

ADVANCED ALGEBRA

Wednesday, June 20, 1945 — 9.15 a. m. to 12.15 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to part II (a) name of school where you have studied, (b) number of weeks and recitations a week in advanced algebra.

The minimum time requirement is five recitations a week for half a school year after the completion of intermediate algebra.

Part II

Answer five questions from part II.

21 Solve the equation $2x^3 - 5x^2 + 8x - 3 = 0$ [10]

22 Find, correct to the *nearest tenth*, the positive root of $x^3 - 6x - 12 = 0$ [10]

23 Prove that if $a + bi$ is a root of a rational integral equation with real coefficients, then $a - bi$ is also a root. [10]

24 a Draw the graph of $y = x^3 - 2x^2 - 4x + 6$ [5]

b From the graph made in answer to a, estimate to the *nearest tenth* the roots of $x^3 - 2x^2 - 4x + 6 = 0$ [3]

c From the graph made in answer to a, read the roots of $x^3 - 2x^2 - 4x + 6 = -2$ [2]

25 a Given the formula $t = a \sqrt{\frac{kp}{s}}$

Using logarithms, find, correct to the *nearest hundredth*, the value of t if $a = 24$, $k = 0.19$, $p = 5.5$ and $s = 10,000$ [5]

b Solve for x correct to the *nearest tenth*: $3^{2x} = 5^{x+1}$ [5]

26 a Prove that the reciprocals of the terms of a geometric progression are in geometric progression. [5]

b Prove that if each term of an arithmetic progression is multiplied by a constant k , the resulting numbers are in arithmetic progression. [5]

27 Two towns on opposite sides of a lake are 33 miles apart by water. At 6 a. m. a boat starts from each town for the other town, traveling at uniform speed. The boats pass each other at 9 a. m. One boat arrives at its destination 1 hour and 6 minutes earlier than the other. Find the time it takes each boat to make the trip. [10]

*28 a Find the slope of the line which is tangent to the parabola $y = 2x^2 - 3x - 1$ at the point whose abscissa is 2. [4]

b A bullet is fired straight upward. Its height h , in feet, after t seconds is given by the formula $h = 960t - 16t^2$

(1) Find its speed at any time (t). [2]

(2) Find its acceleration. [2]

(3) What is the maximum height to which the bullet will rise? [2]

*29 a Find the modulus of $-3 + i$. [1]

b Write $-3 + i$ in the form $\rho (\cos \theta + i \sin \theta)$, expressing θ correct to the nearest degree. [3]

c Write $4(\cos 300^\circ + i \sin 300^\circ)$ in the form $a + bi$. [3]

d Write in polar form the root of $x^5 - 32 = 0$ which is represented graphically by a point in the third quadrant. [3]

* This question is based on one of the optional topics in the syllabus.

Fill in the following lines:

Name of school.....Name of pupil.....

Part I

Answer all questions in this part. Each correct answer will receive 2] credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

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|--|---------|
| 1 Find the product of $(3 + 2i)$ and $(3 - 2i)$. | 1..... |
| 2 If $f(x) = 2x^2 - 5$, find $f(x + 1)$. | 2..... |
| 3 What is the remainder when $x^{3n} - x^n + 5$ is divided by $x - 1$? | 3..... |
| 4 One leg of a right triangle is 4 and the other leg is x . Express the hypotenuse as a function of x . | 4..... |
| 5 What is the real value of y in the equation $y^{-\frac{3}{2}} = 27$? | 5..... |
| 6 Write in the form $x^3 + bx^2 + cx + d = 0$ the equation whose roots are 2, 5 and -3 . | 6..... |
| 7 How many of the roots of the equation $x^4 + 15x^2 + 7x - 11 = 0$ are imaginary? | 7..... |
| 8 Between what two integers does the positive root of $x^3 - 5x - 3 = 0$ lie? | 8..... |
| 9 Transform the equation $2x^3 - 3x^2 - 3x + 5 = 0$ into an equation whose roots are less by 2 than the roots of the given equation. | 9..... |
| 10 If two of the roots of $x^3 + 3x^2 + px + q = 0$ are numerically equal but opposite in sign, what is the third root? | 10..... |
| 11 If the roots of the equation $x^3 + px^2 + qx + t = 0$ are a , b and c , write the equation whose roots are $-a$, $-b$ and $-c$. | 11..... |
| 12 Using logarithms, find, correct to the nearest thousandth, the cube root of .93 | 12..... |
| 13 Write the equation of the straight line which passes through the point $(-3, 4)$ and is parallel to the graph of $2x + y = 3$ | 13..... |
| 14 How many degrees are there in the obtuse angle formed by the x axis and the straight line connecting the origin with the point $-3 + 3i$? | 14..... |
| 15 Write the fourth term of the expansion of $(2 + \frac{x^2}{2})^8$ | 15..... |
| 16 By what number must each of the roots of $x^4 - 12x^3 - 6x^2 + 7x - 3 = 0$ be diminished in order that the resulting equation shall have no term in x^3 ? | 16..... |
| 17 Write the following statement as an equation: The volume V of a gas varies directly as the absolute temperature T and inversely as the pressure P . | 17..... |
| 18 In how many ways can three different prizes be awarded among eight people if no person is to receive more than one prize? | 18..... |
| 19 From 10 different books, how many different selections of 5 books each can be made if one specified book is always included? | 19..... |
| 20 If the probability of a man's living 15 years longer is 0.7, what is the probability of his dying within 15 years? | 20..... |