The University of the State of New York

11

REGENTS HIGH SCHOOL EXAMINATION

ELEVENTH YEAR MATHEMATICS

Wednesday, January 25, 1967 — 1:15 to 4:15 p.m., only

The last page of the booklet is the answer sheet, which is perforated. Fold the last page along the perforation and then, slowly and carefully, tear off the answer sheet. Now fill in the heading of your answer sheet. When you have finished the heading, you may begin the examination immediately.

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Unless otherwise specified, answers may be left in terms of π or in radical form. Write your answers in the spaces provided on the separate answer sheet.

1 Factor: $3 \sin^2 x + \sin x - 4$

2 Find the value of $a^0 + b^{\frac{1}{2}} + c^{-1}$ if $a = 3$, $b = 4$, and $c = 5$.

3 Express as an equivalent fraction in *simplest form*:

$$\frac{\dfrac{1}{b} + a}{\dfrac{1 + ab}{c}}$$

4 Find to the *nearest minute* the positive acute angle θ if $\log \sin \theta = 9.4256 - 10$.

5 Express $\dfrac{2}{2\sqrt{3} + 1}$ as an equivalent fraction with a rational denominator.

6 Find the abscissa of the point at which the graphs of $3x + 5y = 22$ and $2x - 3y = 2$ intersect.

7 Find the positive value of k such that the following equation will have equal roots: $x^2 + kx + 1 = 0$

8 Solve for x: $\sqrt{x^2 - 25} = x + 5$

9 If $ay = by + c$, solve for y in terms of a, b, and c.

10 Write an equation of the straight line whose slope is 2 and which intersects the x-axis at the point whose coordinates are $(-3,0)$.

11 If $\theta = \text{arc sin } \dfrac{\sqrt{3}}{2}$ and θ lies in the first quadrant, find $\sec \left(\text{arc sin } \dfrac{\sqrt{3}}{2} \right)$.

12 If 2 and 4 are the first and fourth terms, respectively, of an arithmetic progression, find the value of the common difference.

13 Find the value of x greater than 90° and less than 180° which satisfies the equation $2 \sin^2 x - \sqrt{3} \sin x = 0$.

14 Express in terms of π the number of radians in an angle of 240°.

Directions (15–30): Write in the space provided on the separate answer sheet the *number* preceding the expression that best completes *each* statement or answers *each* question.

15 The sum and product of the roots of a quadratic equation are -5 and 4, respectively. An equation satisfying these conditions is
(1) $x^2 - 5x - 4 = 0$
(2) $x^2 - 4x - 5 = 0$
(3) $x^2 + 5x + 4 = 0$
(4) $x^2 + 4x + 5 = 0$

16 If the expression $(x - 2)(3 + x)$ is replaced by $(x - 2)(x + 3)$, which of the following laws is demonstrated?
(1) the commutative law for addition
(2) the associative law for multiplication
(3) the associative law for addition
(4) the distributive law for multiplication over addition

17 Consider the graph of $y = \cos x$ as x increases from $-\frac{\pi}{2}$ to $\frac{\pi}{2}$ radians. The value of y

(1) decreases and then increases
(2) increases and then decreases
(3) increases throughout the interval
(4) decreases throughout the interval

18 If $\log 9 = a$, the logarithm of 3 expressed in terms of a is

(1) $\sqrt{a}$ (3) $\frac{1}{2}a$
(2) $2a$ (4) a^2

19 A three-digit number is formed by placing the digit 2 between the tens digit t and units digit u of a two-digit number. The new number thus formed is represented by

(1) $100t + 2t + u$ (3) $10t + u + 20$
(2) $100t + 20 + u$ (4) $tu + 20$

20 The expression $\sqrt[3]{-1} + \sqrt{-1}$ is equivalent to

(1) $1 + i$ (3) $-1 + i$
(2) $-1 - i$ (4) $-2i$

21 The graph of the equation $x^2 - 1 = 2x + 2y$ is a

(1) parabola (3) circle
(2) straight line (4) hyperbola

22 If $\pi = \frac{A}{r^2}$, then

(1) r varies directly as the square of A
(2) r varies inversely as the square of A
(3) A varies directly as the square of r
(4) A varies inversely as the square of r

23 The minimum value of $\frac{1}{4} \sin \frac{1}{2} x$ as x varies from 0 to 4π is

(1) $+\frac{1}{4}$ (3) $+\frac{1}{2}$
(2) $-\frac{1}{4}$ (4) $-\frac{1}{2}$

24 The expression $\sin (270° + A)$ is equivalent to

(1) $-\sin A$ (3) $\sin A$
(2) $-\cos A$ (4) $\cos A$

25 In triangle ABC, if angle $A = 40°$, $a = 5$, and $b = 6$, then angle B

(1) must be acute
(2) must be obtuse
(3) may be a right angle
(4) may be either acute or obtuse

26 The expression $\cos A \cos B + \sin A \sin B$ is equivalent to

(1) $\sin (A - B)$ (3) $\sin (A + B)$
(2) $\cos (A - B)$ (4) $\cos (A + B)$

27 If the vertex angle of an isosceles triangle is 30° and each leg is 6, the area of the triangle is equal to

(1) 9 (3) 18
(2) $9\sqrt{3}$ (4) $18\sqrt{3}$

28 In triangle ABC, if $a = 10$, $A = 45°$, and $\sin B = \frac{1}{2}$, the length of side $b =$

(1) 5 (3) $\frac{10\sqrt{3}}{3}$
(2) $5\sqrt{2}$ (4) $10\sqrt{2}$

29 If $\tan x = m$, the expression $\tan 2x =$

(1) $\frac{2m}{1 - m^2}$ (3) $m^2 + 1$
(2) $\frac{2m}{m^2 + 1}$ (4) $2m$

30 In $\triangle ABC$, if $a = 4$, $b = 3$, and $\cos C = -\frac{1}{4}$, the value of $c =$

(1) 7 (3) $\sqrt{37}$
(2) $\sqrt{13}$ (4) $\sqrt{19}$

Answers to the following questions are to be written on paper supplied by the school.

Part II

Answer four questions from this part. Show all work unless otherwise directed.

31 *a* Find to the *nearest tenth* the values of $\tan\theta$ which satisfy the equation $2\tan^2\theta = 5\tan\theta + 1$. [8]

b How many values of θ are there between 0° and 360° that satisfy the equation $2\tan^2\theta = 5\tan\theta + 1$? [2]

32 *a* On the same set of axes, sketch the graphs of $y = 2\cos x$ and $y = \sin 2x$ as x varies from $-\pi$ to $+\pi$ radians. [Label each curve with its equation.] [4, 4]

b What values of x between $-\pi$ and $+\pi$ satisfy the equation $2\cos x = \sin 2x$? [2]

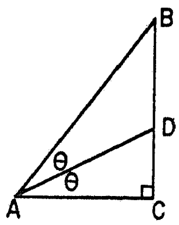
33 The time t in seconds for one complete oscillation of a pendulum of length l feet is given by $t = 2\pi\sqrt{\dfrac{l}{g}}$ where $g = 32.2$ feet per second per second. Using logarithms, find t to the *nearest hundredth of a second* for a pendulum 1.78 feet long. [Use the approximation $\pi = 3.14$.] [10]

34 *a* Prove the identity:

$$\frac{2\sin^2 A}{\sin 2A} + \cot A = \sec A \csc A \qquad [5]$$

b Given in the accompanying diagram right $\triangle ABC$ with angle C the right angle. The bisector of angle BAC intersects BC at D. If angle DAC is represented by θ, show that

$$AD = \frac{AB(\cos^2\theta - \sin^2\theta)}{\cos\theta}. \qquad [5]$$



35 Write an equation or a system of equations that would be used to solve the following problems. In each case, state what the variable or variables represent. [*Solution of the equations is not required.*]

a Two cities are 224 miles apart. If the speed of a train going from City A to City B were to be increased by 8 miles per hour, the train would reach City B $\frac{1}{2}$ hour sooner. How many hours did the trip take originally? [5]

b John, working alone, takes twice as long as Henry to paint a room. Henry starts alone and, after working for 3 hours, is joined by John. They work together for 2 hours to finish the painting. How long would it have taken Henry to paint the room, working alone? [5]

36 Answer *either a* or *b*, but *not* both:

a Two forces F_1 and F_2 have magnitudes of 25 pounds and 40 pounds, respectively, and act upon a body at an angle of 67° between them. Find to the *nearest pound* the resultant force F_3 of these two forces. [10]

OR

b An aircraft carrier is stationed 860 miles from Cape Kennedy, at a bearing from Cape Kennedy of 92° 10′ (S 87° 50′ E). A spaceship "splashes down" at a point due east of Cape Kennedy and at a bearing of 286° (N 74° W) from the carrier. Find to the *nearest mile* the distance from the spacecraft to the aircraft carrier. [5, 5]

37 Solve the following system of equations for x, y, and z: [10]

$$\begin{aligned} x + 6y + 2z &= 3 \\ 3x + 2y - z &= -6 \\ 2x - 4y + 3z &= -3 \end{aligned}$$

206

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Part I Score:....................
Rater's Initials:
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Regents High School Examination

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ANSWER SHEET

Pupil...Teacher...

School..

Your answers to Part I should be recorded on this answer sheet.

Part I

Answer all questions in this part.

1..............................	11..............................	21..............................
2..............................	12..............................	22..............................
3..............................	13..............................	23..............................
4..............................	14..............................	24..............................
5..............................	15..............................	25..............................
6..............................	16..............................	26..............................
7..............................	17..............................	27..............................
8..............................	18..............................	28..............................
9..............................	19..............................	29..............................
10..............................	20..............................	30..............................

Your answers for Part II should be placed on paper supplied by the school.

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208

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FOR TEACHERS ONLY

11

SCORING KEY
ELEVENTH YEAR MATHEMATICS

Wednesday, January 25, 1967 — 1:15 to 4:15 p.m., only

Use only *red* ink or pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use checkmarks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. In problems involving logarithms, answers should be left correct to four significant digits unless directions say otherwise. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow 2 credits for each correct answer; allow no partial credit. For questions 15–30, allow credit if the pupil has written the correct answer instead of the number 1, 2, 3, or 4.

(1) $(3 \sin x + 4)(\sin x - 1)$

(2) $3\frac{1}{5}$

(3) $\frac{c}{b}$

(4) $15° \ 27'$

(5) $\frac{4\sqrt{3} - 2}{11}$

(6) 4

(7) 2

(8) -5

(9) $\frac{c}{a - b}$

(10) $y = 2x + 6$

(11) 2

(12) $\frac{2}{3}$

(13) $120°$

(14) $\frac{4\pi}{3}$

(15) 3

(16) 1

(17) 2

(18) 3

(19) 2

(20) 3

(21) 1

(22) 3

(23) 2

(24) 2

(25) 4

(26) 2

(27) 1

(28) 2

(29) 1

(30) 3

Part II

Please refer to the Department's pamphlet *Suggestions on the Rating of Regents Examination Papers in Mathematics.* Care should be exercised in making deductions as to whether the error is purely a mechanical one or due to a violation of some principle. A mechanical error generally should receive a deduction of 10 percent, while an error due to a violation of some cardinal principle should receive a deduction ranging from 30 percent to 50 percent, depending on the relative importance of the principle in the solution of the problem.

(31) *a* 2.7 and —0.2 [8]
b four [2]

(32) *b* $-\frac{\pi}{2}$ and $\frac{\pi}{2}$ [2]

(33) 1.48 [10]

(35) *a* $x =$ number of hours trip took originally
$\left(\frac{224}{x} + 8\right)\left(x - \frac{1}{2}\right) = 224$ [5]
b $x =$ number of hours Henry takes alone
$\frac{5}{x} + \frac{2}{2x} = 1$ [5]

(36) *a* 55 [10]
b 118 [5, 5]

(37) $x = -2,\ y = \frac{1}{2},\ z = 1$ [10]