June 22, 1961

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed.

1. If $x - 2$ is a factor of $x^3 + hx + 10$, find the value of h. 1_____

2. Find the value of $x^2 - 4^x$ when $x = -\frac{1}{2}$. 2_____

3. Find all of the roots of the equation $x^3 - 2x^2 - 5x + 6 = 0$. 3_____

4. Find the coordinates of a point of intersection of the graphs of $x^2 + y = 10$ and $x + y = 10$. 4_____

5. Write the repeating decimal 0.2555.... as the quotient of two integers. 5_____

6. The parabola $y = x^2 - 6x + k$ has its turning point on the x-axis. What is the value of k? 6_____

7. Three boys agree to divide a bag of marbles in the following manner: The first boy takes one-half of the marbles. The second boy takes one-third of the remaining marbles, leaving 10 marbles for the third boy. Find the original number of marbles. 7_____

8. A box contains 5 red, 4 white and 3 black balls. If two balls are drawn at random, without replacement, what is the probability that both are white? 8_____

9. If $t = x^{1.5}$, find to the *nearest integer* the value of t when $x = 20$. 9_____

10. Find a value of x that satisfies the equation:
$$\log_{10}(x^2 - 3x) = 1$$ 10_____

11. Given the triangle whose vertices are at A (3, 3), B (6, 2) and C (8, −2). What is the slope of the altitude to side AC? 11_____

12. Write an equation of the line tangent to the curve $y = 2x^2 + 6$ at the point (1, 8). 12_____

13. If $f(x) = 2 - x$, express $2f(a) + f(2a)$ in simplest form, 13_____

14. Find in simplest form the 5th term in the expression of
$$\left\{ x^2 + \frac{1}{x} \right\}^8$$ 14_____

15. Write an equation of the circle that passes through the origin and whose center is the point (−3, 4). 15_____

Directions (16-20): Write on the line at the right of *each* of the following the *number* preceding the expression that best completes the statement.

16. If $x^{-1} - 1$ is divided by $x - 1$, the quotient is

(1) $\frac{1}{x-1}$ (2) $-\frac{1}{x-1}$ (3) $\frac{1}{x}$ (4) $-\frac{1}{x}$ 16_____

17. In the equation $x^3 - hx^2 + 2k = 0$, the sum of the roots is 3 and the product of the roots is 4. Then h and k have the values, respectively, (1) −3 and 2 (2) −3 and −2 (3) 3 and −2 (4) 3 and 2 17 3

18. The sum of $\frac{1}{2+\sqrt{2}}$ and $\frac{1}{2-\sqrt{2}}$ is (1) ½ (2) 2 (3) $\sqrt{2}$ (4) $-\sqrt{2}$ 18 2

19. The simplest form of $1 - \frac{1}{1+\frac{a}{1-a}}$ is (1) 1 (2) $1-a$ (3) a (4) 0 19 3

20. The graphs of $3x - 2y = 8$, $2x + 2y = 7$, $x = 3$ and $y = ½$ are drawn on the same set of axes. The number of distinct intersections of all pairs of these straight lines is (1) 1 (2) 2 (3) 6 (4) 4 20 1

21. In how many ways may a committee of three students be selected from a group of six students? 21 20

22. Given that t varies directly as v and inversely as d^3. If $t = ½$ when $v = 25$ and $d = 10$, find the value of the constant of variation. 22 20

23. Solve for x: $3x - 6 < 5x$ 23 $x > 3$

24. By how much does the arithmetic mean between 1 and 25 exceed the positive geometric mean between 1 and 25? 24 8

25. If $(2 + i)^2 = c + di$, find the value of d. 25 4

Part II

*Answer ten questions from this part, 26-40. Each correct answer will receive 2½ credits. No partial credit will be allowed. Questions marked * are based upon optional topics in the syllabus. Write your answer on the line at the right.*

26. Find to the *nearest tenth* the root of $x^3 + 4x - 1 = 0$ which lies between 0 and 1. 26 .2

27. The points $(2, -2)$, $(4, 2)$ and $(5, k)$ are on the same straight line. What is the value of k? 27 4

28. If a certain number is added to each of the numbers 10, 25 and 50, the resulting numbers form a geometric progression. What is this number? 28 12½

29. Solve to the *nearest tenth* the equation: $3^x = 20$ 29 2.7

30. A certain law of motion is given by the equation $S = ½t^3$ where S is in feet and t is in seconds. Find in feet per second the average velocity over the time interval $t = 2$ to $t = 4$. 30 14

31. Express in $a + bi$ form the product of $2(\cos 60° + i \sin 60°)$ and $3(\cos 90° + i \sin 90°)$. 31 $-3\sqrt{3}+3$

32. Find the fractional root of the equation $3x^3 - 5x^2 + 5x - 2 = 0$. 32 ⅔

*33. Transform $r^2 = 4r \cos \theta - r \sin \theta$ from polar to rectangular coordinates. 33 x^2+y^2-4x+y

*34. The value of the determinant $\begin{vmatrix} 1 & 0 & 0 \\ 0 & a & 2 \\ 0 & 0 & 3 \end{vmatrix}$ is 4. Find the value of a. 34 4/3

Directions (35-38): Write on the line at the right of each of those chosen the *number* preceding the expression that best completes the statement or answers the question.

35. Which equality is an example of the distributive law?

(1) $\frac{a+b}{2} = \frac{b+a}{2}$ (2) $a + (b + c) = (a + b) + c$

(3) $\frac{1}{2}(a + b) = \frac{1}{2}a + \frac{1}{2}b$ (4) $a(b + c) = (b + c)a$ 35. 3

36. The logarithm of $\sqrt[4]{8}$ to the base 2 is (1) 1 (2) $\frac{3}{4}$ (3) $\frac{1}{2}$ (4) $\frac{1}{4}$ 36. 2

37. A rectangular field, twice as long as it is wide, is completely enclosed by x yards of fencing. The area in terms of x is

(1) $\frac{x^2}{2}$ (2) $2x^2$ (3) $\frac{2x^2}{9}$ (4) $\frac{x^2}{18}$ 37. 4

38. Given that y varies directly as the square of x. The graph of this relation is (1) a straight line (2) an ellipse (3) a hyperbola (4) a parabola 38. 4

39. Find the coordinates of the point of inflection of the graph of the equation $y = 6 + 3x^2 - x^3$. 39. 1, 8

40. How many even numbers consisting of three digits each can be formed from the digits 1, 2, 3, 4 and 5? [Repetition of digits are permitted.] 40. 50

Part III

*Answer ten questions from this part, 41-55. Each correct answer will receive 2½ credits. No partial credit will be allowed. Questions marked * are based upon optional topics in the syllabus. Write your answer on the line at the right.*

41. If $s = x - x^2$, express x in terms of s. 41. $\frac{1 \pm \sqrt{1-4s}}{2}$

42. Find the abscissa of the point of intersection of the x-axis and the line joining the points $(-1, 1)$ and $(3, 9)$. 42. $-\frac{3}{2}$

43. Write an equation with real coefficients and of lowest possible degree, two of whose roots are 1 and $1 + i$. 43. $x^3 - 3x^2 + 4x - 2 = 0$

44. Find the value of r when $(\sqrt{3} + i)^5$ is expressed in the form $r(\cos\theta + i\sin\theta)$. 44. 32

*45. [Write the *number* preceding the correct answer in the space provided.] Which set of coordinates does *not* satisfy the equation $r = 4(1 - \cos\theta)$? (1) $(8, \pi)$ (2) $(0, 0)$

(3) $\left\{-4, -\frac{\pi}{2}\right\}$ (4) $\left\{4, \frac{\pi}{2}\right\}$ 45. 3

*46. Write in determinant form the value of y for the system

$$x = 1$$
$$x + y = 2$$
$$2x + y + z = 1$$

46 $\begin{vmatrix} 1 & 1 & 0 \\ 1 & 2 & 0 \\ 2 & 1 & 1 \end{vmatrix}$ $\begin{vmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 2 & 1 & 1 \end{vmatrix}$

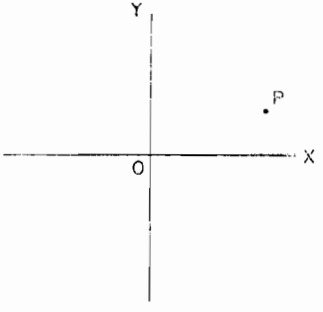
47. How many ounces of water must be added to 9 ounces of a 50% salt solution to change it to a 30% salt solution? 47 6

48. The roots of the equation $x^5 = 1$ are represented in polar form. Find the amplitude of the root that lies in quadrant II. 48 144°

49. A certain law of motion is given by the equation $S = 24t + \frac{1}{2}t^3$ where S is in feet and t is in seconds. Find in feet per second the velocity at $t = 4$. 49 48

50. To solve the inequality $x^2 - x - 6 < 0$, we may consider the values of x for which the graph of $y = x^2 - x - 6$ lies below the x-axis. If the solution has the form $a < x < b$, what is the value of b? 50 3

51. In the accompanying diagram, OX is the axis of reals and OY is the axis of imaginaries. Point P represents the complex number $a + bi$. Indicate *on the diagram* by the letter Q, the point which represents the complex number $c + di$ such that $(a + bi) + (c + di) = 0$.



Directions (52-55): Write on the line at the right of each of those chosen the *number* preceding the expression that best completes the statement.

52. The equation $\sqrt{2 - x} = x$ has (1) 1 as its only root (2) -2 as its only root (3) 1 and -2 as its only roots (4) -1 and 2 as its only roots 52 1

53. If $3^a = k + 1$, then 3^{2a} is equal to (1) $2(k + 1)$ (2) $k^2 + 1$ (3) $k^2 + 2k + 1$ (4) $3(k + 1)$ 53 3

54. The equation $2x + 1 = bx + 3$ has a solution for x (1) for all values of b (2) for no values of b (3) only if b is equal to 2 (4) only if b is not equal to 2 54 2

55. If $\log_a 2 = x$ and $\log_b 3 = y$, then $a^x b^y$ equals (1) $(ab)^{x+y}$ (2) 6 (3) $\log_{ab} 6$ (4) $\log_{ab} 5$ 55 2