TWELFTH YEAR MATHEMATICS

12A (Advanced Algebra)

Wednesday, June 23, 1965—9:15 a.m. to 12:15 p.m., only

Part I

Answer all questions in this part. Each correct answer will receive 2½ credits. No partial credit will be allowed. Write your answers on the lines to the right.

1. Express $\frac{1}{1+i}$ as an equivalent fraction with a real denominator. 1.....

2. Find the value of the remainder obtained when $6x^4 + 5x^3 - 2x + 8$ is divided by $x - \frac{1}{2}$. 2.....

3. Given that T varies inversely as the square root of D and directly as P. If $T = 12$ when $D = 36$ and $P = 8$, find the value of the constant of variation. 3.....

4. Solve the equation: $2x + \sqrt{x} - 1 = 0$ 4.....

5. Given the quadratic function $y = 4x^2 + 2x + 1$. Find the average rate of change of the function as x varies from $x = 1$ to $x = 3$. 5.....

6. If $f(x) = 2x^2$, write $f(x - 2)$ as a polynomial without parentheses. 6.....

7. Solve for t: $27^{6-t} = 9^{t-1}$ 7.....

8. Determine all the values of x which satisfy the inequality $3 + 2(x - 1) > -4$. 8.....

9. For what value of k is the polynomial $kx^2 + kx + 20$ exactly divisible by $x - 2$? 9.....

10. Between what two successive positive integers does a real root of the equation $x^3 - x - 1 = 0$ lie? 10.....

11. A root of the equation $x^3 + 2x^2 - x - 1 = 0$ lies between 0.8 and 0.9. Find this root to the *nearest tenth.* 11.....

12. How many integers greater than 1,000 can be formed from the digits 0, 2, 3, 5 if no digit is repeated in any number? 12.....

13. Express the repeating decimal 0.191919 . . ., in which the digits 1 and 9 are repeated endlessly, in the form $\frac{a}{b}$ where a and b are integers. 13.....

14. Find the radius of the circle whose equation is $x^2 + y^2 - 6x + 8y = 0$. 14.....

15. The slope of the straight line joining the points (5,7) and $(2,y)$ is 1. Find the value of y. 15.....

$y - 7 = 2 - 5$

$y = -1$

Directions (16-24): Indicate the correct completion for *each* of the following by writing the number 1, 2, 3 or 4 on the line to the right.

16. The members of the family of lines having the equation $y = -2(x - a)$, where x, y and a are real, have (1) the same x-intercept (2) the same y-intercept (3) the same slope (4) a common point not on either coordinate axis 16.....

17. If the sum of two consecutive integers is k, then the smaller integer is (1) $k - 1$ (2) $\frac{k}{2}$ (3) $\frac{k-1}{2}$ (4) $\frac{k+1}{2}$ 17.....

18. The sum of $\frac{\sqrt{3}}{2}$ and $\frac{2}{\sqrt{3}}$ is (1) 1 (2) $\frac{5}{2\sqrt{3}}$ (3) $\frac{7\sqrt{3}}{6}$ (4) $\frac{\sqrt{3}+2}{2\sqrt{3}}$ 18.....

19. When drawn on the same axes, the graphs of $x^2 - 3y^2 = 9$ and $(x - 2)^2 + y^2 = 9$ have in common exactly (1) 1 point (2) 2 points (3) 3 points (4) 4 points 19.....

20. The eighth term of the geometric progression $3\sqrt{2}, -6, 6\sqrt{2}, \ldots$ is (1) 144 (2) -144 (3) 48 (4) -48 20.....

21. The fifth term in the expansion of $(a + bi)^7$, where $i = \sqrt{-1}$, is (1) $35a^3b^4$ (2) $-35a^3b^4$ (3) $21a^2b^5i$ (4) $-21a^2b^5i$ 21.....

22. In order that $\frac{x+y}{z} - 1 = \frac{x-z}{y} + 1$, it is sufficient that (1) $x = y$ (2) $y = z$ (3) $x = z$ (4) $x = 0$ 22.....

23. If the equation $x^3 - 6x^2 + px + q = 0$ has 3 equal roots, then (1) $q = 0$ (2) $p = 0$ (3) each root $= 2$ (4) each root $= -2$ 23.....

24. An expression which is a rational, integral function of x is (1) $x^5 - \sqrt{2x^2} + 4$ (2) $x^5 - 2x^{1/2} + 6$ (3) $x^5 - 2x^{-1/2} + \frac{1}{2}$ (4) $x + \frac{1}{x} + 1$ 24.....

Part II

*Answer sixteen questions from this part, 25-48. Each correct answer will receive 2½ credits. No partial credit will be allowed. Questions marked * are based upon optional topics in the syllabus. Write your answers on the lines to the right.*

25. Find the value of $\frac{(x-3)^n + x^{-n}}{n^{x-3}}$ when $x = 3$ and $n = 2$. 25.....

26. If $\log (9!) = 5.5598$, determine the value of $\log (10!)$. 26.....

27. Find the abscissa of the point of inflection of the graph of $y = 2x^3 + 3x^2 + 6x - 4$. 27.....

*28. The straight line whose equation is $\begin{vmatrix} x & y & 1 \\ 2 & 3 & 1 \\ 4 & 7 & 1 \end{vmatrix} = 0$ passes through the point $(a,7)$. Find the value of a. 28.....

29. If $x = 7$ is the equation of the axis of symmetry of the graph of $y = ax^2 + 7x - 14$, find the value of a. 29.....

30. The equation $x^3 - 5x^2 + 8x - C = 0$ has one real root. If $(1 + i)$ is a root, find the value of the real root. 30.....

31. The longer side of a rectangle is represented by s, the perimeter by p and the area by A. Express A in terms of p and s. 31.....

32. How many different chords can be drawn between 10 different points on the circumference of a circle? 32.....

33. From a standard deck of 52 playing cards, one card is drawn at random, and then returned to the deck. A second card is then drawn. What is the probability of drawing a red card followed by a black card? (*Note*: A standard playing deck consists of two black suits and two red suits, each suit containing 13 cards.) 33.....

34. The positive rational root of the equation
$$4x^3 - x^2 + 16x - 4 = 0$$
is a common fraction. Find this root. 34.....

35. Find the value of x, given the simultaneous equations
$$(2^x)(2^y) = 8$$
$$\frac{2^x}{2^y} = 32$$ 35.....

36. An arrow is shot vertically upward. Its height h in feet after t seconds is given by the formula $h = 128t - 16t^2$. Find in feet the maximum height to which the arrow will rise. 36.....

37. A root of $x^5 - 32 = 0$ lies in quadrant II. Write this root in the form $r(\cos\theta + i\sin\theta)$. 37.....

38. Indicate by means of an expression of the form $a < x < b$ all those values of x which satisfy $x^2 < 3x + 10$. 38.....

39. If $x^2 - dx - 2d = 0$, express x in terms of d. 39.....

40. Write the complex number $-2 - 2i$ in the polar form $r(\cos\theta + i\sin\theta)$. 40.....

41. Write a general equation of the family of lines perpendicular to the line $x + 2y + 3 = 0$. 41.....

Directions (42-48): For each *of those chosen,* write on the line to the right the *number* preceding the expression that best completes the statement.

42. If $\frac{1}{x} + y = 2$ and $x + \frac{1}{y} = 3$, then the ratio of x to y is (1) 1:2 (2) 2:3 (3) 3:1 (4) 3:2 42.....

*43. The point whose polar coordinates are $(5,-30°)$ is the same as the point whose polar coordinates are (1) $(-5,30°)$ (2) $(-5,150°)$ (3) $(5,-150°)$ (4) $(-5,-30°)$ 43.....

44. If $0 < x < 1$, then (1) $0 < \log_{10} x < 1$ (2) $\log_{10} x > 1$ (3) $\log_{10} x < 0$ (4) $\log_{10} x = 0$ 44.....

45. A formula that describes the growth of a bacterial colony is given by $N = N_0 e^{rt}$, where N is the number of bacteria at the end of time t, N_0 is the original number, r is the rate of growth and e is a constant. If the formula is solved for t, then t equals

(1) $\dfrac{\log (N - N_0)}{r \log e}$ (2) $\dfrac{\log N - \log N_0}{r \log e}$ (3) $\dfrac{\log (N - N_0)}{(\log e)^r}$

(4) $\dfrac{\log N - \log N_0}{(\log e)^r}$ 45.....

46. If a is the arithmetic mean between two different positive real numbers and g is their positive geometric mean, then a true statement is (1) $a > g$ (2) $g > a$ (3) $a = g$ (4) $a = \frac{1}{2}g$ 46.....

47. If $7^{-x} = 10$, then 7^{2x} is equal to (1) $\dfrac{1}{100}$ (2) $\dfrac{1}{20}$ (3) 20 (4) 100 47.....

48. If the product of the roots of the equation

$$px^3 + qx^2 + rx + s = 0$$

is added to the sum of the roots of the same equation, the result may be expressed as (1) $\dfrac{q+s}{p}$ (2) $\dfrac{q-s}{p}$ (3) $-\dfrac{s+q}{p}$

(4) $\dfrac{s-q}{p}$ 48.....