ADVANCED ALGEBRA

Monday, September 8, 1919—9.15 a.m. to 12.15 p.m., only

Answer eight questions. Each answer should be reduced to its simplest form. Papers entitled to less than 75 credits will not be accepted.

1 Find to two decimal places the positive root of the equation \( x^4 - 8x^3 + 14x^2 + 4x - 8 = 0 \) which lies between 2 and 3.

2 a Write a rational integral equation of the sixth degree which will have for three of its roots \(-1 + \sqrt{3}, 2 + \sqrt{-2}, \text{ and } 3 - \sqrt{-1}\).

b Change the equation \( 2x^4 - 3x^3 + 4x^2 - x + 2 = 0 \) into another, the coefficient of whose highest term is unity and the coefficients of the other terms integers.

3 a Plot the graph of \( 2x^3 - 3x^2 - 12x + 4 = y \)

b If \( 2x^3 - 3x^2 - 12x + 4 = 0 \), find from the graph the roots of the equation correct to the nearest tenth.

4 From 4 Americans, 4 Englishmen, 3 Italians and 5 Frenchmen, how many committees of 8 members can be formed, each nation to be represented by two men?

5 A man invests \( \$3100 \) in Liberty Bonds of the 1st, 2d and 3d issues. Later he converts his 3½% bonds into those of the 2d issue paying 4% and in this way increases his yearly income by \( \$5 \). Still later he exchanges all his 4% bonds for the 4½% bonds of the 3d issue, thus receiving \( \$9.75 \) more each year than he would have received from his original investment. How much did he invest originally in each of the three issues?

6 By means of graphs find the sum of

\( a \quad \frac{-2 - \sqrt{-3}}{2} \) and \( \frac{-2 + \sqrt{-3}}{2} \)

\( b \quad 3 - \sqrt{-2}, -5, \text{ and } 2 - 2 \sqrt{-2} \)

7 Write the derivatives of \( x^4 - 3x^2 + 2x^2 - 7x - 5 \)

8 The hypotenuse of a right-angled triangle is 13 and its area is 30; find the three sides of the triangle.

9 Solve for \( H \) the equation \( 3b = \frac{4\sqrt{H} - 2b}{H} \)

10 Given the equation \( x^6 - 3x^4 + 2x^2 + x^2 - x - 12 = 0 \) Without solving the equation, fill out the following statements by placing the proper number in each parenthesis; justify each statement by quoting the appropriate theorem or principle:

a The equation has ( ) roots.

b It has at most ( ) positive roots and at most ( ) negative roots.

c It has at least ( ) real roots.

d It has ( ) rational fractional roots.

e The sum of the roots is ( )

f The roots are factors of ( ).