

The University of the State of New York

312TH HIGH SCHOOL EXAMINATION

ELEVENTH YEAR MATHEMATICS

Wednesday, June 20, 1951 — 9.15 a. m. to 12.15 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II, III and IV (a) name of school where you have studied, (b) number of weeks and recitations a week in eleventh year mathematics.

The minimum time requirement is four or five recitations a week for a school year after the completion of tenth year mathematics.

Part II

Answer two questions from part II.

26 Find to the *nearest tenth* the value of $\sin x$ which satisfies the equation $\cos^2 x + 3 \sin x - 2 = 0$. [10]

27 Solve the following system of equations and check *one* set of answers: [8, 2]

$$2x + y = 1$$

$$x^2 - xy = 10$$

28 Write the equations that would be used in solving the following problems. In *each* case state what the letter or letters represent. [Solution of the equations is not required.]

a How many pounds of water must be evaporated from 100 pounds of a 3% solution of salt in order to make the result a 5% solution of salt? [5]

b Paul can do a certain job in half the time that Jim requires to do it. Jim worked alone for one hour and stopped; then Paul completed the job in ten hours. How many hours would it take each working alone to do the whole job? [5]

29 The radius R of a circle circumscribed about a regular decagon whose area is A is given by the formula $R = \sqrt{\frac{A}{5 \sin 36^\circ}}$. Using logarithms, find R to the *nearest hundredth of an inch* when $A = 71.70$ square inches. [10]

Part III

Answer three questions from part III.

30 a On the same set of axes, *sketch* the graphs of the equations $y = \sin 2x$ and $y = \cos x$ as x varies from 0 to 2π radians inclusive. [5, 3]

b From the graphs made in answer to a, determine the number of values of x between 0 and 2π radians inclusive which satisfy the equation $\sin 2x = \cos x$. [2]

31 a Starting with the formula for $\cos 2A$, derive the formula for $\cos \frac{1}{2}x$ in terms of $\cos x$. [5]

b Prove that the following equality is an identity: [5]

$$\frac{2 \sin A + \sin 2A}{2 \sin^2 A} = \csc A + \cot A$$

32 Radio station A is 24.1 miles due north of radio station B. The bearing of a ship in distress is found from its signal to be S $79^\circ 40'$ E from A and N $76^\circ 20'$ E from B. Find to the *nearest tenth of a mile* the distance from the ship to station B. [5, 5]

ELEVENTH YEAR MATHEMATICS

33 In triangle ABC , $a = 15$, $b = 23$ and $c = 13$. Find angle B to the nearest degree. [10]

34 In triangle ABC , $B = 90^\circ + A$.

a Using the law of sines, show that $b = a \cot A$. [5]

b Show that $C = 90^\circ - 2A$. [2]

c Show that the area K of triangle ABC is given by the formula $K = \frac{a^2 \cot A \cos 2A}{2}$

[3]

ELEVENTH YEAR MATHEMATICS

Fill in the following lines:

Name of pupil..... Name of school.....

Part I

Answer all questions in part I. Each correct answer will receive 2 credits. No partial credit will be allowed.

- 1 Write the expression $2\sqrt{-25} - 3\sqrt{-1}$ as a single term. 1.....
- 2 Express $\frac{1}{4 + \sqrt{3}}$ as an equivalent fraction with a rational denominator. 2.....
- 3 Write an equation of the straight line whose slope is 2 and whose y-intercept is -3 . 3.....
- 4 If s varies directly as t and $s = 3$ when $t = 5$, find s when $t = 8$. 4.....
- 5 Find the 25th term of the progression $-8, -5, -2, \dots$ 5.....
- 6 Find *two* numbers that when inserted between 2 and 128 form with those numbers a geometric progression of four terms. 6.....
- 7 Find the value of $(27)^{-\frac{1}{3}} + (27)^0$ 7.....
- 8 Simplify the complex fraction $\frac{1 - \frac{1}{m}}{\frac{1}{m}}$ 8.....
- 9 Find the sum of the roots of the equation $2x^2 - 5x + 6 = 0$. 9.....
- 10 Find the product of the roots of the equation $px^2 + qx = 0$. 10.....
- 11 What is the name of the graph of the equation $x^2 - \frac{y^2}{4} = 9$? 11.....
- 12 Find $\log \tan 26^\circ 16'$ 12.....
- 13 Find the number whose logarithm is $9.8030 - 10$ 13.....

ELEVENTH YEAR MATHEMATICS

14 Solve the equation $\sqrt{\cos x + 2} = 1$ for the positive value of x less than 360° . 14.....

15 Solve for $\sin x$, in terms of d , the equation $d = \frac{\sin x}{1 - \sin x}$ 15.....

16 Express $\sin 250^\circ$ as a function of a positive acute angle. 16.....

17 In a circle whose radius is 2 inches, a central angle intercepts an arc whose length is 6 inches. Find the number of radians in the angle. 17.....

18 Express $\sec \theta \tan \theta$ in terms of $\sin \theta$. 18.....

19 In triangle ABC , $a = 12$, $c = 16$, and $\sin B = \frac{3}{4}$. Find the area of triangle ABC . 19.....

20 In triangle ABC , angle $A = 30^\circ$ and angle $C = 45^\circ$. Find the ratio of side a to side c . [Answer may be left in radical form.] 20.....

21 In triangle ABC , $a = 3$, $b = 7$, and $\cos C = \frac{5}{8}$. Find side c to the nearest integer. 21.....

22 Express $\cos (x + y)$ in terms of the sine and cosine of x and of y . 22.....

23 Find the principal value of $\cos^{-1} (-\frac{1}{2})$ 23.....

Directions (questions 24–25) — Indicate the correct answer to *each* question by writing on the line at the right the letter a , b or c .

24 The statement $\sin^2 x = \frac{1 - \cos 2x}{2}$ is (a) true for all values of x
 (b) true for only certain values of x (c) not true for any value of x 24.....

25 Using the data $A = 32^\circ$, $a = 15$, and $c = 20$, it is possible to construct
 (a) no triangle (b) only one triangle (c) two different triangles 25.....