The University of the State of New York

315TH HIGH SCHOOL EXAMINATION

ADVANCED ALGEBRA

Wednesday, June 18, 1952 — 9.15 a. m. to 12.15 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to part II (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in advanced algebra.

The minimum time requirement is four or five recitations a week for half a school year after the completion of intermediate algebra.

Part II

Answer five questions from part II.

21 Find to the *nearest tenth* the real root of the equation $x^3 + x - 41 = 0$. [10]

22 Solve the equation $x^4 - 2x^3 - 3x^2 + 4x - 12 = 0$. [10]

23 *a* State and prove the Remainder Theorem. [1, 4]

b Prove that $\log_b a = \dfrac{\log_{10} a}{\log_{10} b}$ [5]

24 The adiabatic law for a confined volume of air is $PV^{1.4} = 976$, where P is the pressure in centimeters of mercury and V is the volume in cubic centimeters. Find the volume to the *nearest tenth* when $P = 124$ cm. [10]

25 *a* Draw the graph of $y = 3^{x-1}$ for values of x from -1 to $+3$ inclusive. [4]

b On the same axes as used in answer to *a*, draw the graph of $y = 2 + 2x - x^2$ for values of x from -1 to $+3$ inclusive. [4]

c From the graphs made in answer to *a* and *b*, estimate to the *nearest tenth* the positive root of the equation $2 + 2x - x^2 = 3^{x-1}$. [2]

26 The difference between two roots of the equation $x^3 + 3x^2 + k = 0$ is 3. Find the values of k. [10]

27 A piece of work can be done in one day by *a* men and *b* boys, or it can be done in one day by *c* men and *d* boys. Express in terms of *a, b, c* and *d* the number of days it would take one man to do the work alone. [5, 5]

*28 *a* Write in polar form the three roots of $x^3 + 27 = 0$. [6]

b Find the modulus of $\frac{5}{12} - i$. [2]

c Find the amplitude (angle) of $-2 + 2i\sqrt{3}$ [2]

*29 *a* Given $f(x) = 3x - x^2$. Find the average rate of change of $f(x)$ from the point whose abscissa is 3 to the point whose abscissa is 5. [5]

b Find the equation of the line tangent to the curve $y = 3x - x^2$ at the point whose abscissa is 4. [5]

* This question is based upon one of the optional topics in the syllabus.

Fill in the following lines:

Name of pupil.. Name of school..

Part I

Answer all questions in this part. Each correct answer will receive $2\frac{1}{2}$ credits. No partial credit will be allowed.

1 Express $\dfrac{2+i}{1-i}$ as an equivalent fraction with a real denominator. 1................

2 Express the repeating decimal 0.454545...as a common fraction. 2................

3 If $f(x) = x^{14} - 2x^7 - 4$, find the remainder when $f(x)$ is divided by $x + 1$. 3................

4 Write in simplest form the third term in the expansion of $\left(2a - \dfrac{1}{a}\right)^5$. 4................

5 Using k as the constant of variation, write an equation expressing the following relationship: The number of units H of heat generated by an electric current in a circuit varies directly as the product of the resistance R, the time t and the square of the current A. 5................

6 Solve for x: $8^x = \frac{1}{4}$. 6................

7 Find x if $\log_4 x = -1$. 7................

8 Find the slope of the line $x - 4y - 12 = 0$. 8................

9 Write the equation of lowest degree with real coefficients, three of whose roots are 1, -1 and i. 9................

10 Given the equation $x^3 - x^2 + 4x - 12 = 0$, whose roots are represented by a, b and c. Transform this equation into an equation whose roots are $a + 2$, $b + 2$ and $c + 2$. 10................

11 Transform the equation $x^3 - 8x + 1 = 0$ into an equation whose roots are each one half the roots of the given equation. 11................

12 For what value of k will the graph of $y = x^2 - 6x + k$ be tangent to the x-axis? 12................

13 How many imaginary roots has the equation $x^6 + 3x^4 + 2x - 6 = 0$? 13................

14 A man travels m miles in t hours and then returns over the same route in h hours. Express his average rate for the entire trip in terms of m, h and t. 14................

15 If the graphs of $x^2 + y^2 = 4$ and $y = 8x - x^2$ are drawn on the same set of axes, how many points do the graphs have in common? 15................

16 Express the sum of the first n positive odd integers as a function of n. 16................

17 Solve the equation $\sqrt{x^2 + xy} = 3 - x$ for x in terms of y. 17................

18 How many code words of five different letters each can be made from the letters of the word *taken* if each word is to begin and end with a vowel? 18................

19 There are 10 men qualified to run a machine that requires 3 operators at a time. How many different crews of 3 can be selected to run the machine? 19................

20 A bag contains 9 balls numbered from 1 to 9. Two balls are drawn from the bag. Find the probability that the numbers on the balls drawn are both odd. 20................