TWELFTH YEAR MATHEMATICS

12A (Advanced Algebra)

Friday, June 19, 196-—9:15 a.m. to 12:15 p.m., only

Part I

Answer all questions in this part. Each correct answer will receive 2½ credits. No partial credit will be allowed. Write your answers on the line to the right.

1. Find the integral root of $x^3 - 2x^2 - x - 6 = 0$. 1.....

2. If $x = \sqrt{t}$ and $y = t - 3$, express y in terms of x. 2.....

3. If $f(x) = \dfrac{1-x}{x}$, find, in *simplest* form, $f\left(\dfrac{1}{t}\right)$. 3.....

4. Find the product of $4(\cos 30° + i \sin 30°)$ and $3(\cos 90° + i \sin 90°)$. [Answer may be left in polar form.] 4.....

5. Find the y-intercept of the graph of $y = (2 - x)^3$. 5.....

6. Combine into a single term involving only one radical: $\dfrac{\sqrt{3}}{2} + \dfrac{3}{\sqrt{3}}$ 6.....

7. Solve for the positive value of x: $x^{2/3} = 64$ 7.....

8. Given that $f(x) = kx^3 + 2x^2 + 3x + 6$. Find k such that if $f(x)$ is divided by $x - 2$, the remainder will be 24. 8.....

9. A committee is to consist of 5 seniors. How many ways may the committee be selected from the group of 10 seniors who volunteer for the task? 9.....

10. The real root of $x^3 - 3x^2 + x + 7 = 0$ lies between two consecutive integers. Find these two integers. 10.....

11. Express $\dfrac{2 - i\sqrt{2}}{1 - i\sqrt{2}}$ as an equivalent fraction with a real denominator. 11.....

12. Write an equation of the locus of points equidistant from the two lines whose equations are $y = 2x + 4$ and $y = 2x - 4$. 12.....

13. If $x - 2$ is a factor of $x^4 - x^3 - mx^2 - 4$, find the value of m. 13.....

14. Express in $a + bi$ form the cube of $(1 + i)$. 14.....

15. Find the sum of the infinite geometric progression, the first two terms of which are 2 and $-\frac{1}{2}$. 15.....

16. Each of a group of n boys plans to pay d dollars in order to purchase a certain boat. If one boy drops out of the group and the remaining boys wish to share equally the cost of the boat, express in terms of n and d the number of dollars each of the remaining boys must pay. 16.....

17. Write an equation of the line parallel to $2x - 5y = 6$ that passes through the point $(1,-3)$. 17.....

18. How many different code words, using all of the letters at a time, can be made from the letters of the word BANANA? [Any arrangement of letters is considered a code word.] 18.....

19. The equation of a circle is $x^2 - 4x + y^2 + 6y - 36 = 0$. Find the coordinates of the center. 19.....

Directions (20-24): Write on the line to the right the *number* preceding the expression that best completes the statement or answers the question.

20. The value of $4 \log_4 2$ equals (1) $\frac{1}{2}$ (2) 2 (3) $-\frac{1}{2}$ (4) -2 20.....

21. If x and y are real numbers, and if $x + 3yi = 6$, then (1) $x = 0$ (2) $y = 2$ (3) $y = -2$ (4) $x = 6$ 21.....

22. If the graphs of $x^2 + y^2 = 9$ and $9x^2 + 4y^2 = 36$ are drawn on the same set of axes, how many distinct points will they have in common? (1) 1 (2) 2 (3) 3 (4) 4 22.....

23. If $a^x = b$, then a^{x+3} is equal to (1) $3 + b$ (2) a^3b (3) b^3 (4) $3b$ 23.....

24. If a, b and c are real numbers and $c < 0$ and $a < b$, then (1) $a + c > b + c$ (2) $a - c > b - c$ (3) $ac > bc$ (4) $ac < bc$ 24.....

Part II

*Answer sixteen questions from this part, 25-48. Each correct answer will receive 2½ credits. No partial credit will be allowed. Questions marked * are based upon optional topics in the syllabus. Write your answers on the line to the right.*

25. Find $\log_3 4$ to the *nearest tenth*. 25.....

26. Write an equation of the straight line whose x-intercept is 3 and whose y-intercept is -2. 26.....

27. If $(n - 1)$, $(n + 2)$ and $(2n + 4)$ are positive numbers which form a geometric progression in that order, find the value of the first term. 27.....

28. The point $(h,-1)$ lies on the straight line through the two points $(1,5)$ and $(-3,-3)$. Determine the value of h. 28.....

29. Two roots of the equation $x^3 + 3x^2 + kx + q = 0$ are 3 and 1. Find the third root. 29.....

30. Solve for x the inequality $4 - 3x \leqq 12 + x$. 30.....

31. If a buyer samples two items from a lot of 10 items that contains 3 defective items, what is the probability that both will be defective? 31.....

*32. Transform the equation $x^2 + y^2 = 2x + 20$ from rectangular to polar coordinates. 32.....

33. If the sum of two consecutive integers is k, express the smaller integer in terms of k. 33.....

34. Solve for x in terms of y and a: $y = \dfrac{a - x}{2a - x}$ 34.....

*35. If $\begin{vmatrix} a & 1 & 0 \\ 2 & 1 & 1 \\ 1 & 3 & 1 \end{vmatrix} = 0$, find the value of a. 35.....

36. Write in *simplest* form the fifth term *only* in the expansion of $\left(x + \dfrac{1}{x^3}\right)^{10}$. 36.....

37. Write in the form $r(\cos\theta + i\sin\theta)$ the cube root of $8(\cos 72° + i \sin 72°)$, the amplitude of which lies in the first quadrant. 37.....

38. Find the point of inflection for the following curve: $y = 2x^3 - 6x^2 - 5x + 11$ 38.....

39. A body moves along a line in such a way that its distance S in feet from a fixed point on the line at the end of t seconds is given by the equation $S = t^3 - 5t^2$. Find the number of feet per second in its velocity at the end of 5 seconds. 39.....

40. Solve the following equation for the positive real value of x: $\log \dfrac{x^4}{2} = \log 8$ 40.....

41. Express the repeating decimal 0.5444 . . . where the digit 4 repeats endlessly in the form $\dfrac{a}{b}$ where a and b are integers. 41.....

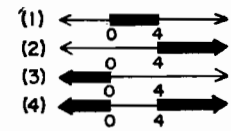
42. Write a general equation of the family of lines perpendicular to $3x + 2y = 6$. 42.....

43. Given that $f(x) = x^3 + x^2 + 2x + 6$. Find $f(i)$ in $a + bi$ form where $i = \sqrt{-1}$ and a and b are real. 43.....

44. If $s = 2 - 3t + 2t^2$, find the average rate of change of s with respect to t as t increases from $t = 0$ to $t = 2$. 44.....

Directions (45-48): For each *of those chosen*, write in the space provided on the line to the right the *number* preceding the expression that best completes the statement or answers the question.

45. The graph of the solution of the inequality $x^2 - 4x \geqq 0$ is to be shown by a heavy line or lines on the real number axis. Which graph is correct?



45.....

46. Which value of x satisfies the equation $x^2 - 2x + 1 = k$?
(1) $-1 + \sqrt{k}$ (2) $1 + \sqrt{k}$ (3) $1 + i\sqrt{k}$ (4) $1 - i\sqrt{k}$ 46.....

47. Given that y varies inversely as the square of x. If x is doubled, then y is (1)multiplied by 2 (2)divided by 2 (3)multiplied by 4 (4)divided by 4 47.....

48. The graph of which of the following functions contains points in quadrants I and IV? (1) $y = 2^x$ (2) $y = 2^{-x}$ (3) $y = 2x$ (4) $y = \log_2 x$ 48.....