Instructions

Do not open this sheet until the signal is given.

Group I

This group is to be done first and the maximum time allowed for it is one and one half hours.

If you finish group I before the signal to stop is given you may begin group II. However, it is advisable to look your work over carefully before proceeding, since no credit will be given any answer in group I which is not correct and in its simplest form.

When the signal to stop is given at the close of the one and one half hour period, work on group I must cease and this sheet of the question paper must be detached. The sheets will then be collected and you should continue with the remainder of the examination.

Group II

Write at top of first page of answer paper to group II (a) name of school where you have studied, (b) number of weeks and recitations a week in advanced algebra.

The minimum time requirement is five recitations a week for half a school year after the completion of intermediate algebra.

The use of the slide rule will be allowed for checking but all computations with tables must be shown on the answer paper.
Fill in the following lines:

Name of school...........................................Name of pupil...........................................

Detach this sheet and hand it in at the close of the one and one half hour period.

Group I

Answer all questions in this group. Each correct answer will receive 2½ credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

1. Write \(a^{n-1}b(a + b)\) as the sum of two terms.

2. Write the equation for the following statement: The ordinate \((y)\) of a point varies directly as the square of the abscissa \((x)\).

3. Write in the form \(a + bi\), the product of \(1 + 3i\) and \(-1 - 3i\).

4. Write the equation of the line passing through the point \((1, -2)\) and parallel to the line whose equation is \(y = 2x - 3\).

5. Write the third term of the expansion \((\frac{1}{x} + x)^4\)

6. What real value of \(x\) satisfies the equation \(x^2 + 3 = 8\)?

7. Given \(x^2 - 4x - y = 0\); solve for \(x\) in terms of \(y\).

8. How many positive roots has the equation \(x^6 - 7x^2 - 11 = 0\)?

9. What is the sum of the roots of the equation \(3x^4 - 5x^3 + 7x - 6 = 0\)?

10. Write the equation whose roots are twice the roots of the equation \(x^3 - x + 1 = 0\)

11. Write the equation whose roots are greater by 2 than the roots of the equation \(y^9 - 3y - 1 = 0\)

12. How many chords are determined by seven points on a circle?

13. How many even numbers of four different digits each can be formed from the digits 3, 5, 6, 7 and 9?

14. There are three times as many white balls as black balls in a box. If one ball is drawn at random, what is the probability that it will be black?

15. For what value of \(m\), other than 1, are the roots of the equation \(x^2 - (m + 3)x + 4m = 0\) equal?

16. Find, correct to the nearest hundredth, the real root of the equation \(x^6 - 37.2 = 0\)

17. Does the graph of \(y^2 = 8(x - 3)\) intersect the graph of \(y = x^2\)? [Answer yes or no.]
18 If \( a = 10^x \) and \( b = 10^y \), does \( x \log b = y \log a \)? [Answer yes or no.]

19 Is it possible for the geometric mean between two unequal numbers to equal their arithmetic mean? [Answer yes or no.]

20 In the diagram below subtract graphically \( 7 + 3i \) from 9.
Group II

Answer five questions from this group. Full credit will not be granted unless all operations (except mental ones) necessary to find results are given; simply indicating the operations is not sufficient. Each answer should be reduced to its simplest form. Purely arithmetical solutions for problems will not be accepted.

21 Find, correct to the nearest tenth, the real root of the equation $2x^3 + 5x - 15 = 0$ \[10\]

22 Solve completely the equation $2x^4 - 3x^3 + 3x^2 + 5x - 3 = 0$ \[10\]

23 It has been determined that if a white oak beam supported at both ends is uniformly loaded, the maximum safe load $W$ in pounds is given by the formula $W = \frac{(10)^3 \sqrt{17}}{3} \times \frac{B D^3}{L}$ where the distance between the supports ($L$), the breadth of the beam ($B$) and the depth of the beam ($D$) are measured in inches. Using logarithms, find, correct to the nearest hundredth of an inch, the proper depth of the beam, if the maximum safe load desired is 3000 pounds, the distance between the supports is 141 inches and the breadth of the beam is 6 inches. \[10\]

24 a Prove the Remainder Theorem. \[7\]

b If $x^3 - 3x^2 + 4x + d$ is divided by $x - 1$, the remainder is 10; find $d$. \[3\]

25 Two cottages on opposite sides of a lake are 20 miles apart by water. At noon a boat starts from each cottage and travels toward the other at a uniform speed. The boats pass each other at 2 p.m. One boat arrives at its destination 1 hour and 40 minutes earlier than the other. Find the rate of the faster boat. \[6, 4\]

26 A rectangular box with a square base of side $s$ feet has a volume $V$ cubic feet. If the cost of the material used for the base and top is $a$ cents per square foot and the cost of the material used for the sides is $b$ cents per square foot, derive a formula for the total cost $C$ (in cents) of the material used in the construction. \[10\]

27 a On the same set of axes, plot the graphs of $y = x^3 - 3x + 3$ and $x + y = 3$ \[7, 1\]

b From the graphs made in answer to $a$, determine the values of $x$ and $y$ common to the two equations. \[Estimate the irrational values correct to the nearest tenth.] \[2\]

28 The motion of a point is given by the formula $S = 2t^3 - 15t^2 + 24t + 60$, where $S$ is measured in feet and $t$ in seconds. Find the velocity and acceleration of the point at the end of two seconds. \[10\]

29 The frequency distribution of percentages (grades) of an advanced algebra class on a certain test is given below:

<table>
<thead>
<tr>
<th>Grade (to nearest 5% interval)</th>
<th>55</th>
<th>60</th>
<th>65</th>
<th>70</th>
<th>75</th>
<th>80</th>
<th>85</th>
<th>90</th>
<th>95</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Pupils</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>6</td>
<td>9</td>
<td>2</td>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>

a From the table determine (1) the mean, correct to the nearest per cent, (2) the intervals in which the median and the mode fall. \[3, 2, 1\]

b Represent the data in the table graphically. \[3\]

c Does the distribution follow the normal probability? \[1\]

* This question is based on one of the optional topics in the syllabus.