The University of the State of New York
248th High School Examination

ADVANCED ALGEBRA

Thursday, June 19, 1930 — 9.15 a. m. to 12.15 p. m., only

Instructions

Do not open this sheet until the signal is given.

Answer all questions in part I and five questions from part II.

Part I is to be done first and the maximum time to be allowed for this part is one and one half hours. Merely place the answer to each question in the space provided; no work need be shown.

If you finish part I before the signal to stop is given you may begin part II. However, it is advisable to look your work over carefully before proceeding to part II, since no credit will be given any answer in part I which is not correct and reduced to its simplest form.

When the signal to stop is given at the close of the one and one half hour period, work on part I must cease and this sheet of the question paper must be detached. The sheets will then be collected and you should continue with the remainder of the examination.
NAME OF SCHOOL........................................................................NAME OF PUPIL

DETACH THIS SHEET AND HAND IT IN AT THE CLOSE OF THE ONE AND ONE HALF HOUR PERIOD.

PART I

Answer all questions in this part. Each question has 2½ credits assigned to it; no partial credit should be allowed. Each answer must be reduced to its simplest form.

1. Express .270270 . . . as a common fraction.

2. In an arithmetic series of 5 terms, the first term is 3 and the last term is 12; what is the second term?

3. For what values of k will the equation \(x^2 + (1 - k)x + 1 = 0\) have two equal roots?

4. If \(x\) must be a real number, can the fraction \(\frac{x^2}{x - 3}\) have the value 2?

[Answer yes or no.]

5. If \(x\) represents the number of boys in a class, the equation \(2x^2 - 11x - 30 = 0\)

   a. can not possibly be true.
   b. must necessarily be true.
   c. may be either true or false.

Which statement is correct, \(a\), \(b\) or \(c\)?

6. Give, after simplifying, the third term in the expansion of \((\sqrt{x} + \frac{1}{3x^2})^{10}\)

7. If \(x = 1 + 3i\), what is the value of \(\frac{x^2}{5 - x}\)?

8. If \(y = \log_{10} 3\), what is the value of \(10^{xy}\)?

9. In a baseball league of 6 teams, each team plays each of the other teams twice; how many games are played?

10. Write an equation with integral coefficients whose roots are 

    \(-1 + \sqrt{3}, -1 - \sqrt{3}\) and \(\frac{1}{2}\)

11. What is the exact number of imaginary roots of the equation \(x^4 = 1\)?

12. One root of the equation \(24x^3 + 5x^2 + 96x + 20 = 0\) is \(2i\); what are the two remaining roots?

13. What is the rational root of the equation \(3x^3 + 5x^2 + 5x + 2 = 0\)?

14. Write an equation whose roots are half the roots of 

    \(3x^3 - 10x^2 + 24 = 0\)

15. How many times does the graph of \(y = x^4 + 6x^2 - x - 11\) cut the \(x\)-axis?

16. How many odd numbers of three different digits can be written with the digits 1, 2, 4, 6, 8?

17. If \(y = 2 + \frac{1}{1 - x}\), does \(y\) increase or decrease as \(x\) increases from 2 to 10?
Write at top of first page of answer paper (a) name of school where you have studied, (b) number of weeks and recitations a week in (1) elementary algebra, (2) intermediate algebra, (3) advanced algebra. The minimum time requirement is five recitations a week in algebra for two school years.

Part II

Answer five questions from this part. Full credit will not be granted unless all operations (except mental ones) necessary to find results are given; simply indicating the operations is not sufficient. Each answer should be reduced to its simplest form.

In the examination in advanced algebra the use of the slide rule will be allowed for checking, provided all computations with tables are shown on the answer paper.

21 If the interest at 6% is compounded annually, in how many years will $1 amount to $100? [10]

22 Find the four roots of \( x^4 + 2x^3 - 4x^2 - 5x - 6 = 0 \) [10]

23 A rectangular bin with a square base has a depth 2 feet greater than one side of the base. If the capacity of the bin is 500 cubic feet, find its dimensions to the nearest tenth of a foot. [10]

24 Does the graph of \( y = x^4 - 8x^2 + 16 \) intersect the graph of \( y = -2x^2 \)? Show reason for your answer. [10]

25 a Transform the equation \( x^3 + 6x^2 + 2x - 5 = 0 \) into an equation in which the term of the second degree is missing. [5]

b What are the roots of the equation \( x^3 - 5x^2 + 6x + k = 0 \), if the graph of \( y = x^3 - 5x^2 + 6x + k \) passes through the origin? [5]

26 a Prove that an equation in one unknown all of whose coefficients are integers, the coefficient of the term of highest degree being 1, can not have a rational fraction for a root. [5]

b Find to the nearest hundredth \( \sqrt[3]{45} \) [Use logarithms.] [5]

27 At noon A starts from a town and walks at the rate of 2 miles an hour until 2.30 p.m., rests until 3.30 p.m. and then walks on at the rate of 4 miles an hour. B sets out from the town on the same road at 1 p.m. and walks steadily at the rate of 3 miles an hour.

a On the same set of axes represent these facts graphically for each man for the interval from noon to 7 p.m. inclusive. [7]

b From the graph made in answer to a, determine

(1) how far from the starting point each man is at 4 o'clock. [1]

(2) at what time each of them passes a point on the road 7 miles from the town. [1]

(3) when and where B passes A. [1]
18 If A’s age now is \( x \), and A is twice as old as B was 10 years ago, express the difference \( y \) between their present ages as a function of \( x \), assuming that A is older than B. 

\[ \text{Ans.} \]

19. On the diagram below, plot the points that represent the complex numbers \( 5 + \sqrt{-4} \) and \( -1 + \sqrt{-25} \).

20. Determine graphically the point that represents the sum of the two complex numbers given in question 19.