The University of the State of New York
245th High School Examination
ADVANCED ALGEBRA
Thursday, June 20, 1929 — 9.15 a. m. to 12.15 p. m., only

Instructions

_Do not open this sheet until the signal is given._

_Answer all questions in part I and five questions from part II._

_Part I is to be done first and the maximum time to be allowed for this part is one and one half hours._ Merely write the answers to each question in the space at the right; no work need be shown.

If you finish part I before the signal to stop is given you may begin part II. However, it is advisable to look your work over carefully before proceeding to part II, since _no credit will be given any answer in part I which is not correct and reduced to its simplest form._

When the signal to stop is given at the close of the one and one half hour period, work on part I must cease and this sheet of the question paper must be detached. The sheets will then be collected and you should continue with the remainder of the examination._
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Fill in the following lines:

Name of school..............................................Name of pupil..............................................

Detach this sheet and hand it in at the close of the one and one half hour period.

Part I

Answer all questions in this part. Each question has 2½ credits assigned to it; no partial credit should be allowed. Each answer must be reduced to its simplest form.

1 Express as a common fraction .1717 . . .

2 Form the cubic equation whose roots are \(1 + \sqrt{2}\), \(1 - \sqrt{2}\) and 0.

3 Find by the use of logarithms the value of \(12^{\frac{1}{3}}\) to the nearest tenth.

4 What is the possible maximum number of negative roots of \(2x^4 + x^3 + 5x^2 + 3x - 4 = 0\)?

5 What is the exact number of positive roots of the equation given in question 4?

6 One root of \(2x^4 - 6x^3 + 3x^2 + 6x - 5 = 0\) is \(\frac{1}{2} - \frac{1}{2}i\); what is another root?

7 If the roots of \(x^3 - 3x^2 - x + 3 = 0\) are arranged in order of increasing algebraic value, they are in arithmetic progression; find the difference \(d\) of the progression.

8 Write the fourth term of \((a + x)^{-1}\)

9 If \(y^2 + 2y - 3 + x^2 = 0\), express \(y\) as a function of \(x\); that is, solve the equation for \(y\) in terms of \(x\).

10 Find the equation whose roots are 2 less than the roots of \(x^3 + 2x^2 - 19 = 0\)

11 Find an equation whose roots are twice the roots of \(2x^3 - x^2 + 6x - 3 = 0\)

12 Express \(\frac{5 + i}{1 - i}\) in the form \(a + bi\)

13 What is the angle between the lines joining the origin to the points representing \(3 + 4i\) and \(-4 + 3i\)?

14 There are 4 clothes hooks on a wall. In how many ways can 3 coats be hung upon them, one coat on a hook?

15 A girl has 8 gowns and 3 coats. In how many ways can she select 3 gowns and 1 coat for a week-end trip?

16 Solve for \(y\): \(\log_{10} y - \log_{10} a = c\)

17 If \(\log a^n = 0.3257\), find \(\log a\)

18 The graph of \(y = 11x^2 - 15x + 5\) must (a) cut the \(x\)-axis, (b) be tangent to the \(x\)-axis or (c) lie wholly on one side of the \(x\)-axis. Which is true, (a), (b) or (c)\?

19 If \(x\) is one side of a rectangle whose area is 20, express the perimeter \(y\) as a function of \(x\).

20 If \(r\) is a root of \(ax^2 + bx + a = 0\), is \(\frac{1}{r}\) also a root? [Answer yes or no]

Ans.
Write at top of first page of answer paper (a) name of school where you have studied, (b) number of weeks and recitations a week in (1) elementary algebra, (2) intermediate algebra, (3) advanced algebra. The minimum time requirement is five recitations a week in algebra for two school years.

Part II

Answer five questions from this part. Full credit will not be granted unless all operations (except mental ones) necessary to find results are given; simply indicating the operations is not sufficient. Each answer should be reduced to its simplest form.

In the examination in advanced algebra the use of the slide rule will be allowed for checking, provided all computations with tables are shown on the answer paper.

21 Find the four roots of the equation

\[ 2x^4 - x^3 - 23x^2 + 18x + 18 = 0 \]  \[ 10 \]

22 Find to the nearest tenth the real root of

\[ x^3 + 2x^2 - 19 = 0 \]  \[ 10 \]

23 Two sums of money are placed at interest at the same time and 10 years later the two amounts (principal plus interest) are equal. One sum is \( P \) dollars and bears simple interest at 6%; the other sum is \( \$1200 \) and is placed at compound interest at 5%, the interest being compounded annually. Find \( P \).  \[ 10 \]

24 Find the values of \( k \) for which the graph of \( y = x^3 - 3x^2 + x + k \) is tangent to the graph of \( y = x + 3 \); that is, find the value of \( k \) that will make two values of \( x \) equal in the solution of the two equations.  \[ \text{Do not plot the graphs.} \]  \[ 10 \]

25 A and B own cottages 12 miles apart on the bank of a river in which the speed of the current is 2 miles an hour. Each has a small motor boat. In racing down the river from one cottage to the other, A wins by one fifth of an hour. In racing up the river the same distance, A wins by half an hour. Find the speed of each boat.  \[ 7, 3 \]

26 Two householders, A and B, compare notes on heating their homes for a winter. It is found that B runs his furnace one month longer than A, burns 2 tons of coal more than A and pays \$4 a ton less than A. A’s total expense for coal is \$16 greater than B’s and his expense per month is \$4 greater than B’s. The number of dollars A pays a ton is twice the number of months he runs his furnace. Find how many tons of coal A uses, how much he pays a ton and how many months he heats his home.  \[ 6, 4 \]

27 An automobile is moving on a straight road. During an interval of 5 seconds from \( t = 0 \) to \( t = 5 \) its velocity \( v \), in feet per second, is given by the formula \( v = 2t^3 - 15t^2 + 24t + 9 \)

a Plot the graph of this formula for values of \( t \) from \( t = 0 \) to \( t = 5 \) inclusive, plotting values of \( t \) along the horizontal axis and values of \( v \) along the vertical axis.  \[ 6 \]

b What is the velocity of the automobile when we begin to consider its motion?  \[ 1 \]

c What is the greatest velocity during these 5 seconds?  \[ 1 \]

d How many times did the automobile stop during these 5 seconds?  \[ 1 \]

e Was the automobile moving in the same direction during the second and fourth seconds?  \[ 1 \]