ALGEBRA 2/TRIGONOMETRY

The University of the State of New York
REGENTS HIGH SCHOOL EXAMINATION

ALGEBRA 2/TRIGONOMETRY

Friday, June 19, 2015 — 9:15 a.m. to 12:15 p.m., only

Student Name: Mr. Sibol
School Name: JMAP

The possession or use of any communications device is strictly prohibited when taking this examination. If you have or use any communications device, no matter how briefly, your examination will be invalidated and no score will be calculated for you.

Print your name and the name of your school on the lines above.

A separate answer sheet for Part I has been provided to you. Follow the instructions from the proctor for completing the student information on your answer sheet.

This examination has four parts, with a total of 39 questions. You must answer all questions in this examination. Record your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in Parts II, III, and IV directly in this booklet. All work should be written in pen, except for graphs and drawings, which should be done in pencil. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc.

The formulas that you may need to answer some questions in this examination are found at the end of the examination. This sheet is perforated so you may remove it from this booklet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. You may remove this sheet from this booklet. Any work done on this sheet of scrap graph paper will not be scored.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

Notice...
A graphing calculator and a straightedge (ruler) must be available for you to use while taking this examination.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.
Part I

Answer all 27 questions in this part. Each correct answer will receive 2 credits. For each statement or question, choose the word or expression that, of those given, best completes the statement or answers the question. Record your answers on your separate answer sheet.

1 Which list of ordered pairs does not represent a one-to-one function?
   (1) (1, -1), (2, 0), (3, 1), (4, 2)
   (2) (1, 2), (2, 3), (3, 4), (4, 6)
   (3) (1, 3), (2, 4), (3, 3), (4, 1)
   (4) (1, 5), (2, 4), (3, 1), (4, 0)

2 The terminal side of an angle measuring \( \frac{4\pi}{5} \) radians lies in Quadrant
   (1) I
   (2) II
   (3) III
   (4) IV

3 If \( f(x) = 2x^2 + 1 \) and \( g(x) = 3x - 2 \), what is the value of \( f(g(-2)) \)?
   (1) -127
   (2) -23
   (3) 25
   (4) 129

4 The expression \( \sqrt[3]{27a^3} \cdot \sqrt[4]{16b^8} \) is equivalent to
   (1) \( 6ab^3 \)
   (2) \( 6ab^4 \)
   (3) \( 12ab^2 \)
   (4) \( 12ab^4 \)

5 If \( x^2 = 12x - 7 \) is solved by completing the square, one of the steps in the process is
   (1) \( (x - 6)^2 = -43 \)
   (2) \( (x + 6)^2 = -43 \)
   (3) \( (x - 6)^2 = 29 \)
   (4) \( (x + 6)^2 = 29 \)
6 Which expression is equivalent to \( \frac{x^{-1}y^2}{x^2y^{-4}} \)?

(1) \( \frac{x}{y^2} \)  
(2) \( \frac{x^3}{y^6} \)  
(3) \( \frac{y^2}{x} \)  
(4) \( \frac{y^6}{x^3} \)

7 What is the solution of the inequality \( 9 - x^2 < 0 \)?

(1) \( \{x \mid -3 < x < 3\} \)  
(2) \( \{x \mid x > 3 \text{ or } x < -3\} \)  
(3) \( \{x \mid x > 3\} \)  
(4) \( \{x \mid x < -3\} \)

8 What is the area of a parallelogram that has sides measuring 8 cm and 12 cm and includes an angle of 120°?

(1) \(24\sqrt{3}\)  
(2) \(48\sqrt{3}\)  
(3) \(83\sqrt{3}\)  
(4) \(96\sqrt{3}\)

9 The expression \( \frac{5}{4 - \sqrt{11}} \) is equivalent to

(1) \(4 + \sqrt{11}\)  
(2) \(\frac{20 + 5\sqrt{11}}{27}\)  
(3) \(4 - \sqrt{11}\)  
(4) \(\frac{20 - 5\sqrt{11}}{27}\)

10 Given \(y\) varies inversely as \(x\), when \(y\) is multiplied by \(\frac{1}{2}\), then \(x\) is multiplied by

(1) \(\frac{1}{2}\)  
(2) \(2\)  
(3) \(-\frac{1}{2}\)  
(4) \(-2\)

Use this space for computations.
11 What is the total number of different nine-letter arrangements that can be formed using the letters in the word “TENNESSEE”?

(1) 3,780  (2) 15,120  (3) 45,360  (4) 362,880

\[
\frac{9!}{4! 2! 2!} = 3780
\]

12 What is the fourth term of the sequence defined by

\[
a_1 = 3xy^5 \\
a_n = \left( \frac{2x}{y} \right) a_{n-1}^2
\]

(1) \(12x^3y^3\)  (2) \(24x^2y^4\)  (3) \(24x^4y^2\)  (4) \(48x^5y\)

\[
3xy^5 \cdot \left( \frac{2x}{y} \right)^3 = 24x^4y^2
\]

13 What is the solution set of \(|x - 2| = 3x + 10|\)?

(1) \{ \}  (2) \{-2\}  (3) \{-6\}  (4) \{-2, -6\}

\[
\begin{align*}
|x - 2| &= 3x + 10 \\
x - 2 &= 3x + 10 \\
-12 &= 2x \\
x &= -6
\end{align*}
\]

\[
\begin{align*}
|4x| &= 2 \\
4x &= -8 \\
x &= -2
\end{align*}
\]

14 By law, a wheelchair service ramp may be inclined no more than 4.76°. If the base of a ramp begins 15 feet from the base of a public building, which equation could be used to determine the maximum height, \(h\), of the ramp where it reaches the building’s entrance?

(1) \(\sin 4.76^\circ = \frac{h}{15}\)  (3) \(\tan 4.76^\circ = \frac{h}{15}\)

(2) \(\sin 4.76^\circ = \frac{15}{h}\)  (4) \(\tan 4.76^\circ = \frac{15}{h}\)

\[
\tan 4.76^\circ = \frac{h}{15}
\]

\[
3(-2) + 10 = -8
\]
15 When \( \frac{7}{8}x^2 - \frac{3}{4}x \) is subtracted from \( \frac{5}{8}x^2 - \frac{1}{4}x + 2 \), the difference is

(1) \(-\frac{1}{4}x^2 - x + 2\)  
(2) \(\frac{1}{4}x^2 - x + 2\)  
(3) \(-\frac{1}{4}x^2 + \frac{1}{2}x + 2\)  
(4) \(\frac{1}{4}x^2 - \frac{1}{2}x - 2\)

16 Which transformation of \( y = f(x) \) moves the graph 7 units to the left and 3 units down?

(1) \(y = f(x + 7) - 3\)  
(2) \(y = f(x + 7) + 3\)  
(3) \(y = f(x - 7) - 3\)  
(4) \(y = f(x - 7) + 3\)

17 If \( \log x = 2 \log a + \log b \), then \( x \) equals

(1) \(a^2b\)  
(2) \(2ab\)  
(3) \(a^2 + b\)  
(4) \(2a + b\)

18 Which value is in the domain of the function graphed below, but is not in its range?

(1) 0  
(2) 2  
(3) 3  
(4) 7
19 How many full cycles of the function \( y = 3 \sin 2x \) appear in \( \pi \) radians?

- (1) 1
- (2) 2
- (3) 3
- (4) 4

20 A theater has 35 seats in the first row. Each row has four more seats than the row before it. Which expression represents the number of seats in the \( n \)th row?

- (1) 35 + (n + 4)
- (2) 35 + (4n)
- (3) 35 + (n + 1)
- (4) 35 + (n - 1)

21 What is the inverse of the function \( f(x) = \log_4 x \)?

- (1) \( f^{-1}(x) = x^4 \)
- (2) \( f^{-1}(x) = 4^x \)
- (3) \( f^{-1}(x) = \log_x 4 \)
- (4) \( f^{-1}(x) = -\log_4 x \)

22 The expression \( \frac{1 + \cos 2A}{\sin 2A} \) is equivalent to

- (1) \( \cot A \)
- (2) \( \tan A \)
- (3) \( \sec A \)
- (4) \( 1 + \cot 2A \)

23 A video-streaming service can choose from six half-hour shows and four one-hour shows. Which expression could be used to calculate the number of different ways the service can choose four half-hour shows and two one-hour shows?

- (1) \( 6P_4 \cdot 4P_2 \)
- (2) \( 6P_4 + 4P_2 \)
- (3) \( 6C_4 \cdot 4C_2 \)
- (4) \( 6C_4 + 4C_2 \)
24 The roots of $3x^2 + x = 14$ are

(1) imaginary
(2) real, rational, and equal
(3) real, rational, and unequal
(4) real, irrational, and unequal

25 Circle $O$ has a radius of 2 units. An angle with a measure of \( \frac{\pi}{6} \) radians is in standard position. If the terminal side of the angle intersects the circle at point $B$, what are the coordinates of $B$?

(1) \( \left( \frac{\sqrt{3}}{2}, \frac{1}{2} \right) \)
(2) \( \left( \sqrt{3}, 1 \right) \)
(3) \( \left( \frac{1}{2}, \frac{\sqrt{3}}{2} \right) \)
(4) \( \left( 1, \sqrt{3} \right) \)

26 What is the value of \( \sum_{x=0}^{2} (3 - 2a)^x \)?

(1) \( 4a^2 - 2a + 12 \)
(2) \( 4a^2 - 2a + 13 \)
(3) \( 4a^2 - 14a + 12 \)
(4) \( 4a^2 - 14a + 13 \)

27 A population, $p(x)$, of wild turkeys in a certain area is represented by the function $p(x) = 17(1.15)^{2x}$, where $x$ is the number of years since 2010. How many more turkeys will be in the population for the year 2015 than 2010?

(1) 46
(2) 49
(3) 51
(4) 68

\[ 17(1.15)^{2(5)} - 17(1.15)^{2(0)} = 68.8 - 17 = 51.8 \]
Part II

Answer all 8 questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [16]

28 Solve algebraically for \( x \):

\[
5^{4x} = 125^x - 1
\]

\[
5^{4x} = (5^3)^x - 1
\]

\[
4x = 3x - 3
\]

\[
x = 3
\]
29 In triangle $ABC$, determine the number of distinct triangles that can be formed if $\angle A = 85$, side $a = 8$, and side $c = 2$. Justify your answer.

\[
\frac{8}{\sin 85} = \frac{2}{\sin C}
\]

\[
C = \sin^{-1} \left( \frac{2 \cdot \sin 85}{8} \right)
\]

$C \approx 14.4$

$85 + 14.4 < 180$  $85 + 165.6 > 180$

1 triangle
30 The probability that Kay and Joseph Dowling will have a redheaded child is 1 out of 4. If the Dowlings plan to have three children, what is the exact probability that only one child will have red hair?

\[
3 \cdot \binom{3}{1} \left(\frac{1}{4}\right)^1 \left(\frac{3}{4}\right)^2
\]

\[
3 \cdot \frac{1}{4} \cdot \frac{9}{16}
\]

\[
\frac{27}{64}
\]
31 If \( \log_{(x + 1)} 64 = 3 \), find the value of \( x \).

\[
(x + 1)^3 = 64
\]

\[
x + 1 = 4
\]

\[
x = 3
\]
32 Factor completely:

\[ x^3 - 6x^2 - 25x + 150 \]

\[ x^2(x - 6) - 25(x - 6) \]

\[ (x^2 - 25)(x - 6) \]

\[ (x + 5)(x - 5)(x - 6) \]
33 Express $x^8 - y^6$ in simplest form.

$x(1) - y(-1)$

$x + y$

34 Given the equation $3x^2 + 2x + k = 0$, state the sum and product of the roots.

$S = \frac{-b}{a} = \frac{-2}{3}$

$P = \frac{c}{a} = \frac{k}{3}$
35 Determine which set of data given below has the stronger linear relationship between $x$ and $y$. Justify your choice.

<table>
<thead>
<tr>
<th>Set A</th>
<th>x</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>y</td>
<td>24</td>
<td>30</td>
<td>36</td>
<td>51</td>
<td>70</td>
<td>86</td>
</tr>
<tr>
<td>r</td>
<td></td>
<td>.976</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Set B</th>
<th>x</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>y</td>
<td>31</td>
<td>64</td>
<td>49</td>
<td>36</td>
<td>25</td>
<td>16</td>
</tr>
<tr>
<td>r</td>
<td></td>
<td>.994</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Part III

Answer all 3 questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [12]

36 Find the measure of the smallest angle, to the nearest degree, of a triangle whose sides measure 28, 47, and 34.

\[ 28^2 = 47^2 + 34^2 - 2(47)(34) \cos A \]
\[ 784 = 3365 - 3196 \cos A \]
\[ -2481 = -3196 \cos A \]
\[ \frac{2481}{3196} = \cos A \]
\[ 36 \approx A \]
Solve algebraically for \( x \):

\[
\frac{3}{x} + \frac{x}{x+2} = \frac{2}{x+2}
\]

\[
\frac{3(x+2)}{x(x+2)} + \frac{x^2}{x(x+2)} = \frac{-2x}{x(x+2)}
\]

\[
3x + 6 + x^2 = -2x
\]

\[
x^2 + 5x + 6 = 0
\]

\[
(x + 3)(x + 2) = 0
\]

\[
x = -3 \quad \text{or} \quad x = -2
\]

\[\text{extraneous}\]
The table below shows the final examination scores for Mr. Spear's class last year.

<table>
<thead>
<tr>
<th>Test Score</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>72</td>
<td>1</td>
</tr>
<tr>
<td>76</td>
<td>1</td>
</tr>
<tr>
<td>79</td>
<td>4</td>
</tr>
<tr>
<td>83</td>
<td>5</td>
</tr>
<tr>
<td>85</td>
<td>7</td>
</tr>
<tr>
<td>88</td>
<td>5</td>
</tr>
<tr>
<td>94</td>
<td>3</td>
</tr>
</tbody>
</table>

Find the population standard deviation based on these data, to the nearest hundredth.

5.17

Determine the number of students whose scores are within one population standard deviation of the mean.

84.46 ± 5.17
79.29 - 89.63
517 + 5 = 17
39 In the interval $0^\circ \leq \theta < 360^\circ$, solve the equation $5 \cos \theta = 2 \sec \theta - 3$ algebraically for all values of $\theta$, to the nearest tenth of a degree.

\[
5 \cos \theta - \frac{2}{\cos \theta} + 3 = 0
\]

\[
5 \cos^2 \theta - 2 + 3 \cos \theta = 0
\]

\[
5x^2 + 3x - 2 = 0
\]

\[
(5x - 2)(x + 1) = 0
\]

\[
x = \frac{2}{5}, \quad x = -1
\]

\[
\cos \theta = \frac{2}{5}, \quad \cos \theta = -1
\]

$66.4^\circ, 293.6^\circ, 180^\circ$