The University of the State of New York
254th High School Examination
ADVANCED ALGEBRA
Thursday, June 23, 1932 — 9.15 a.m. to 12.15 p.m. only

Instructions

Do not open this sheet until the signal is given.

Answer all questions in part I and five questions from part II.

Part I is to be done first and the maximum time to be allowed for this part is one and one half hours. Merely place the answer to each question in the space provided; no work need be shown.

If you finish part I before the signal to stop is given you may begin part II. However, it is advisable to look your work over carefully before proceeding to part II, since no credit will be given any answer in part I which is not correct and reduced to its simplest form.

When the signal to stop is given at the close of the one and one half hour period, work on part I must cease and this sheet of the question paper must be detached. The sheets will then be collected and you should continue with the remainder of the examination.
Fill in the following lines:

Name of school.............................................Name of pupil..........................................

Detach this sheet and hand it in at the close of the one and one half hour period.

Part I

Answer all questions in this part. Each question has 2\(\frac{1}{2}\) credits assigned to it; no partial credit should be allowed. Each answer must be reduced to its simplest form.

1 What is the value of \(2^x - i + x\) when \(x = 4\)?
   \[\text{Ans.}\]

2 Is the value of the repeating decimal \(.0009\) . . . greater or less than the fraction \(\frac{1}{1000}\)?
   \[\text{Ans.}\]

3 Does the graph of the equation \(x = y + 5\) cut the graph of the equation \(x^2 + 4y = 16\)? [Drawing of the graphs not required]
   \[\text{Ans.}\]

4 For what values of \(m\) will the roots of the equation \(x^2 - mx + m + 3 = 0\) be equal?
   \[\text{Ans.}\]

5 Express \(\frac{1 + i}{2i}\) in the form \(a + bi\)
   \[\text{Ans.}\]

6 Give in simplified form the seventh term of the expansion of \((x^2 - \frac{1}{x})^9\)
   \[\text{Ans.}\]

7 If the complex numbers \(9 - i\) and \(6 + 7i\) are represented graphically by the points \(P\) and \(Q\) respectively, which point is at the greater distance from the origin?
   \[\text{Ans.}\]

8 If \(\log_{10} 2 = .3010\), what is the value of \(10^{- .3010}\)?
   \[\text{Ans.}\]

9 If \(\log_{10} 5 = x\), express \(\log_{10} 2\) as a function of \(x\).
   \[\text{Ans.}\]

10 By means of logarithms, find the fifth root of 2 correct to the nearest hundredth.
   \[\text{Ans.}\]

11 What is the sum of the two complex roots of the equation \(x^2 - 8 = 0\)?
   \[\text{Ans.}\]

12 One root of the equation \(20x^3 - 24x^2 + 5x - 6 = 0\) is \(\frac{1}{2}\); what are the other two roots?
   \[\text{Ans.}\]

13 Write the equation with real coefficients and of lowest possible degree that has \(-2\) and \(\sqrt{-3}\) as two of its roots.
   \[\text{Ans.}\]

14 Form an equation whose roots are twice those of the equation \(2x^3 - x + 3 = 0\)
   \[\text{Ans.}\]

15 Form an equation whose roots are less by 2 than those of the equation \(2x^3 - 6x^2 + 6x - 5 = 0\)
   \[\text{Ans.}\]

16 The equation \(x^3 - 2x^2 - x - 7 = 0\) has only one real root; is this root positive or negative?
   \[\text{Ans.}\]
17 If \( x = y - \sqrt{y^2 - y} \), express \( y \) as a function of \( x \).

18 How many integers greater than 300 and less than 1000 can be made with the digits 1, 2, 3, 4, 5, no digit being repeated in any number?

19 How many different committees of three can be chosen from a class of 11 if a certain boy must be included in the committee?

20 If the area of a rectangle is 9 square inches, express the length of a diagonal \( y \) as a function of the length of one side \( x \).
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Write at top of first page of answer paper (a) name of school where you have studied, (b) number of weeks and recitations a week in (1) elementary algebra, (2) intermediate algebra, (3) advanced algebra. The minimum time requirement is five recitations a week in algebra for two school years.

Part II

Answer five questions from this part. Full credit will not be granted unless all operations (except mental ones) necessary to find results are given; simply indicating the operations is not sufficient. Each answer should be reduced to its simplest form.

In the examination in advanced algebra the use of the slide rule will be allowed for checking, provided all computations with tables are shown on the answer paper.

21 Find all the roots of the equation $2x^2 - 13x + 18x - 6 = 0$ [10]

22 How many terms of the arithmetic series 9, 11, 13, ... must be added in order that the sum should equal the sum of nine terms of the geometric series 3, −6, 12, −24, ... ? [10]

23 Find two unequal numbers each of which is the square of the other. [10]

24 Prove that if the fraction $\frac{p}{q}$ (p and q being integers with no common factor) is a root of a polynomial equation in x having integral coefficients, then q must be a factor of the coefficient of the highest power of x. [10]

25 A rectangular box has a volume of 98 cubic inches, its length being 5 inches greater, and its width 2 inches greater, than its depth. Find its dimensions correct to the nearest tenth of an inch. [10]

26 A has a number of nickels and B a number of coins of a higher denomination; the two together have a total of 16 coins amounting to $1.60. If they exchange one coin each, A giving B a nickel and B giving one of his coins to A, they will then have equal amounts of money. How many coins has each and what kind of coins has B? [7, 3]

27 Assuming that all men work at the same speed and that 5 men can do a certain piece of work in 24 days, let y represent the number of days required for x men to do the piece of work and express y as a function of x. Plot the graph of this function from $x = 4$ to $x = 12$. Indicate on your graph the point that shows the number of men needed to do the work in 15 days. [10]