The University of the State of New York
249th High School Examination
ADVANCED ALGEBRA
Wednesday, August 20, 1930 — 8.30 to 11.30 a. m., only

Instructions

Do not open this sheet until the signal is given.

Answer all questions in part I and five questions from part II.

Part I is to be done first and the maximum time to be allowed for this part is one and one half hours. Merely place the answer to each question in the space provided; no work need be shown.

If you finish part I before the signal to stop is given you may begin part II. However, it is advisable to look your work over carefully before proceeding to part II, since no credit will be given any answer in part I which is not correct and reduced to its simplest form.

When the signal to stop is given at the close of the one and one half hour period, work on part I must cease and this sheet of the question paper must be detached. The sheets will then be collected and you should continue with the remainder of the examination.
Fill in the following lines:

Name of school........................................Name of pupil

Detach this sheet and hand it in at the close of the one and one half hour period.

Part I

Answer all questions in this part. Each question has 2 1/2 credits assigned to it; no partial credit should be allowed. Each answer must be reduced to its simplest form.

1-2 Given the equation \(5x^2 - 3x + \frac{1}{2} = 0\)
   a What is the product of the roots of this equation?
   b Are the roots real or imaginary?

3 By the use of logarithms find the value of \(2^{10}\) to the nearest million.

4 If \(y = 10^x\), what is the value of \(y\) when \(x = \log_{10}3\)?

5 Find the value of \(x\) if \(10^x = 100^{1-x}\)

6 If \(x^2 - 4xy + y^2 = 1\), express \(y\) as a function of \(x\); that is, solve the equation for \(y\) in terms of \(x\).

7 A man makes part of a 75-mile trip on foot at the rate of 3 miles an hour and the remaining distance in a car at 25 miles an hour. If \(y\) represents the total time for the trip (in hours), and \(x\) the distance he walks (in miles), express \(y\) as a function of \(x\).

8 To what common fraction is the repeating decimal \(0.1818 \ldots\) equal?

9 Find and simplify the fifth term in the expansion of \((x + \sqrt{x})^7\).

10 What is the value of \(\frac{x^2}{3x - 5}\) if \(x = 3 - i\)?

11 What is the distance from the origin to the point representing the complex number \(\frac{5}{4 - 3i}\)?

12 Is it possible to seat a class of six pupils in a row of six seats in a different order every day (except Sundays) for two years? [Answer yes or no.]

13 In how many different ways may a committee of two be selected from a senior class of 100?

14 If the sum of the roots of the following equation in \(x\) is 6, find the product of the roots:
   \(kx^3 + (k + 1)x^2 + (k + 2)x + 5 = 0\)

15 Form an equation whose coefficients are integers and whose roots are 2, -3 and \(\frac{1}{3}\).

16 How many complex roots has the equation \(x^3 + 2x^2 + 7x - 5 = 0\)?

17 Write an equation whose roots are half as large as the roots of
   \(3x^3 - 2x^2 + 40 = 0\)

18 Write an equation whose roots are 3 less than the roots of
   \(2x^3 + 5x - 1 = 0\)

19 Form an equation whose coefficients are real and two of whose roots are \(\sqrt{-1}\) and \(\sqrt{-3}\).

20 The graph of \(y = 3x^3 + 2x - 5\) cuts the x-axis (a) not at all, (b) only once, (c) more than once. Which is correct, (a), (b) or (c)?

\[3\]
Write at top of first page of answer paper (a) names of schools where you have studied, (b) number of weeks and recitations a week in (1) elementary algebra, (2) intermediate algebra, (3) advanced algebra previous to entering summer high school, (c) number of recitations in this subject attended in summer high school of 1930.

The minimum time requirement previous to entering summer high school is five recitations a week in algebra for two school years.

For those pupils who have met the time requirement previous to entering summer high school the minimum passing mark is 65 credits; for all others 75 credits.

For admission to this examination attendance on at least 30 recitations in this subject in a registered summer high school in 1930 is required.

Part II

Answer five questions from this part. Full credit will not be granted unless all operations (except mental ones) necessary to find results are given; simply indicating the operations is not sufficient. Each answer should be reduced to its simplest form.

In the examination in advanced algebra the use of the slide rule will be allowed for checking, provided all computations with tables are shown on the answer paper.

21 Find all the roots of the equation \(3x^4 - 4x^3 + 5x^2 - 16x - 28 = 0\) \[10\]

22 State and prove the Remainder Theorem for any polynomial in \(x\). \[10\]

23 Find to the nearest hundredth the real root of \(2x^3 - 5x^2 = 10\) \[10\]

24 A sum of $1000 is divided into two portions. The first portion is placed at simple interest at 6%, and the second at compound interest at 4%, compounded annually. At the end of 10 years, the total amount (principal and interest) is $1539.50. Find the two portions. \[10\]

25 A man can row 24 miles down a river in one hour less time than he requires to row 12 miles down and back; he can row 12 miles down and back in exactly the same time he needs to row 20 miles upstream. Find his rate of rowing in still water and the rate of the current. \[7, 3\]

26 If the roots of \(x^2 - 12x + k = 0\) are in arithmetic progression, show that \(k + 4h = 128\) \[10\]

27 One leg of a right triangle is to be one inch shorter than the hypotenuse. Letting \(y\) represent the area of the triangle and \(x\) the hypotenuse, express \(y\) as a function of \(x\). Plot the graph of this equation for values of \(x\) from \(x = 1\) to \(x = 5\) inclusive, calculating the values of \(y\) to the nearest tenth. From your graph determine approximately the hypotenuse of such a triangle when the area is 5 square inches. \[3, 6, 1\]