

ADVANCED ALGEBRA

Wednesday, June 23, 1954—9.15 a. m. to 12.15 p. m., only

Part I

Answer all questions in this part. Each correct answer will receive 2½ credits. No partial credit will be allowed.

1. Write the *third term* in the expansion of $(\frac{1}{a} - b)^9$. 1.....
2. If $(2 - i)(3 + 2i) = a + bi$, find the value of a . 2.....
3. Find the slope of the straight line that passes through the points $(3,2)$ and $(8,4)$. 3.....
4. Write an equation of the straight line passing through the point $(-1,2)$ and parallel to the line $3y = 2x - 6$. 4.....
5. Write an equation of the straight line passing through the point $(2,-3)$ and perpendicular to the x -axis. 5.....
6. For what value of k will the graph of $y = kx^2 - 2x + 3$ be tangent to the x -axis? 6.....
7. Find the value of m for which $x + 2$ is a factor of $x^3 + x^2 + mx - 4$. 7.....
8. Using k as the constant of variation, write an equation representing the following relationship: P varies directly as the product of R and S and inversely as the square of T . 8.....
9. If $f(x) = 3x - 2$, find $f(2 - h)$. 9.....
10. An equation with real coefficients has i and $2i$ among its roots. What is the lowest possible degree of the equation? 10.....
11. Solve the following equation for x : $4^{2x} = 8^{2x-1}$ 11.....
12. If $\log_b \sqrt{N} = 0.3961$, find $\log_b N$. 12.....
13. If n is a negative integer, then (a) $\frac{6}{n^2}$ is less than $\frac{4}{n}$
(b) $\frac{n}{4}$ is greater than $\frac{-6}{n}$ (c) $\frac{6}{n}$ is less than $\frac{4}{n}$
[Which is correct (a), (b) or (c)?] 13.....
14. How many different committees each consisting of two teachers and three students can be formed from a group of four teachers and seven students? 14.....
15. The order in which four students, A , B , C and D , are to speak at an assembly program is to be determined by lot. What is the probability that A will speak first and B second? 15.....
16. Find the sum of the infinite progression $2, 1.2, 0.72, \dots$ 16.....
17. Solve the equation $x^2 = \frac{1-y}{1+y}$ for y in terms of x . 17.....
18. How many real roots has the equation $x^3 + 2x^2 + 4 = 0$? 18.....

19. Transform the equation $x^3 - 4x^2 + 6x - 4 = 0$ into an equation whose roots are those of the original equation each diminished by 1. 19.....
20. Transform the equation $x^3 - 6x^2 - 9x + 54 = 0$ into an equation whose roots are those of the original equation each multiplied by $1/3$. 20.....

Part II

Answer five questions from part II. All work, including computation, should be shown.

21. Solve the following equation: $2x^3 + x^2 - 13x + 6 = 0$. [10]
22. Find to the nearest tenth the smaller positive root of $x^3 - 6x^2 + 8x + 2 = 0$. [10]
23. a. Derive the formula for the roots of the equation $ax^2 + bx + c = 0$ in terms of a , b , and c . [7]
 b. Write in the form $ax^2 + bx + c = 0$ an equation whose roots are $\frac{2 \pm \sqrt{3}}{2}$. [3]
24. Given $D = 0.4 \sqrt[3]{\frac{P^2}{R^5}}$. Find D to the nearest thousandth when $P = 18.4$ and $R = 5.89$. [10]
25. a. Draw the graph of $y = \log_2 x$. [Use the following values for x : $\frac{1}{4}$, $\frac{1}{2}$, 1, 2, 4.] [5]
 b. On the same set of axes used in answer to a , draw the graph of $(x - 1)^2 + y^2 = 4$. [3]
 c. From the graphs drawn in answer to a and b , estimate to the nearest tenth the values of x and y common to both equations. [2]
26. Three numbers are in geometric progression and their sum is 19. If 1 is subtracted from the first number, the numbers then are in arithmetic progression. Find the original numbers. [10]
27. A park superintendent has a 25-pound sack of grass seed containing 70% permanent grasses. How much of this seed should he replace by seed containing 53% permanent grasses in order to make 25 pounds of seed containing 60% permanent grasses? [Give your answer to the nearest tenth of a pound.] [10]
- *28. Answer either a or b :
 a. (1) Find the modulus of $3 - 2i$. [2]
 (2) Find to the nearest degree the amplitude (angle) of $3 - 2i$. [3]
 (3) Express $2(\cos 150^\circ + i \sin 150^\circ)$ in the form $a + bi$. [2]
 (4) Write in polar form one of the real fourth roots of 4. [3]
 b. Given the equation $y = 2x^3 - 3x^2 - 12x + 10$.
 (1) Find the coordinates of
 (a) the maximum point [3]
 (b) the minimum point [3]
 (c) the point of inflection [2]
 (2) Find the slope of the line tangent to the curve at the point where the curve crosses the y axis. [2]

*This question is based upon optional topics in the syllabus.