ADVANCED ALGEBRA

Wednesday, June 23, 1948 — 9.15 a. m. to 12.15 p. m., only

Part I

Answer all questions in this part. Each correct answer will receive 2\% credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

1. Express \( \frac{2-3i}{3-2i} \) as a fraction with a real denominator.

2. Write the equation of the straight line parallel to the line \( 2x - 3y = 4 \) and passing through the origin.

3. Given the equation \( kx^2 - 4x + k = 0 \), find the positive value of \( k \) which will make the roots equal.

4. If \( f(x) = x^2 - 3x + 2 \), find \( f(2a) \).

5. Given \( R = \frac{1}{R_1} + \frac{1}{R_2} \); solve for \( R_2 \).

6. Find the remainder when \( x^{75} - 2 \) is divided by \( x + 1 \).

7. Write the equation of lowest degree possible with real coefficients which has for two of its roots \( 1 \) and \( 2 + 5i \).

8. Find the sum of the roots of the equation \( 2x^3 - x^2 + 8 = 0 \).

9. Transform the equation \( 2x^3 - x^2 + 8 = 0 \) into an equation whose roots are the negatives of the roots of the given equation.

10. The equation \( 2x^3 - x^2 + 8 = 0 \) has one and only one negative root. [Answer true or false.]

11. Which of the following is a rational integral equation in \( x \)?

\[
\begin{align*}
(a) & \quad \frac{1}{x^2} + 7 = 0 \\
(b) & \quad x^2 - x^{-\frac{1}{2}} + 7 = 0 \\
(c) & \quad \frac{1}{x} = \sqrt{2x} - \sqrt{7} = 0
\end{align*}
\]

12. Solve \( 3x + 4 = 9x \).

13. Find the \( x \)-intercept of the graph of \( y = \log_{10} x \).

14. If \( y = -ax^2 + bx + c \), in which \( a \), \( b \) and \( c \) are positive integers, then there is always a value of \( x \) which will make \( y \) negative. [Answer true or false.]

15. Find the logarithm of \( (100)^{1.4} \).

16. Express \( .23333... \) as a common fraction.

17. Express in simplest form the third term of the expansion of \( \left(2x^3 - \frac{2}{3}\right)^5 \).
18. If the probability of a man's living for at least 10 years is .8, find the probability of his dying within these 10 years.  

19. If \( C_2 = 45 \), find \( n \).  

20. How many numbers of 3 digits each can be written with the digits 1, 5, 6, 8, 9 if repetition is allowed?  

Part II

Answer five questions from Part II.

21. Find to the nearest tenth the real root of the equation  
\[ x^3 + 3x - 20 = 0 \]  
[10]

22. Solve the equation  
\[ 2x^4 + 9x^3 + 15x^2 + 14x - 12 = 0 \]  
[10]

23. Given \( V = \frac{\pi r^3}{540} \); if \( V = 146 \) and \( r = 173 \), find \( r \) to the nearest tenth. [Use \( \pi = 3.14 \)]  
[10]

24. Solve graphically the following pair of equations: [Estimate your answer to the nearest tenth.]
\[ \begin{align*} 6x + 7y &= 42 \\ y &= 2x \end{align*} \]  
[6, 2, 2]

25. a. The amount of electrical current required to melt a fuse wire varies as the three-halves power of the diameter. If the current required to melt a wire of diameter 0.09 in. is 27 amperes, what current will melt a wire of diameter 0.04 in.?  
\[ \frac{\pi PHA^\frac{3}{2}}{10} \]  
[6]

b. Given \( G = \frac{\pi PHA^2}{3} \); express \( G \) in terms of \( M \) and \( A \) if  
\[ M = \frac{\pi PHA^2}{3} \]  
[4]

26. The workers in a certain trade have been receiving $77 a week. They are asking for the same weekly wage for 4 fewer hours per week, which would increase their hourly wage by 17\( \frac{1}{2} \) cents. What is their present hourly wage?  
[10]

27. Prove that if \(-, a \) a rational fraction in its lowest terms, is a root of  
\[ q + bx + c = 0 \]  
in which \( a, b \) and \( c \) are integers, then \( p \) is a factor of \( c \). [A statement that this is a special case of the more general theorem will not be accepted.]  
[10]

28. a. Express in polar form the complex number \( 2 - 2i \).  
[3]

b. Express \( 2(\cos 120^\circ + i \sin 120^\circ) \) in the form \( a + bi \).  
[3]

c. Express in polar form one of the imaginary roots of the equation  
\[ x^4 - 1 = 0 \]  
[4]

29. Find the equation of the tangent to the curve \( y = x^3 - 2x^2 + 2x - 8 \) at the point whose abscissa is 2.  
[10]

This question is based on one of the optional topics in the syllabus.