1 - \cos^2 \theta$
(4) $1 + \cos^2 \theta$

29 If $\angle A = 60, a = 5$, and $b = 15$, then the triangle $ABC$
(1) must be an acute triangle
(2) must be an obtuse triangle
(3) may be either an acute or an obtuse triangle
(4) cannot be constructed

30 The expression $\log \sin 2x$ is equivalent to
(1) $2 \log \sin x$
(2) $2 (\log \sin x + \log \cos x)$
(3) $\log 2 + \log \sin x + \log \cos x$
(4) $\log \sin x + \log \cos x$
Answers to the following questions are to be written on paper provided by the school.

Part II

Answer four questions from this part. Show all work unless otherwise directed.

31 Solve the following equation for all values of \( \theta \) to the nearest degree in the interval \( 0^\circ < \theta < 360^\circ \): [10]
\[
5 \sin \theta + 2 \cos 2\theta - 3 = 0
\]

32 Solve the following system of equations and check in both equations: [8, 2]
\[
x^2 - 2y^2 = 7
\]
\[
x - 2y = 5
\]

33 Answer both a and b:
   a On the same set of axes, sketch the graphs of 
   \( y = \sin \frac{1}{2}x \) and \( y = \cos x \) for all values of \( x \) which 
   are in the interval \( 0 \leq x \leq 2\pi \). [4, 4]
   b From the graphs made in part a, determine the 
   number of values of \( x \) that satisfy the equation 
   \( \sin \frac{1}{2}x = \cos x \) in the interval \( 0 \leq x \leq 2\pi \). [2]

34 Write an equation or system of equations which can be used to solve each of the following problems. In each case, state what the variable or variables represent. [Solution of the equations is not required.] [10]
   a A class consists of 14 boys and 19 girls. How 
   many girls should be taken out of this class so that 
   the remaining girls will make up only 30% of the 
   total number of pupils left in the class?
   b The rate of a stream in which a man rows a boat 
   is 2 miles an hour. If it takes the man twice as 
   much time to row upstream a distance of 30 miles 
   as it takes him to return the same distance down- 
   stream, what is the rate of the boat in still water?

35 Using logarithms, find, to the nearest hundredth, the value of: [10]
\[
\frac{\sqrt{93.1 \sin 29^\circ 40'}}{7.74}
\]

36 Answer a or b but not both: [10]
   a A canoe race is to be run over a triangular course 
   marked by buoys \( A, B, \) and \( C \). The distance be- 
   tween \( A \) and \( B \) is 100 yards, that between \( B \) and \( C \) 
   is 160 yards, and that between \( C \) and \( A \) is 220 
   yards. Find, to the nearest degree, the angle \( ABC \).

   OR

   b From a point, \( A \), on one bank of a river, the angle 
   of elevation of the top of a tree directly across the 
   river on the other bank is 43° 20'. As shown in the 
   figure, point \( B \) is 100 feet behind \( A \) and in the 
   same straight line as point \( A \) and point \( C \) (the foot 
   of the tree). From point \( B \) the angle of elevation 
   of the top of the tree is 32° 30'.

   [Diagram]

   What is the length of \( \overline{AC} \) to the nearest foot?

37 a On a graph, indicate the solution set of 
\[
\{(x, y) \mid xy \geq 8 \text{ and } y \leq \frac{1}{2}x + 2\}. \quad [9]
\]

   b State the coordinates of one point which is in the 
   solution set of \( \{(x, y) \mid y = \frac{1}{2}x + 2\} \), but is not 
   included in the solution set of \( \{(x, y) \mid xy \geq 8 \text{ and } 
   y \leq \frac{1}{2}x + 2\} \). [1]

* Based on an optional topic in the syllabus.
Use only red ink or pencil in rating Regents papers. Do not attempt to correct the pupil's work by making insertions or changes of any kind. Use checkmarks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. In problems involving logarithms, answers should be left correct to four significant digits unless directions say otherwise. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow 2 credits for each correct answer; allow no partial credit. For questions 14–30, allow credit if the pupil has written the correct answer instead of the number 1, 2, 3, or 4.

(1) 45
(2) \( \frac{4\pi}{3} \)
(3) \( \frac{\sqrt{3}}{2} \) or 0.866
(4) \( d = \frac{t - a}{n - 1} \)
(5) \( \frac{1}{8} \)
(6) 36
(7) \( -2 \)
(8) 30
(9) 6
(10) 0

(11) 0.7463
(12) \( -\sin 42^\circ \) or \( -\cos 48^\circ \)
(13) 7
(14) 2
(15) 3
(16) 2
(17) 1
(18) 1
(19) 3
(20) 4
(21) 1
(22) 1
(23) 3
(24) 4
(25) 3
(26) 2
(27) 4
(28) 2
(29) 4
(30) 3

[OVER]
Eleventh Year Mathematics — concluded

Part II

Please refer to the Department’s pamphlet Suggestions on the Rating of Regents Examination Papers in Mathematics. Care should be exercised in making deductions as to whether the error is purely a mechanical one or due to a violation of some principle. A mechanical error generally should receive a deduction of 10 percent, while an error due to a violation of some cardinal principle should receive a deduction ranging from 30 percent to 50 percent, depending on the relative importance of the principle in the solution of the problem.

(31) 14, 90, 166 [10]
(32) (3, -1) and (-13, -9) [8]
(33) b 2 [2]
(34) \( a \ x = \) number of girls to be taken out
\[ 19 - x = 0.3(33 - x) \] [5]
\( b \ x = \) rate of boat in still water
\[ 2 \left( \frac{30}{x + 2} \right) = \frac{30}{x - 2} \] [5]
(35) 0.46 [10]
(36) a 114 OR b 208 [10]

DO YOU KNOW . . .

Who writes the questions used on Regents examinations?
1 the members of the Board of Regents
2 the subject supervisors in the State Education Department
3 college professors in the various disciplines
4 classroom teachers from schools throughout New York State

The correct answer is 4. Last year more than 400 classroom teachers were involved in the preparation of Regents examination questions, and many other teachers served on the committee that assembled the examinations.