Instructions

Do not open this sheet until the signal is given.

Answer all questions in part I and five questions from part II.

Part I is to be done first and the maximum time to be allowed for this part is one and one half hours. Merely write the answers to each question in the space at the right; no work need be shown.

If you finish part I before the signal to stop is given you may begin part II. However, it is advisable to look your work over carefully before proceeding to part II, since no credit will be given any answer in part I which is not correct and reduced to its simplest form.

When the signal to stop is given at the close of the one and one half hour period, work on part I must cease and this sheet of the question paper must be detached. The sheets will then be collected and you should continue with the remainder of the examination.
Part I

Answer all questions in this part. Each question has $\frac{2}{3}$ credits assigned to it; no partial credit should be allowed. Each answer must be reduced to its simplest form.

1. Find the sum of the infinite geometric series $3 + 1 + \frac{1}{3} + \ldots$

2. In a geometric progression of five terms the last term is 16 and the second term is $-2$; find the third term.

3. Find the seventh term of the expansion $(1 + x)^{10}$

4. Given the equation $x^4 - 3x - 4 = 0$; is $1 + \sqrt{2}$ a root? [Answer yes or no.]

5. If $2 - \sqrt{-3}$ is one root of an algebraic equation with integral coefficients, what must be another root of the equation?

6. Form the cubic equation whose roots are 2, $-2$ and 0.

7. In the equation $x^3 - 4x^2 - 2x^2 - x^2 + 2x - 1 = 0$, what is the greatest possible number of positive roots?

8. Given the equation $x^4 - 2x^3 + 3x^2 + 5x - 2 = 0$; determine the number of negative roots.

9. Transform the equation $2x^2 - 1 = 0$ into an equation in which the coefficients are all integers, that of the highest-degree term being unity.

10. Transform the equation $x^2 - 4x = 0$ into an equation whose roots are 2 less than the roots of the given equation.

11. Express $\frac{2 + 3i}{i}$ in the form $a + bi$

12. If $x^2 - 2x + 1 - y = 0$, solve the equation for $x$ in terms of $y$; that is, express $x$ as a function of $y$.

13. Find the values of $x$ for the points at which the graph of $y = x^2 - 9$ cuts the $x$-axis.

14. A trust company allows on deposits 4% interest, compounded annually. If $\$250$ is deposited, indicate by substitution in the proper formula the amount at the end of 8 years. [Do not solve.]

15. If $10^x = 200$, find $x$ by the use of logarithms.

16. Find by the use of logarithms the value of $\sqrt[3]{268}$ to the nearest tenth.

17. In how many ways can a battery (that is, a pitcher and a catcher) be chosen, if a baseball team has 4 pitchers and 2 catchers?

18. How many different numbers of 5 digits each can be formed with the digits 2, 3, 5, 7 and 9, if no digit is repeated?

19. If the complex number $3 + 4i$ is represented graphically by the point $P$, how far is $P$ from the origin?

20. Two towns, $A$ and $B$, are 20 miles apart. A man leaves $A$ and walks toward $B$ at the uniform rate of 3 miles an hour. He travels for $t$ hours. Express in terms of $t$ his distance from $B$ at the end of this time.
Write at top of first page of answer paper (a) name of school where you have studied, (b) number of weeks and recitations a week in (1) elementary algebra, (2) intermediate algebra, (3) advanced algebra.
The minimum time requirement is five recitations a week in algebra for two school years.

Part II

Answer five questions from this part. Full credit will not be granted unless all operations (except mental ones) necessary to find results are given; simply indicating the operations is not sufficient. Each answer should be reduced to its simplest form.

In the examination in advanced algebra the use of the slide rule will be allowed for checking, provided all computations with tables are shown on the answer paper.

21 Find the four roots of the equation
   \[2x^4 - x^3 - 13x^2 + 5x + 15 = 0\] [10]

22 Find to the nearest tenth the positive root of
   \[x^3 + x^2 - 2x - 11 = 0\] [10]

23 Find the values of \(k\) for which the graphs of \(y = x^3 - 4x\) and \(x + y = k\) are tangent; that is, find the values of \(k\) that will make two values of \(x\) equal in the solution of the two equations. [Do not plot the graphs.] [10]

24 If interest is compounded annually, at what rate must $500 be invested so that it will amount to $1078 in 12 years? [10]

25 P and Q are two cities 300 miles apart. A leaves P for Q at the same time that B leaves Q for P. Both travel at uniform rates. After they meet, it takes A 9 hours to get to Q and B 4 hours to get to P. Find the rates of A and B. [10]

26 Two men, A and B, buy different grades of coal. Each spends $126, but A buys 4 tons more than B. If A pays $1 less for 5 tons than B pays for 4 tons, what price per ton does each man pay? [10]

27 A steam roller is moving back and forth on a road across which there is a “Road closed” sign. During an interval of 6 minutes from \(t = 0\) to \(t = 6\) the distance \(d\) from the sign to the roller is given by the formula \(d = t^3 - 9t^2 + 15t + 30\)

- \(a\) Plot the graph of this formula for values of \(t\) from \(t = 0\) to \(t = 6\) inclusive, plotting values of \(t\) along the horizontal axis and values of \(d\) along the vertical axis. [6]
- \(b\) How far is the roller from the sign when we begin to consider its motion? [1]
- \(c\) What is the greatest distance from the sign to the roller during these 6 minutes? [1]
- \(d\) When is the roller nearest the sign? [1]
- \(e\) How many times does the roller stop and reverse the direction of the motion? [1]