

ALGEBRA 2/TRIGONOMETRY

The University of the State of New York
REGENTS HIGH SCHOOL EXAMINATION

ALGEBRA 2/TRIGONOMETRY

Wednesday, January 23, 2013 — 1:15 to 4:15 p.m., only

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School Name: JMAP

The possession or use of any communications device is strictly prohibited when taking this examination. If you have or use any communications device, no matter how briefly, your examination will be invalidated and no score will be calculated for you.

Print your name and the name of your school on the lines above.

A separate answer sheet for Part I has been provided to you. Follow the instructions from the proctor for completing the student information on your answer sheet.

This examination has four parts, with a total of 39 questions. You must answer all questions in this examination. Record your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in Parts II, III, and IV directly in this booklet. All work should be written in pen, except for graphs and drawings, which should be done in pencil. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc.

The formulas that you may need to answer some questions in this examination are found at the end of the examination. This sheet is perforated so you may remove it from this booklet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. You may remove this sheet from this booklet. Any work done on this sheet of scrap graph paper will *not* be scored.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

Notice...

A graphing calculator and a straightedge (ruler) must be available for you to use while taking this examination.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.

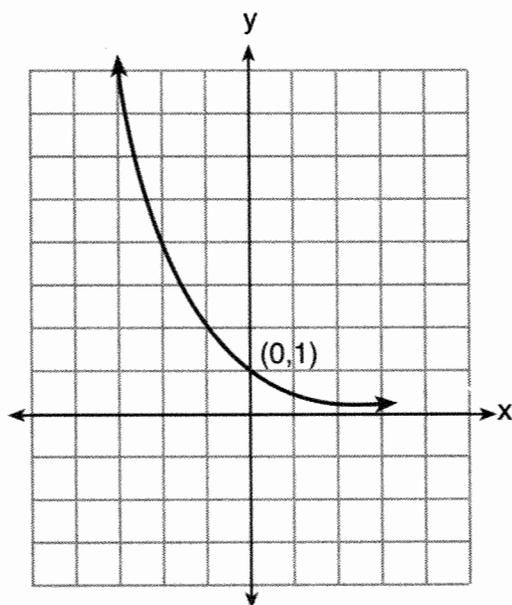
ALGEBRA 2/TRIGONOMETRY

Part I

Answer all 27 questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. For each statement or question, choose the word or expression that, of those given, best completes the statement or answers the question. Record your answers on your separate answer sheet. [54]

Use this space for computations.

1 What is the equation of the graph shown below?



- (1) $y = 2^x$
 (2) $y = 2^{-x}$

- (3) $x = 2^y$
 (4) $x = 2^{-y}$

2 Which ordered pair is a solution of the system of equations shown below?

$$x + y = 5$$

$$(x + 3)^2 + (y - 3)^2 = 53$$

- (1) (2,3)
 (2) (5,0)

- (3) (-5,10)
 (4) (-4,9)

$$(x+3)^2 + (-x+5-3)^2 = 53$$

$$x^2 + 6x + 9 + x^2 - 4x + 4 = 53$$

$$2x^2 + 2x - 40 = 0$$

$$x^2 + x - 20 = 0$$

$$(x+5)(x-4) = 0$$

$$x = -5 \quad x = 4$$

$$-5 + y = 5 \quad 4 + y = 5$$

$$y = 10 \quad y = 1$$

Use this space for
computations.

3 The relationship between t , a student's test scores, and d , the student's success in college, is modeled by the equation $d = 0.48t + 75.2$. Based on this linear regression model, the correlation coefficient could be

- (1) between -1 and 0 (3) equal to -1
(2) between 0 and 1 (4) equal to 0

4 What is the common ratio of the geometric sequence shown below?

$-2, 4, -8, 16, \dots$

- (1) $-\frac{1}{2}$ (3) -2
(2) 2 (4) -6

5 Given the relation $\{(8,2), (3,6), (7,5), (k,4)\}$, which value of k will result in the relation *not* being a function?

- (1) 1 (3) 3
(2) 2 (4) 4

6 Which expression is equivalent to $(9x^2y^6)^{-\frac{1}{2}}$?

- (1) $\frac{1}{3xy^3}$ (3) $\frac{3}{xy^3}$
(2) $3xy^3$ (4) $\frac{xy^3}{3}$

$$9^{-1/2} x^{-1} y^{-3}$$
$$\frac{1}{3xy^3}$$

Use this space for computations.

7 In a certain high school, a survey revealed the mean amount of bottled water consumed by students each day was 153 bottles with a standard deviation of 22 bottles. Assuming the survey represented a normal distribution, what is the range of the number of bottled waters that approximately 68.2% of the students drink?

- (1) 131-164
(2) 131-175
(3) 142-164
(4) 142-175

$$\begin{aligned} & \mu \pm \sigma \\ & 153 \pm 22 \\ & 131 - 175 \end{aligned}$$

8 What is the fourth term in the binomial expansion $(x - 2)^8$?

- (1) $448x^5$
(2) $448x^4$
(3) $-448x^5$
(4) $-448x^4$

$$\begin{aligned} & 8C_3 x^{8-3} (-2)^3 \\ & 56x^5 \cdot -8 \\ & -448x^5 \end{aligned}$$

9 Which value of k satisfies the equation $8^{3k+4} = 4^{2k-1}$?

- (1) -1
(2) $-\frac{9}{4}$
(3) -2
(4) $-\frac{14}{5}$

$$\begin{aligned} & (2^3)^{3k+4} = (2^2)^{2k-1} \\ & 2^{9k+12} = 2^{4k-2} \\ & 9k+12 = 4k-2 \\ & 5k = -14 \\ & \frac{5k}{5} = \frac{-14}{5} \end{aligned}$$

10 There are eight people in a tennis club. Which expression can be used to find the number of different ways they can place first, second, and third in a tournament?

- (1) ${}_8P_3$
(2) ${}_8C_3$
(3) ${}_8P_5$
(4) ${}_8C_5$

Use this space for computations.

11 If $\sin A = \frac{1}{3}$, what is the value of $\cos 2A$? $\cos 2A = 1 - 2\sin^2 A$

(1) $-\frac{2}{3}$

(3) $-\frac{7}{9}$

(2) $\frac{2}{3}$

(4) $\frac{7}{9}$

$$= 1 - 2\left(\frac{1}{3}\right)^2$$

$$= 1 - \frac{2}{9}$$

$$= \frac{7}{9}$$

12 In the interval $0^\circ \leq x < 360^\circ$, $\tan x$ is undefined when x equals

(1) 0° and 90°

(3) 180° and 270°

(2) 90° and 180°

(4) 90° and 270°

13 If $f(x) = \sqrt{9 - x^2}$, what are its domain and range?

(1) domain: $\{x \mid -3 \leq x \leq 3\}$; range: $\{y \mid 0 \leq y \leq 3\}$

(2) domain: $\{x \mid x \neq \pm 3\}$; range: $\{y \mid 0 \leq y \leq 3\}$

(3) domain: $\{x \mid x \leq -3 \text{ or } x \geq 3\}$; range: $\{y \mid y \neq 0\}$

(4) domain: $\{x \mid x \neq 3\}$; range: $\{y \mid y \geq 0\}$

14 When $x^2 + 3x - 4$ is subtracted from $x^3 + 3x^2 - 2x$, the difference is

(1) $x^3 + 2x^2 - 5x + 4$

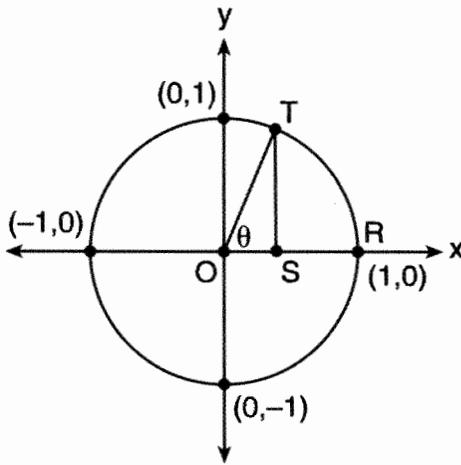
(3) $-x^3 + 4x^2 + x - 4$

(2) $x^3 + 2x^2 + x - 4$

(4) $-x^3 - 2x^2 + 5x + 4$

Use this space for computations.

15 In the diagram below, the length of which line segment is equal to the exact value of $\sin \theta$?



- (1) \overline{TO} (3) \overline{OR}
 (2) \overline{TS} (4) \overline{OS}

16 The area of triangle ABC is 42. If $AB = 8$ and $m\angle B = 61$, the length of \overline{BC} is approximately

- (1) 5.1 (3) 12.0
 (2) 9.2 (4) 21.7

$$42 = \frac{1}{2} (a)(8) \sin 61$$

$$42 \approx 3.5 a$$

$$12 \approx a$$

17 When factored completely, the expression $3x^3 - 5x^2 - 48x + 80$ is equivalent to

- (1) $(x^2 - 16)(3x - 5)$
 (2) $(x^2 + 16)(3x - 5)(3x + 5)$
 (3) $(x + 4)(x - 4)(3x - 5)$
 (4) $(x + 4)(x - 4)(3x - 5)(3x - 5)$

$$x^2(3x - 5) - 16(3x - 5)$$

$$(x^2 - 16)(3x - 5)$$

$$(x + 4)(x - 4)(3x - 5)$$

18 The value of $\sin(180 + x)$ is equivalent to

- (1) $-\sin x$ (3) $\sin x$
 (2) $-\sin(90 - x)$ (4) $\sin(90 - x)$

Use this space for computations.

$$\sin(180+x)$$

$$\sin 180 \cos x + \cos 180 \sin x$$

$$0 \cos x - 1 \sin x$$

$$-\sin x$$

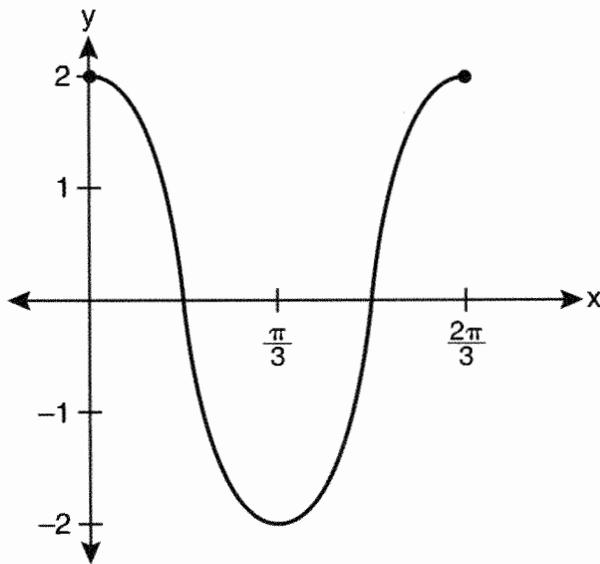
19 The sum of $\sqrt[3]{6a^4b^2}$ and $\sqrt[3]{162a^4b^2}$, expressed in simplest radical form, is

- (1) $\sqrt[6]{168a^8b^4}$ (3) $4a\sqrt[3]{6ab^2}$
 (2) $2a^2b\sqrt[3]{21a^2b}$ (4) $10a^2b\sqrt[3]{8}$

$$\sqrt[3]{6ab^2} + 3a\sqrt[3]{6ab^2}$$

$$4a\sqrt[3]{6ab^2}$$

20 Which equation is represented by the graph below?



- (1) $y = 2 \cos 3x$ (3) $y = 2 \cos \frac{2\pi}{3}x$
 (2) $y = 2 \sin 3x$ (4) $y = 2 \sin \frac{2\pi}{3}x$

Use this space for computations.

21 The quantities p and q vary inversely. If $p = 20$ when $q = -2$, and $p = x$ when $q = -2x + 2$, then x equals

- (1) -4 and 5
(2) $\frac{20}{19}$
(3) -5 and 4
(4) $-\frac{1}{4}$

$$\begin{aligned}20(-2) &= x(-2x+2) \\ -40 &= -2x^2+2x \\ 2x^2-2x-40 &= 0 \\ x^2-x-20 &= 0 \\ (x+4)(x-5) &= 0\end{aligned}$$

22 What is the solution set of the equation $-\sqrt{2} \sec x = 2$ when $0^\circ \leq x < 360^\circ$?

- (1) $\{45^\circ, 135^\circ, 225^\circ, 315^\circ\}$
(2) $\{45^\circ, 315^\circ\}$
(3) $\{135^\circ, 225^\circ\}$
(4) $\{225^\circ, 315^\circ\}$

$$\begin{aligned}x &= -4, 5 \\ \sec x &= \frac{-2}{\sqrt{2}} \\ \cos x &= \frac{-\sqrt{2}}{2}\end{aligned}$$

23 The discriminant of a quadratic equation is 24. The roots are

- (1) imaginary
(2) real, rational, and equal
(3) real, rational, and unequal
(4) real, irrational, and unequal

$$b^2 - 4ac$$

24 How many different six-letter arrangements can be made using the letters of the word "TATTOO"?

- (1) 60
(2) 90
(3) 120
(4) 720

$$\frac{6P_6}{3!2!} = \frac{720}{12} = 60$$

Use this space for computations.

25 Expressed in simplest form, $\frac{3y}{2y-6} + \frac{9}{6-2y}$ is equivalent to

(1) $\frac{-6y^2 + 36y - 54}{(2y-6)(6-2y)}$

(3) $\frac{3}{2}$

$$\frac{3y}{2y-6} - \frac{9}{2y-6} = \frac{3y-9}{2y-6} = \frac{3(y-3)}{2(y-3)}$$

(2) $\frac{3y-9}{2y-6}$

(4) $-\frac{3}{2}$

26 If $\log 2 = a$ and $\log 3 = b$, the expression $\log \frac{9}{20}$ is equivalent to

(1) $2b - a + 1$

(3) $b^2 - a + 10$

$$\log 9 - \log 20$$

(2) $2b - a - 1$

(4) $\frac{2b}{a+1}$

$$\log 3^2 - \log(10 \cdot 2)$$

$$2\log 3 - (\log 10 + \log 2)$$

27 The expression $(x + i)^2 - (x - i)^2$ is equivalent to

(1) 0

(3) $-2 + 4xi$

$$2b - (1 + a)$$

(2) -2

(4) $4xi$

$$2b - a - 1$$

$$x^2 + 2xi + i^2 - (x^2 - 2xi + i^2)$$

$$4xi$$

Part II

Answer all 8 questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [16]

- 28 Determine the sum of the first twenty terms of the sequence whose first five terms are 5, 14, 23, 32, and 41.

$$a_n = 9n - 4$$

$$a_1 = 5$$

$$a_{20} = 176$$

$$S_n = \frac{20(5+176)}{2}$$
$$= 1810$$

29 Determine the sum and the product of the roots of $3x^2 = 11x - 6$.

$$3x^2 - 11x + 6 = 0$$

$$a = 3 \quad b = -11 \quad c = 6$$

$$\text{sum} = \frac{-b}{a} = \frac{11}{3}$$

$$\text{product} = \frac{c}{a} = \frac{6}{3} = 2$$

30 If $\sec(a + 15)^\circ = \csc(2a)^\circ$, find the smallest positive value of a , in degrees.

$$a + 15 + 2a = 90$$

$$3a + 15 = 90$$

$$3a = 75$$

$$a = 25$$

31 The heights, in inches, of 10 high school varsity basketball players are 78, 79, 79, 72, 75, 71, 74, 74, 83, and 71. Find the interquartile range of this data set.

71 71 (72) 74 74 | 75 78 (79) 79 83

$$79 - 72 = 7$$

32 Solve the equation $6x^2 - 2x - 3 = 0$ and express the answer in simplest radical form.

$$\frac{2 \pm \sqrt{(-2)^2 - 4(6)(-3)}}{2(6)}$$

$$\frac{2 \pm \sqrt{76}}{12}$$

$$\frac{2 \pm \sqrt{4\sqrt{19}}}{12}$$

$$\frac{2 \pm 2\sqrt{19}}{12}$$

$$\frac{1 \pm \sqrt{19}}{6}$$

- 33 The number of bacteria present in a Petri dish can be modeled by the function $N = 50e^{3t}$, where N is the number of bacteria present in the Petri dish after t hours. Using this model, determine, to the nearest hundredth, the number of hours it will take for N to reach 30,700.

$$\frac{30,700}{50} = \frac{50e^{3t}}{50}$$

$$614 = e^{3t}$$

$$\ln 614 = \ln e^{3t}$$

$$\frac{\ln 614}{3} = \frac{\cancel{3t} \ln \cancel{e}}{\cancel{3}}$$

$$2.14 \approx t$$

34 Determine the solution of the inequality $|3 - 2x| \geq 7$.

[The use of the grid below is optional.]

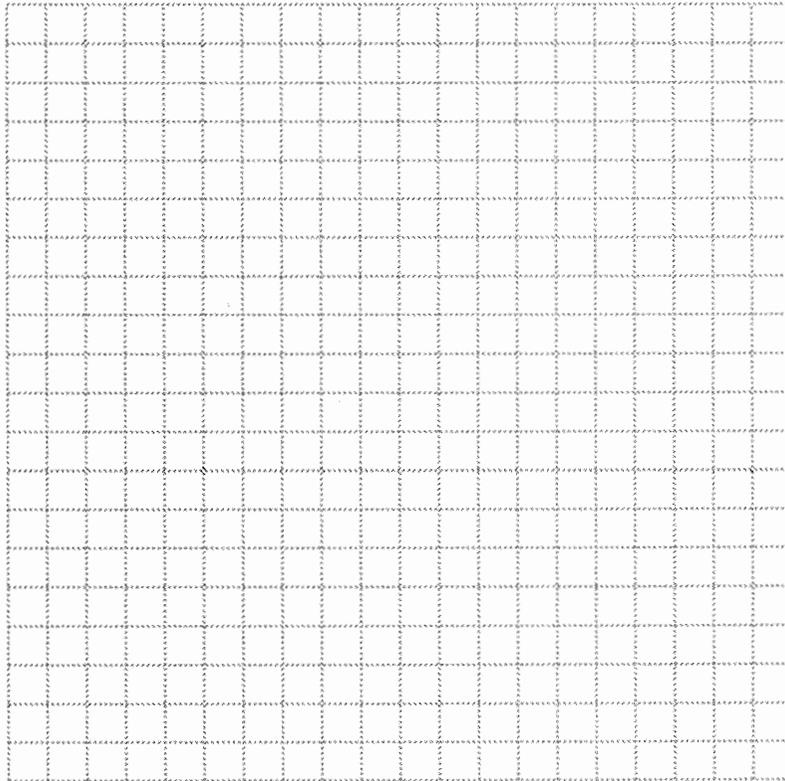
$$3 - 2x \geq 7$$

$$3 - 2x \leq -7$$

$$-2x \geq 4$$

$$-2x \leq -10$$

$$x \leq -2 \quad \text{or} \quad x \geq 5$$



35 Convert 3 radians to degrees and express the answer to the *nearest minute*.

$$3 \cdot \frac{180}{\pi} \approx 171.89 \approx 171^{\circ} 53'$$

Part III

Answer all 3 questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [12]

36 Solve algebraically for all values of x :

$$\log_{(x+4)}(17x - 4) = 2$$

$$(x+4)^2 = 17x - 4$$

$$x^2 + 8x + 16 = 17x - 4$$

$$x^2 - 9x + 20 = 0$$

$$(x-4)(x-5) = 0$$

$$x = 4, 5$$

37 The data collected by a biologist showing the growth of a colony of bacteria at the end of each hour are displayed in the table below.

Time, hour, (x)	0	1	2	3	4	5
Population (y)	250	330	580	800	1650	3000

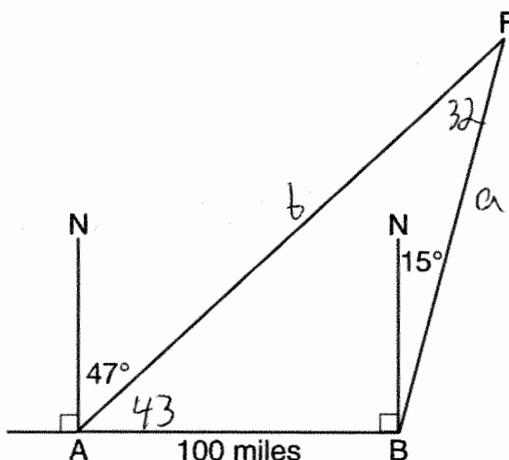
Write an exponential regression equation to model these data. Round *all* values to the *nearest thousandth*.

$$y = 215.983(1.652)^x$$

Assuming this trend continues, use this equation to estimate, to the *nearest ten*, the number of bacteria in the colony at the end of 7 hours.

$$215.983(1.652)^7 \approx 7250$$

- 38 As shown in the diagram below, fire-tracking station A is 100 miles due west of fire-tracking station B. A forest fire is spotted at F, on a bearing 47° northeast of station A and 15° northeast of station B. Determine, to the nearest tenth of a mile, the distance the fire is from both station A and station B. [N represents due north.]



$$\frac{100}{\sin 32} = \frac{b}{\sin 105}$$

$$b \approx 182.3$$

$$\frac{100}{\sin 32} = \frac{a}{\sin 43}$$

$$a \approx 128.7$$

Part IV

Answer the question in this part. A correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. A correct numerical answer with no work shown will receive only 1 credit. The answer should be written in pen. [6]

39 Solve algebraically for x : $\sqrt{x^2 + x - 1} + 11x = 7x + 3$

$$\sqrt{x^2 + x - 1} = -4x + 3$$

$$x^2 + x - 1 = 16x^2 - 24x + 9$$

$$0 = 15x^2 - 25x + 10$$

$$0 = 3x^2 - 5x + 2$$

$$0 = (3x - 2)(x - 1)$$

$$x = \frac{2}{3}$$

~~$$x = 1$$~~

$$-4\left(\frac{2}{3}\right) + 3 \geq 0$$

$$-4(1) + 3 < 0$$