ADVANCED ALGEBRA

Wednesday, June 19, 1957—9:15 a.m. to 12:15 p.m., only

Part I

Answer all questions in this part. Each correct answer will receive 2½ credits. No partial credit will be allowed.

1. Write an equation of the line passing through the point $(0, -4)$ and parallel to the line whose equation is $3x - 2y = 7$. 1.....

2. If $f(x) = \frac{3}{x^2} - 4x^0 + x^{-\frac{1}{2}}$, find $f(4)$. 2.....

3. Express $\frac{}{3 - 2i}$ as a fraction with a real denominator. 3.....

4. Find n if ${}_nC_2 = 66$. 4.....

5. Write in *simplest form* the fifth term *only* in the expansion of $(a + \sqrt{a})^8$. 5.....

6. Find the arithmetic mean between $\frac{x+1}{x}$ and $\frac{x-1}{x}$. 6.....

7. Solve for x: $x^{\frac{2}{3}} = 0.25$ 7.....

8. Solve for x: $\log_3 5x = 2$ 8.....

9. Express the repeating decimal 0.171717 . . . as a common fraction. 9.....

10. If $y = \frac{x}{x+1}$, express x as a function of y. 10.....

11. The product of two of the roots of the equation $x^3 - 3x^2 - 2x + 6 = 0$ is 3. Find the third root. 11.....

12. Ann has 6 different blouses and 3 different skirts. In how many ways may she select 3 blouses and 2 skirts to take on a weekend trip? 12.....

13. One letter is selected at random from the word *trip* and one letter is selected at random from the word *chop*. What is the probability that the same letter is selected from each word? 13.....

Directions (14-18): Indicate the correct completion for *each* of the following by writing the letter *a*, *b*, *c* or *d* on the line at the right.

14. If the graphs of the equations, $xy = 6$ and $y = x^2$, are drawn on the same set of axes, the number of points common to the two graphs is (*a*) one (*b*) two (*c*) three (*d*) four 14.....

15. If y varies inversely as the square of x and if x is doubled, y is multiplied by (*a*) ¼ (*b*) 2 (*c*) ½ (*d*) 4 15.....

16. An equation whose graph is tangent to the x-axis is (*a*) $x^2 + y^2 = 9$ (*b*) $y = x^2 - 6x$ (*c*) $y = x^2 - 6x + 9$ (*d*) $y = x^2 + 9$ 16.....

17. The equation $\sqrt{3x + 4} - x = 0$ has (*a*) 4 and −1 as its roots (*b*) 4 as its only root (*c*) −1 as its only root (*d*) neither 4 nor −1 as a root 17.....

18. The equation $x^3 + 3x - 2 = 0$ has (*a*) one positive and two negative roots (*b*) one positive and two imaginary roots (*c*) one negative and two positive roots (*d*) one negative and two imaginary roots 18.....

19. Write an equation whose roots are 2 less than the roots of the equation $x^3 + 2x - 5 = 0$. 19.....

20. Write an equation whose roots are one-half the roots of the equation $x^3 - 4x^2 + 4x + 8 = 0$. 20.....

Part II

Answer five questions from this part. Show all work.

21. Find, to the *nearest tenth*, the positive root of the equation $2x^3 + x^2 - 10x - 4 = 0$. [10]

22. Solve completely the equation $2x^4 - 5x^3 + 11x^2 - 3x - 5 = 0$. [10]

23. (1) Draw the graph of $y = -x^2 + 8$ [4]
(2) On the set of axes used in part (1) draw the graph of $x^2 + (y - 4)^2 = 16$. [3]
(3) If the graphs of $x^2 + (y - 4)^2 = 16$ and $y = -x^2 + K$ are drawn on the same set of axes, the graphs will have *no* point in common if $K =$ (*a*) 8 (*b*) 4 (*c*) 0 (*d*) —2. [3]

24. *a.* If \$3,750, invested at interest which is compounded annually, amounts to \$5,000 in 10 years, what is the annual rate of interest to the *nearest tenth of a percent?* [Use the formula $A = P(1 + r)^n$.] [7]
b. Find x to the *nearest tenth*: $1.91^x = 54.2$ [3]

25. *a.* State and prove the Remainder Theorem. [1, 5]
b. If n is an integer, find the remainder when $x^{2n+1} - 3x^{2n} + 6$ is divided by $x + 1$. [2]
c. For what value of K is $(x - 2)$ a factor of $x^3 + Kx^2 - 5x - 14$? [2]

26. In the equation $x^3 + 4x^2 + x + K = 0$, the sum of two of the roots equals the third root. Find the roots and the value of K. [10]

27. City A is 40 miles south of city B. Two cars start at the same time, one from A and the other from B, and travel toward each other at uniform rates. At the end of one-half hour they meet. If both cars had traveled north, the car from A would have overtaken the car from B at a point 140 miles north of B. Find the rate of the car from A. [7, 3]

*28. *a.* Find the modulus of $5 - 12i$. [2]
b. Find the amplitude of $-\sqrt{2} + i\sqrt{6}$. [2]
c. Express $2(\cos 210° + i \sin 210°)$ in the form $a + bi$. [3]
d. Write in polar form one of the imaginary roots of $x^3 + 8 = 0$. [3]

*29. Given $y = x^3 - 3x^2 + 5$.
a. Find the derivative of y with respect to x. [2]
b. Find the coordinates of the maximum point. [2]
c. Find the coordinates of the minimum point. [2]
d. Find the coordinates of the point of inflection. [2]
e. *Sketch* the graph of $y = x^3 - 3x^2 + 5$. [2]

* These questions are based upon the optional topics in the syllabus.