TWELFTH YEAR MATHEMATICS

12A (Advanced Algebra)

Thursday, June 20, 1963—9:15 a.m. to 12:15 p.m., only

Part I

Answer all questions in this part. Each correct answer will receive 2½ credits. No partial credit will be allowed. Write your answers on the line to the right.

1. What is the remainder when $2x^3 - 3x^2 - 5x + 4$ is divided by $x + 2$? 1.....

2. Find the nonintegral rational root of $5x^3 - 8x^2 - 2x + 3 = 0$. 2.....

3. Find the radius of the circle whose equation is $x^2 - 4x + y^2 + 6y - 36 = 0$. 3.....

4. Find the value of $3x^0 - 54x^3 + 64^x$ when $x = -1/3$. 4.....

5. When the number $\frac{2 - i}{3 + i}$ is expressed in $a + bi$ form, what is the value of b? 5.....

6. Find all values of x for which $4x - 3 > x + 5$. 6.....

7. Find the area of the triangle bounded by the coordinate axes and the line $\frac{x}{5} + \frac{y}{12} = 1$. 7.....

8. Write in simplest form the fourth term *only* of the expansion of $\left(\frac{1}{x} - \frac{x}{2}\right)^9$. 8.....

9. Express the repeating decimal 0.272727 . . . in the form $\frac{a}{b}$ where a and b are integers. 9.....

10. The graph of $y = x^2 - 6x + k$ is tangent to the x-axis. Find the value of k. 10.....

11. Between what two consecutive positive integers does the real root of $4x^3 - 10x^2 + 8x - 3 = 0$ lie? 11.....

12. A bag contains buttons that are identical except that some are red and all the rest are white. There are 6 times as many white buttons as red. If one button is taken out at random, what is the probability that it is white? 12.....

13. In how many different ways may the seven letters of the word MINIMUM be arranged if all of the letters are used each time? 13.....

14. Write an equation of the straight line which is tangent to the graph of $y = x^2 - 2x + 9$ at the point (2,9). 14.....

15. Find an amplitude of the complex number which is the product of $2(\cos 10° + i \sin 10°)$ and $3(\cos 128° + i \sin 128°)$. 15.....

16. In the equation $x^4 + p_1x^3 + p_2x^2 + p_3x + p_4 = 0$, the coefficients are rational numbers. If two roots are $3 + \sqrt{2}$ and $5 - i$, find the value of p_4. 16.....

17. If x varies directly as y^2 and inversely as z and if $x = 4$ when $y = 0.2$ and $z = 0.7$, find the value of the constant of variation. 17.....

18. Write an equation of the axis of symmetry of the graph of $y = 3x^2 - 5x + 9$. 18.....

19. Solve for n (a positive integer) ${}_nC_{n-2} = 36$. 19.....

20. The sum of the first four terms of a geometric progression is 65. If the common ratio is 2/3, find the first term of the progression. 20.....

21. Write an equation of the straight line which is perpendicular to the line $2x - 3y = 5$ at the point (4,1). 21.....

Directions (22-24): Indicate the correct completion for *each* of the following by writing the number 1, 2, 3 or 4 in the space provided.

22. The graph of $y = 2^{-x}$ lies in quadrants (1) I and II (2) II and III (3) III and IV (4) IV and I 22.....

23. If $s = 5t^2 + t - 3$, the average rate of change of s with respect to t for the interval from $t = 0$ to $t = 4$ is (1) 1 (2) 21 (3) 101 (4) 41 23.....

24. If n is divided by d, the quotient is q and the remainder is r. This may be expressed by (1) $\frac{n}{d} = q + r$ (2) $n = \frac{q}{d} + r$ (3) $\frac{n}{d} = q\left(\frac{r}{d}\right)$ (4) $n = dq + r$ 24.....

Part II

*Answer sixteen questions from this part, 25-48. Each correct answer will receive 2½ credits. No partial credit will be allowed. Questions marked * are based upon optional topics in the syllabus. Write your answers on the line at the right.*

25. Eleven different points lie on a circle. With these points as vertices, how many different triangles can be drawn? 25.....

26. Solve for x: $8^{x+3} = 2^{4x}$ 26.....

27. If $f(x) = x^2 + x - 2$, find $4f(-a)$. 27.....

28. Express in simplest form: $\dfrac{\dfrac{x-2}{3}+1}{\dfrac{x+3}{2}-1}$ 28.....

29. One root of $x^5 + 32 = 0$ lies in quadrant IV. Write this root in the form $r(\cos\theta + i\sin\theta)$. 29.....

30. A point moves along a line so that its distance from the starting point at the time t is given by the equation $s = 13t^3 - 4t^2$. Find the acceleration of the point when $t = 1$. 30.....

31. How many quarts of water must be added to 9 quarts of a 30% solution of antifreeze to obtain a 15% solution? 31.....

32. The solution of the inequality $x^2 + 2x - 15 > 0$ has the form $x > a$ or $x < b$. What is the value of b? 32.....

33. The sum of an infinite geometric series is 6. The first term is a and the common ratio is r. Express r in terms of a. 33.....

34. Find to the *nearest tenth*: $\log_8 7$ 34.....

35. Solve for x: $\log(3x + 2) - \log x = 1$ 35.....

*36. Solve for y: $\begin{vmatrix} 2 & 1 & 1 \\ 5 & -1 & 1 \\ 4 & y & 1 \end{vmatrix} = 16$ 36.....

37. Find the value of k in $x^3 - 3x^2 - 49x + k = 0$ if the sum of two of the roots is zero. 37.....

38. The arithmetic mean between two numbers is 7 and their positive geometric mean is $3\sqrt{5}$. Find *both* numbers. 38.....

39. A root of $x^3 - 2x^2 + 3x - 4 = 0$ lies between 1 and 2. Find this root to the *nearest integer*. 39.....

*40. Transform the equation $r = 5\sin\theta$ from polar to rectangular coordinates. 40.....

41. Write a general equation of the family of lines parallel to $4x - 2y + 5 = 0$. 41.....

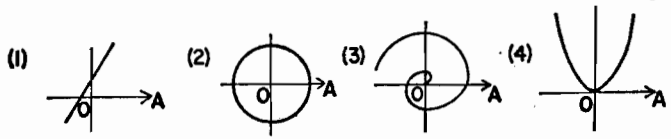
42. Find the abscissa of the point of inflection of the graph of $y = x^3 - 5x^2 + 3x - 7$. 42.....

Directions (43-48): For each *of those chosen,* write in the space provided the *number* preceding the expression that best completes the statement or answers the question.

43. Which of the following defines a rational integral function of x? (1) $y = \sqrt{x - 1}$ (2) $y = \frac{1}{x^2}$ (3) $y = x^{-2} + 4$ (4) $y = x^{11} - \sqrt{3}$ 43.....

44. If $y = (x + 3)^2$ and $(x + 3)^2 + y^2 = 4$ are graphed on the same axes, the total number of points of intersection is (1) 1 (2) 2 (3) 3 (4) 0 44.....

*45. The graph in polar coordinates of $r = 2\theta$ most closely resembles 45.....



46. The equation $3x - \sqrt{x} - 10 = 0$ has (1) the two roots 4 and $-\frac{25}{9}$ (2) the two roots 4 and $\frac{25}{9}$ (3) the single root $\frac{25}{9}$ (4) the single root 4 46.....

47. The equation $x^2 + \frac{b}{a}x = -\frac{c}{a}$ can be put in the form $(x + k)^2 = k^2 - \frac{c}{a}$ by completing the square. The value of k in terms of a and b is (1) $\frac{b}{a}$ (2) $\frac{b}{2a}$ (3) $\frac{b^2}{a^2}$ (4) $\frac{b^2}{4a^2}$ 47.....

48. The complex numbers $a + bi$, $c + di$, $e + fi$ may be considered graphically or algebraically. Which statement is *not* true for all real values of a, b, c, d, e and f? (1) $(a + bi) + (c + di) + (e + fi) = 0$ (2) $(a + bi) + (c + di) = (c + di) + (a + bi)$ (3) $3(a + bi) = 3a + 3bi$ (4) $(a + bi) + [(c + di) + (e + fi)] = [(a + bi) + (c + di)] + (e + fi)$ 48.....