

# ALGEBRA 2/TRIGONOMETRY

The University of the State of New York  
REGENTS HIGH SCHOOL EXAMINATION

# ALGEBRA 2/TRIGONOMETRY

Tuesday, January 25, 2011 — 1:15 to 4:15 p.m., only

Student Name: Mr. Sibol

School Name: HSCR

Print your name and the name of your school on the lines above. Then turn to the last page of this booklet, which is the answer sheet for Part I. Fold the last page along the perforations and, slowly and carefully, tear off the answer sheet. Then fill in the heading of your answer sheet.

This examination has four parts, with a total of 39 questions. You must answer all questions in this examination. Write your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in Parts II, III, and IV directly in this booklet. All work should be written in pen, except graphs and drawings, which should be done in pencil. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc.

The formulas that you may need to answer some questions in this examination are found at the end of the examination. This sheet is perforated so you may remove it from this booklet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. You may remove this sheet from this booklet. Any work done on this sheet of scrap graph paper will *not* be scored.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

**Notice...**

A graphing calculator and a straightedge (ruler) must be available for you to use while taking this examination.

The use of any communications device is strictly prohibited when taking this examination. If you use any communications device, no matter how briefly, your examination will be invalidated and no score will be calculated for you.

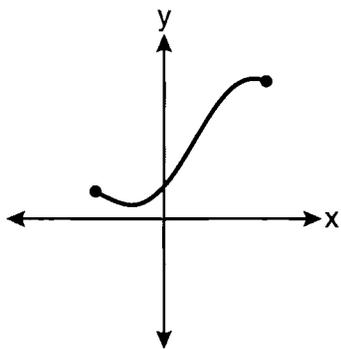
**DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.**

## Part I

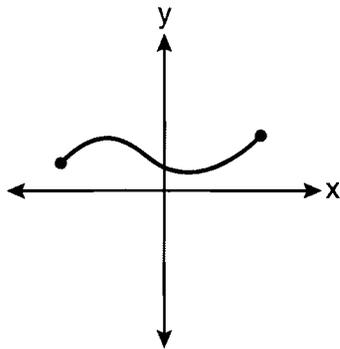
Answer all 27 questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. For each question, write on the separate answer sheet the numeral preceding the word or expression that best completes the statement or answers the question. [54]

Use this space for computations.

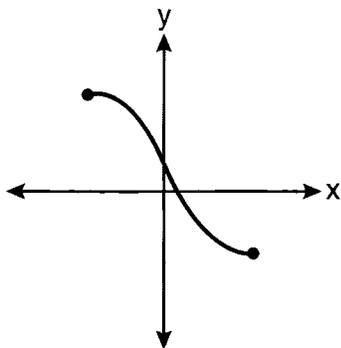
1 Which graph does *not* represent a function?



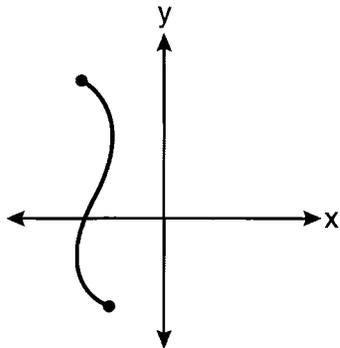
(1)



(3)



(2)



(4)

2 The roots of the equation  $x^2 - 10x + 25 = 0$  are

- (1) imaginary
- (2) real and irrational
- (3) real, rational, and equal
- (4) real, rational, and unequal

$$\begin{aligned} & b^2 - 4ac \\ & (-10)^2 - 4(1)(25) \\ & 100 - 100 \\ & 0 \end{aligned}$$

Use this space for computations.

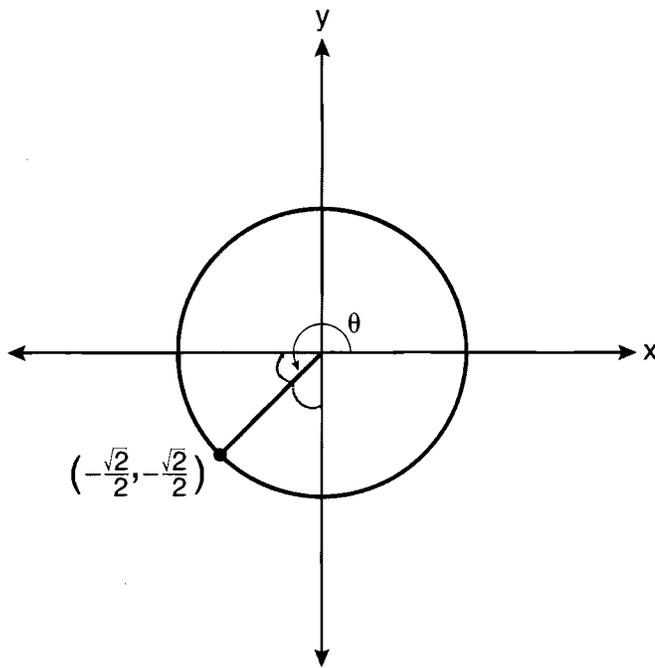
3 Which values of  $x$  are solutions of the equation  $x^3 + x^2 - 2x = 0$ ?

- (1) 0, 1, 2  
(2) 0, 1, -2

- (3) 0, -1, 2  
(4) 0, -1, -2

$$\begin{aligned}x(x^2 + x - 2) &= 0 \\x(x+2)(x-1) &= 0 \\0 \quad -2 \quad 1\end{aligned}$$

4 In the diagram below of a unit circle, the ordered pair  $\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$  represents the point where the terminal side of  $\theta$  intersects the unit circle.



What is  $m\angle\theta$ ?

- (1) 45  
(2) 135

- (3) 225  
(4) 240

Use this space for computations.

5 What is the fifteenth term of the sequence 5, -10, 20, -40, 80, ...?

- (1) -163,840  
(2) -81,920

- (3) 81,920  
(4) 327,680

$$a_n = 5(-2)^{n-1}$$

$$a_{15} = 5(-2)^{15-1} = 81,920$$

6 What is the solution set of the equation  $|4a + 6| - 4a = -10$ ?

- (1)  $\emptyset$   
(2)  $\{0\}$

- (3)  $\{\frac{1}{2}\}$   
(4)  $\{0, \frac{1}{2}\}$

$$4a+6 = 4a-10$$

$$6 \neq -10$$

$$4a+6 = -4a+10$$

$$8a = 4$$

$$a = \frac{4}{8} = \frac{1}{2}$$

7 If  $\sin A = \frac{2}{3}$  where  $0^\circ < A < 90^\circ$ , what is the value of  $\sin 2A$ ?

- (1)  $\frac{2\sqrt{5}}{3}$   
(2)  $\frac{2\sqrt{5}}{9}$

- (3)  $\frac{4\sqrt{5}}{9}$   
(4)  $-\frac{4\sqrt{5}}{9}$

$$14\left(\frac{1}{2}\right) + 6\left(-4\left(\frac{1}{2}\right)\right) = -10$$

$$8 - 2 \neq -10$$

$$\sin^2 A + \cos^2 A = 1$$

$$\left(\frac{2}{3}\right)^2 + \cos^2 A = 1$$

$$\frac{4}{9} + \cos^2 A = 1$$

$$\cos^2 A = \frac{5}{9}$$

$$\cos A = +\frac{\sqrt{5}}{3}$$

A is acute

$$\sin 2A = 2 \sin A \cos A$$
$$2\left(\frac{2}{3}\right)\left(\frac{\sqrt{5}}{3}\right)$$

$$\frac{4\sqrt{5}}{9}$$



Use this space for  
computations.

11 The conjugate of  $7 - 5i$  is

(1)  $-7 - 5i$

(3)  $7 - 5i$

(2)  $-7 + 5i$

(4)  $7 + 5i$

12 If  $\sin^{-1}\left(\frac{5}{8}\right) = A$ , then

(1)  $\sin A = \frac{5}{8}$

(3)  $\cos A = \frac{5}{8}$

(2)  $\sin A = \frac{8}{5}$

(4)  $\cos A = \frac{8}{5}$

13 How many distinct triangles can be formed if  $m\angle A = 35$ ,  $a = 10$ , and  $b = 13$ ?

(1) 1

(3) 3

(2) 2

(4) 0

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\frac{10}{\sin 35} = \frac{13}{\sin B}$$

$$B \approx 48$$

$$35 + 48 < 180$$

$$B \approx 132$$

$$35 + 132 < 180$$

Use this space for computations.

14 When  $\frac{3}{2}x^2 - \frac{1}{4}x - 4$  is subtracted from  $\frac{5}{2}x^2 - \frac{3}{4}x + 1$ , the difference is

(1)  $-x^2 + \frac{1}{2}x - 5$

(3)  $-x^2 - x - 3$

(2)  $x^2 - \frac{1}{2}x + 5$

(4)  $x^2 - x - 3$

15 The solution set of the inequality  $x^2 - 3x > 10$  is

(1)  $\{x \mid -2 < x < 5\}$

(3)  $\{x \mid x < -2 \text{ or } x > 5\}$

(2)  $\{x \mid 0 < x < 3\}$

(4)  $\{x \mid x < -5 \text{ or } x > 2\}$

$$x^2 - 3x - 10 > 0$$

$$(x-5)(x+2) > 0$$

$$x-5 > 0 \text{ or } x+2 > 0$$

$$\underline{x > 5} \text{ or } \underline{x > -2}$$

16 If  $x^2 + 2 = 6x$  is solved by completing the square, an intermediate step would be

(1)  $(x + 3)^2 = 7$

(3)  $(x - 3)^2 = 11$

(2)  $(x - 3)^2 = 7$

(4)  $(x - 6)^2 = 34$

$$x^2 - 6x = -2$$

$$x^2 - 6x + 9 = -2 + 9$$

$$(x-3)^2 = 7$$

17 Three marbles are to be drawn at random, without replacement, from a bag containing 15 red marbles, 10 blue marbles, and 5 white marbles. Which expression can be used to calculate the probability of drawing 2 red marbles and 1 white marble from the bag?

(1)  $\frac{15^2 \cdot 5}{30^3}$

(3)  $\frac{15^2 \cdot 5}{30^3}$

(2)  $\frac{15^2 \cdot 5}{30^3}$

(4)  $\frac{15^2 \cdot 5}{30^3}$

Use this space for computations.

18 The expression  $x^{-\frac{2}{5}}$  is equivalent to

(1)  $-\sqrt{x^5}$

(3)  $\frac{1}{\sqrt[2]{x^5}}$

(2)  $-\sqrt[5]{x^2}$

(4)  $\frac{1}{\sqrt[5]{x^2}}$

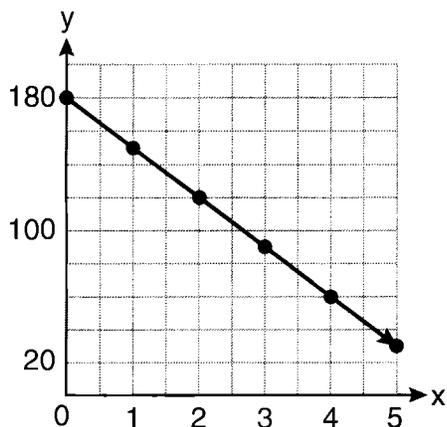
$x^{-\frac{2}{5}} =$

$\frac{1}{x^{\frac{2}{5}}} = \frac{1}{\sqrt[5]{x^2}}$

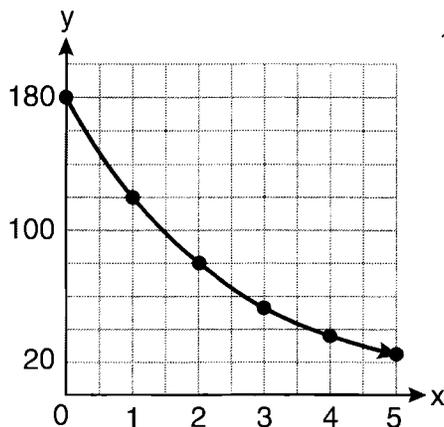
19 On January 1, a share of a certain stock cost \$180. Each month thereafter, the cost of a share of this stock decreased by one-third. If  $x$  represents the time, in months, and  $y$  represents the cost of the stock, in dollars, which graph best represents the cost of a share over the following 5 months?

Exponential

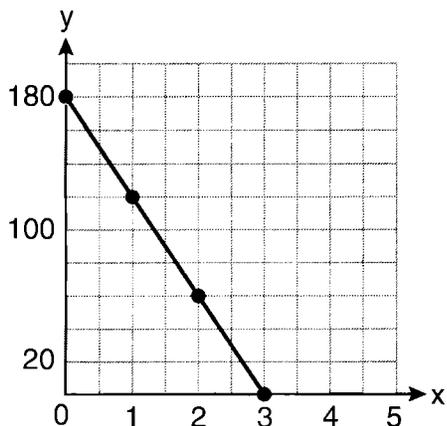
After one month, the stock decreases by \$60 to \$120



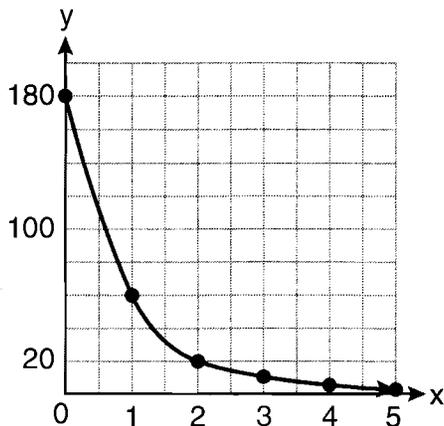
(1)



(3)



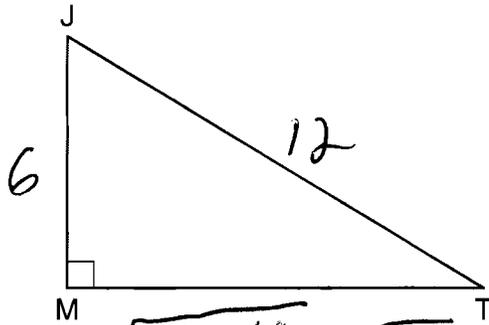
(2)



(4)

Use this space for computations.

- 20 In the diagram below of right triangle  $JTM$ ,  $JT = 12$ ,  $JM = 6$ , and  $m\angle JMT = 90^\circ$ .



$$\sqrt{12^2 - 6^2} = \sqrt{108} = \sqrt{36 \cdot 3} = 6\sqrt{3}$$

What is the value of  $\cot J$ ?

(1)  $\frac{\sqrt{3}}{3}$

(2) 2

(3)  $\sqrt{3}$

(4)  $\frac{2\sqrt{3}}{3}$

$$\cot J = \frac{\text{ADJ}}{\text{OPP}}, \quad \frac{6}{6\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

- 21 For which equation does the sum of the roots equal  $-3$  and the product of the roots equal  $2$ ?

(1)  $x^2 + 2x - 3 = 0$

(2)  $x^2 - 3x + 2 = 0$

(3)  $2x^2 + 6x + 4 = 0$

(4)  $2x^2 - 6x + 4 = 0$

$$-\frac{b}{a} = -3$$

$$\frac{c}{a} = 2$$

- 22 The expression  $\frac{2x+4}{\sqrt{x+2}}$  is equivalent to

(1)  $\frac{(2x+4)\sqrt{x-2}}{x-2}$

(2)  $\frac{(2x+4)\sqrt{x-2}}{x-4}$

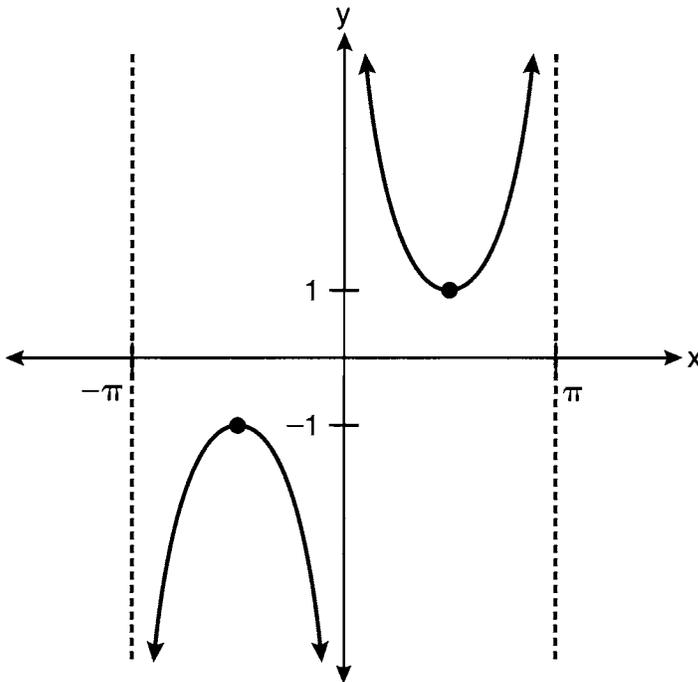
(3)  $2\sqrt{x-2}$

(4)  $2\sqrt{x+2}$

$$\frac{2x+4}{\sqrt{x+2}} \cdot \frac{\sqrt{x+2}}{\sqrt{x+2}} = \frac{2(x+2)\sqrt{x+2}}{x+2}$$

Use this space for computations.

23 Which equation is sketched in the diagram below?



- (1)  $y = \csc x$   
(2)  $y = \sec x$

- (3)  $y = \cot x$   
(4)  $y = \tan x$

24 The expression  $\log_5 \left( \frac{1}{25} \right)$  is equivalent to

- (1)  $\frac{1}{2}$                       (3)  $-\frac{1}{2}$   
(2) 2                              (4) -2

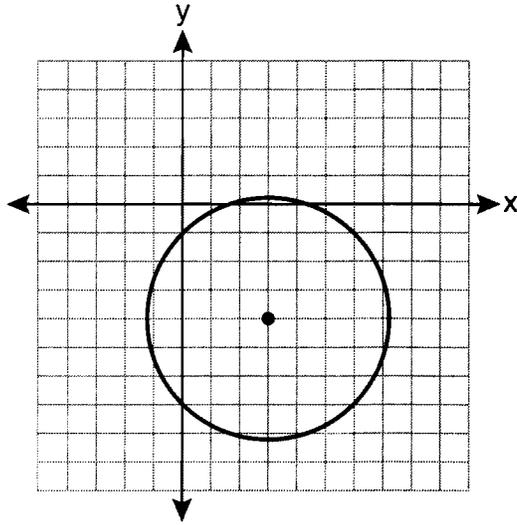
25 A four-digit serial number is to be created from the digits 0 through 9. How many of these serial numbers can be created if 0 can *not* be the first digit, no digit may be repeated, and the last digit must be 5?

- (1) 448                              (3) 2,240  
(2) 504                              (4) 2,520

8 · 8 · 7 · 1

Use this space for  
computations.

- 26 Which equation represents the circle shown in the graph below that passes through the point  $(0, -1)$ ?



- (1)  $(x - 3)^2 + (y + 4)^2 = 16$   
(2)  $(x - 3)^2 + (y + 4)^2 = 18$   
(3)  $(x + 3)^2 + (y - 4)^2 = 16$   
(4)  $(x + 3)^2 + (y - 4)^2 = 18$
- 27 Which task is *not* a component of an observational study?
- (1) The researcher decides who will make up the sample.  
(2) The researcher analyzes the data received from the sample.  
(3) The researcher gathers data from the sample, using surveys or taking measurements.  
(4) The researcher divides the sample into two groups, with one group acting as a control group.
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## Part II

Answer all 8 questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [16]

28 Solve algebraically for  $x$ :  $16^{2x+3} = 64^{x+2}$

$$(4^2)^{2x+3} = (4^3)^{x+2}$$

$$4x+6 = 3x+6$$

$$x = 0$$

29 Find, to the *nearest tenth of a degree*, the angle whose measure is 2.5 radians.

$$2.5 \cdot \frac{180}{\pi} \approx 143.2^\circ$$

30 For a given set of rectangles, the length is inversely proportional to the width. In one of these rectangles, the length is 12 and the width is 6. For this set of rectangles, calculate the width of a rectangle whose length is 9.

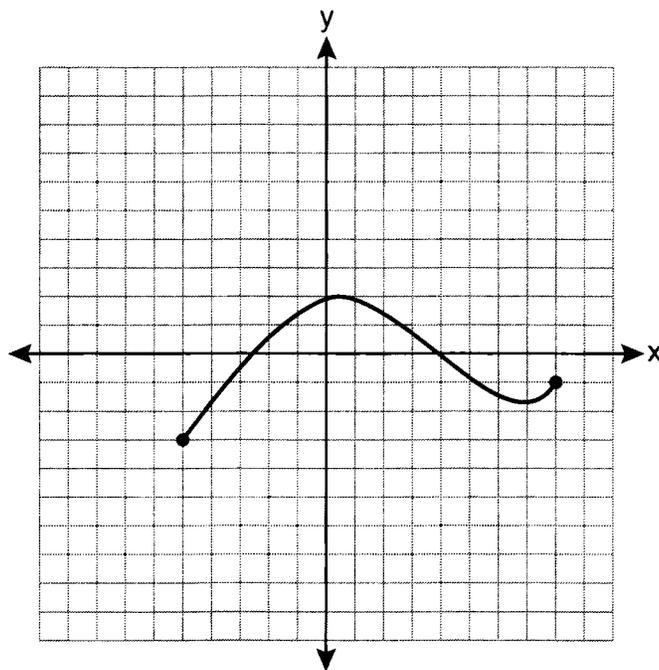
$$\begin{aligned}l_1 w_1 &= l_2 w_2 \\12 \cdot 6 &= 9 w_2 \\8 &= w_2\end{aligned}$$

31 Evaluate:  $10 + \sum_{n=1}^5 (n^3 - 1)$

$n$	$n^3 - 1$
1	0
2	7
3	26
4	63
5	124

$220 + 10 = 230$

32 The graph below represents the function  $y = f(x)$ .



State the domain and range of this function.

$$-5 \leq x \leq 8 \quad -3 \leq y \leq 2$$

33 Express  $\frac{\sqrt{108x^5y^8}}{\sqrt{6xy^5}}$  in simplest radical form.

$$\sqrt{18x^4y^3} = 3x^2y\sqrt{2y}$$

- 34 Assume that the ages of first-year college students are normally distributed with a mean of 19 years and standard deviation of 1 year.

To the *nearest integer*, find the percentage of first-year college students who are between the ages of 18 years and 20 years, inclusive.

68% of the students are within one standard deviation of the mean

To the *nearest integer*, find the percentage of first-year college students who are 20 years old or older.

16% of the students are more than one standard deviation above the mean.

35 Starting with  $\sin^2 A + \cos^2 A = 1$ , derive the formula  $\tan^2 A + 1 = \sec^2 A$ .

$$\frac{\sin^2 A}{\cos^2 A} + \frac{\cos^2 A}{\cos^2 A} = \frac{1}{\cos^2 A}$$

$$\tan^2 A + 1 = \sec^2 A$$

### Part III

Answer all 3 questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [12]

36 Write the binomial expansion of  $(2x - 1)^5$  as a polynomial in simplest form.

$$\begin{aligned} & {}_5C_0 (2x)^5 (-1)^0 + {}_5C_1 (2x)^4 (-1)^1 + {}_5C_2 (2x)^3 (-1)^2 + \\ & {}_5C_3 (2x)^2 (-1)^3 + {}_5C_4 (2x)^1 (-1)^4 + {}_5C_5 (2x)^0 (-1)^5 \\ & 32x^5 - 80x^4 + 80x^3 - 40x^2 + 10x - 1 \end{aligned}$$

37 In  $\triangle ABC$ ,  $m\angle A = 32$ ,  $a = 12$ , and  $b = 10$ . Find the measures of the missing angles and side of  $\triangle ABC$ . Round each measure to the nearest tenth.

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\frac{12}{\sin 32^\circ} = \frac{10}{\sin B}$$

$$B \approx 26.2^\circ$$

$$C \approx 180 - (32 + 26.2) = 121.8^\circ$$

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

$$\frac{12}{\sin 32^\circ} \approx \frac{c}{\sin 121.8^\circ}$$

$$c \approx 19.2$$

38 The probability that the Stormville Sluggers will win a baseball game is  $\frac{2}{3}$ . Determine the probability, to the nearest thousandth, that the Stormville Sluggers will win at least 6 of their next 8 games.

$${}^8C_6 \left(\frac{2}{3}\right)^6 \left(\frac{1}{3}\right)^2 \approx .27313$$

$${}^8C_7 \left(\frac{2}{3}\right)^7 \left(\frac{1}{3}\right)^1 \approx .15607$$

$${}^8C_8 \left(\frac{2}{3}\right)^8 \left(\frac{1}{3}\right)^0 \approx .03902$$

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$$.468$$

### Part IV

Answer the question in this part. A correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. A correct numerical answer with no work shown will receive only 1 credit. The answer should be written in pen. [6]

- 39 The temperature,  $T$ , of a given cup of hot chocolate after it has been cooling for  $t$  minutes can best be modeled by the function below, where  $T_0$  is the temperature of the room and  $k$  is a constant.

$$\ln(T - T_0) = -kt + 4.718$$

A cup of hot chocolate is placed in a room that has a temperature of  $68^\circ$ . After 3 minutes, the temperature of the hot chocolate is  $150^\circ$ . Compute the value of  $k$  to the *nearest thousandth*. [Only an algebraic solution can receive full credit.]

Using this value of  $k$ , find the temperature,  $T$ , of this cup of hot chocolate if it has been sitting in this room for a total of 10 minutes. Express your answer to the *nearest degree*. [Only an algebraic solution can receive full credit.]

$$\begin{aligned}\ln(150 - 68) &= -k(3) + 4.718 \\ 4.407 &\approx -3k + 4.718 \\ k &\approx .104\end{aligned}$$

$$\begin{aligned}\ln(T - 68) &= -.104(10) + 4.718 \\ \ln(T - 68) &= 3.678 \\ T - 68 &= 39.6 \\ T &\approx 108\end{aligned}$$